FOR·THE·INCREASE·AND·DIFFVSION·OF·KNOWLEDGE·AMONG·MEN·
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"EVERY MAN IS A VALUABLE MEMBER OF SOCIETY WHO BY HIS OBSERVATIONS, RESEARCHES AND EXPERIMENTS PROCURES KNOWLEDGE FOR MEN."—SMITHSON.

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S. P. LANGLEY

Secretary S. I.

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INSTITUTED TO ENCOURAGE THE DISCOVERY OF NEW TRUTHS
FOR THE ADVANCEMENT OF MEDICINE.

LECTURE IX.

MENTAL OVER-WORK AND PREMATURE DISEASE AMONG PUBLIC AND PROFESSIONAL MEN.

BY

CHARLES K. MILLS, M. D.,

PROFESSOR OF DISEASES OF THE MIND AND NERVOUS SYSTEM, IN THE PHILADELPHIA POLYCLINIC
AND COLLEGE FOR GRADUATES IN MEDICINE.

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ADVERTISEMENT.

THE "Toner Lectures" have been instituted at Washington, D. C., by JOSEPH M. TONER, M. D., of this city, for the promotion of medical science. With this object the founder has placed in charge of a Board of Trustees, consisting of the Secretary of the Smithsonian Institution, the Surgeon-General of the United States Army, the Surgeon-General of the United States Navy, and the President of the Medical Society of the District of Columbia, a fund, "the interest of which is to be applied for memoirs or essays relative to some branch of medical science, and containing some new truth fully established by experiment or observation."

The publication of these Lectures has been undertaken by the Smithsonian Institution, as falling legitimately within its fundamental purpose, "the increase and diffusion of knowledge among men." The series of "Toner Lectures," published by the Institution in pamphlet form, is as follows; and they are also included in the "Smithsonian Miscellaneous Collections."

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II. "Dual Character of the Brain." By Dr. C. E. BROWN-SÉQUARD. Delivered April 22, 1874. Published January, 1877. 8vo., 23 pp.

III. "On Strain and Over-Action of the Heart." By Dr. J. M. DA COSTA. Delivered May 14, 1874. Published August, 1874. 8vo., 30 pp.

IV. "A Study of the Nature and Mechanism of Fever." By Dr. H. C. WOOD. Delivered January 20, 1875. Published February, 1875. 8vo., 47 pp.

V. "On the Surgical Complications and Sequels of the Continued Fevers." By Dr. WILLIAM W. KEEN. Delivered February 17, 1876. Published March, 1877. 8vo., 70 pp.

VI. "Sub-cutaneous Surgery." By Dr. WILLIAM ADAMS. Delivered September 13, 1876. Published April, 1877. 8vo., 17 pp.

VII. "The Nature of Reparatory Inflammation in Arteries after Ligatures, Acupressure, and Torsion." By EDWARD O. SHAKESPEARE. Delivered June 27, 1878. Published March, 1879. 8vo., 70 pp. and 7 plates.

VIII. "Suggestions for the Sanitary Drainage of Washington City." By GEORGE E. WARING, Jr. Delivered May 26, 1880. Published June, 1880. 8vo., 24 pp.

IX. "Mental Over-Work and Premature Disease among Public and Professional Men." By Dr. CHARLES K. MILLS. Delivered March 19, 1884. Published January, 1885. 8vo., 36 pp.

As it has been found quite impossible to supply gratuitously the large demand from medical men and others for these Lectures, (in addition to the liberal grant to the leading public Libraries and other Institutions in this and foreign countries,) the uniform price of 25 cents has been fixed for each, by which probably their more equitable personal distribution is secured.

SPENCER F. BAIRD,
Secretary Smithsonian Institution.

SMITHSONIAN INSTITUTION,
WASHINGTON, *January*, 1885

LECTURE IX.

Delivered March 19, 1884.

MENTAL OVER-WORK AND PREMATURE DISEASE AMONG PUBLIC AND PROFESSIONAL MEN.

By Charles K. Mills, M. D.

For my subject this evening I am indebted to the suggestion of the public-spirited founder of the "Toner Lectures," who, during his long residence in Washington, having seen many striking instances of break-down among public and professional men, had been led to feel that a study of the causes and the earliest indications of over brain work in these walks of life might prove an interesting investigation, and assist in the development of some new facts.

Extreme mental activity, overstrain, and excitement must be regarded as characteristics of American civilization. In this country every one feels that he is an important possibility in politics, law, medicine, theology, business, science, or literature, so that our very liberties and opportunities become sources of peril to health and life. From the cradle to the grave the American too often lives in an atmosphere of unnatural emulation, while, in other countries, the traditional usages and the more absolute divisions of society into grades and castes prevent so fierce a struggle among the many for high position.

Mr. Herbert Spencer, whom all admit to be entitled to consideration as a close observer of human nature, during his visit to this country, was everywhere struck with the number of faces he met which spoke in strong lines of the burdens that had to

be borne. In every circle he saw the sufferers from nervous collapse, or heard of the victims of over-work. Mitchell, Beard, Jewell, and others have dwelt upon the same fact, and have shown that brain work and brain strain are, in this country, the not infrequent cause of the downfall of health..

That intellectual work *per se* does not injure health or shorten life may, I think, at once be admitted. The longevity of intellectual workers is a subject that has frequently claimed the attention of statisticians, psychologists, and alienists. Madden[1] gives a series of tables showing the relative longevity of medical authors, philologists, authors on revealed and natural religion, and on law and jurisprudence, miscellaneous and novel writers, moral philosophers, dramatists, natural philosophers, poets, artists, and musical composers. The general average age at death for the whole list is 66 years.

Tuke[2] has collected from various sources the ages at death of fifty-four men who were distinguished for intellectual achievements. These ages gave an average of 80 years.

Caspar (quoted by Tuke) gives the average age of clergymen at 65; merchants, 62; clerks and farmers, 61 each; military men, 59; lawyers, 58; artists, 57; medical men, 56.

Beard[3] ascertained the longevity of five hundred of the greatest men of history—poets, philosophers, authors, scientists, lawyers, statesmen, generals, physicians, inventors, musicians, actors, orators, and philanthropists. His list was prepared impartially, and included those who, like Byron, Raphael, Pascal, Mozart, Keats, etc., died young. The average age was found to be 64.20 years. Sherwood (quoted by Beard) ascertained at great labor the ages at death of ten thousand clergymen, the average being 64 years. The average

[1] Infirmities of Genius.

[2] Illustrations of the Influence of the Mind upon the Body in Health and Disease. By Daniel Hack Tuke, M. D., etc. Second Edition. 1884.

[3] A Practical Treatise on Nervous Exhaustion (Neurasthenia). By George M. Beard, A. M., M. D. Second and Revised Edition. 1880.

age at death of all classes of those who live over twenty is about 50 years.

Statistics of this kind, which could be multiplied without limit, are decisive as to the beneficial rather than injurious effects of pure mental labor, conducted upon a proper basis, upon longevity. In our public and professional classes, nevertheless, every physician of experience has seen instances of premature break-down from causes peculiar to these largely intellectual vocations. Even if the instances were few, as claimed by some, their discussion would still be of interest, because any sources of peril to those in the front ranks of society must always demand earnest attention.

In all I have collected a series of sixty cases in which loss of health or life has been largely attributable to excessive brain work and brain strain incident to the callings of those considered. These cases may be arranged into two classes: (1,) Men in political and official life, including cabinet officers, senators, representatives, department officials, governors, and candidates for office; (2,) Professional men, including physicians, lawyers, clergymen, journalists, scientists, and teachers. I have drawn not alone from my own experience, but have obtained the records of cases and corroborative facts from professional friends.[1] The inferences and conclusions of this paper are largely based upon a study of these cases, although time will permit details to be given in but a few instances.

With a subject so wide in scope, limitations must be set, in order to arrive at any practical conclusions in a single lecture. In the first place, then, will be considered some of the causes which lead to mental over-work and break-down in American public and professional life; and, secondly, the early warnings of such over-work, and the forms of disease most likely to result.

Men engaged in commerce and speculation have not been included in the present study, although, by including them, the list

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[1] Especial obligations are due to Drs. S. Weir Mitchell, W. A. Hammond J. M. Toner, A. Y. P. Garnett, D. L. Huntington, J. H. Baxter, H. C. Yarrow, and J. T. Johnson.

of cases could have been largely increased. Premature failure of health, and especially sudden and severe collapse are quite as likely to occur in business life as in any other sphere of action, owing to the protracted labors, and great anxieties and excitements attendant upon pursuits involving the getting and losing of wealth. Brain work and brain strain of a peculiarly destructive kind attend upon the devotees of the counting-house and the exchange; but our present design is to deal only with those whose vocations are in major part intellectual, in the higher meaning which is given to the word intellectual—the men of affairs, of books, and of the laboratory.

The actual occupations embraced within my study were cabinet officer, 1; senators, 8; representatives in Congress, 10; department officials, 5; governors, 2; candidates for important offices, 2; physicians, 6; lawyers, 7; clergymen, 2; journalists, 4; scientists, 6; teachers, 7.

Twenty-eight of the sixty, therefore, were men in political and official life, and eighteen of these were members of Congress.

The average longevity of men in the higher walks of political life in this country is, I am inclined to believe, considerably below the average of those who occupy similar positions in England. Comparing, so far as information was available, the ages at death of United States Congressmen and members of the English Parliament, who have died since 1860, I obtained the following results:[1]

Fifty-nine United States Senators gave an average of 61 years; one hundred and forty-six United States Representatives an average of 55 years; the average for both being, therefore, 58 years. One hundred and twenty-one members of Parliament gave the remarkable average age at death of 68 years.

Taking twenty-five of those that might be regarded as the most eminent American statesmen of the last one hundred years and

[1] The sources of information for these statistics were chiefly, as follows: Lanman's Biographical Annals of the United States Civil Government; Ben Perley Poore's Registry of the United States Government; the Congressional Directories; Foster's Collectanea Genealogica; the British Almanac and Companion; and the Statesman's Year Book.

comparing their ages at death with those of the same number of the most distinguished English statesmen, the United States gave an average of 69 years, and Great Britain of 70—no practical difference. It is noticeable, however, that much of the best work of the great English statesmen—of Palmerston, Derby, and Beaconsfield, for instance—has been done at an advanced age, when most of our American public men have ceased to do anything important.

A little searching will show, in the first place, some *general* causes for these differences. While politics in America may, in the spoilsman's sense, be regarded by many as a business, it is not, as in England, a true vocation followed in the main by those prepared by inheritance, education, and training.

In England we not infrequently see men entering on public careers, usually by a seat in the House of Commons, shortly after attaining their majority, or, at least, at a comparatively early age. Pitt, the elder, for instance, came into Parliament at 27, Pitt, the younger, at 21, Palmerston and Gladstone at 23, and Disraeli, after several attempts, at 32. They come, however, at these early ages to Parliament, usually well-endowed mentally, and as to a training school for their life work. By a gradual process they become accustomed to their duties and their labors, and their responsibilities increase with their years and mental strength. Great responsibilities are not, as a rule, entrusted to them until their powers are matured. In the few instances in which English statesmen have assumed the highest positions early in life, as in the cases, for example, of the younger Pitt, and of Fox, they have usually paid the penalty of premature death. An American, because of constitutional restrictions, cannot reach the lower house of Congress until twenty-five years old, and the Senate until thirty. This ought to be to our advantage, but there are many counterbalancing drawbacks. Many Americans who enter the public arena comparatively young have made and finished their public careers at an age when the British statesman is beginning to reap his reward.

Others come to high political and official position at or after the

meridian of life, and find themselves confronted with brain work, and with duties and responsibilities to which they are unequal, because for them they have had no preparation at all.

The occupations of the members of the Forty-Eighth Congress, as given by the Congressional Directory, were as follows: Lawyers, 197; manufacturers, 24; journalists, 22; farmers or planters, 19; merchants, 16; bankers, 11; physicians, 6; mining capitalists, 5; mining engineers, 3; railroad managers, 3; clergymen, 2; army officers, 2; stenographers, 2; architect, 1; pharmacist, 1; railroad ticket agent, 1; hatter, 1; zoologist, 1; and unclassified, 8.

The legal profession furnishes by far the largest number of Senators and Representatives, and of others holding public places; and among these are to be found our brightest political luminaries. Even legal studies, however, do not necessarily fit men for public position; in special instances, they rather unfit them. It must be borne in mind also that the term "lawyer," as applied to those in political station, is often a mere fiction, those holding it often being men half-trained, or not trained at all, who have assumed the legal role by the easy methods which prevail throughout our land. A lawyer or editor, a banker or merchant, a farmer or planter, a manufacturer or railway magnate, a physician or preacher, finds himself in Congressional halls by virtue of wealth, or local fame, or fortuitous circumstances, and absolutely without any appreciation of or fitness for the labors and responsibilities of his new calling. Ambition, self-esteem, and the instinct for praise impel him to strenuous exertion to compensate for his deficiencies, an effort which leaves him sometimes a mental and physical wreck.

Whether they have entered public life in youth or middle age, and whether prepared or not, mental overwork is particularly dangerous to men beyond the prime of life. Of the sixty cases, breakdown occurred between twenty-five and thirty years in 2 cases; between thirty and forty in 14 cases; between forty and fifty in 18 cases; between fifty and sixty in 17 cases; between sixty and seventy in 9 cases.

Thirteen of the seventeen cases between fifty and sixty, and eight of the nine cases between sixty and seventy, were in political or official life. It is premature decay for the man who should live to eighty or ninety to die at seventy, or for one who should die at seventy to pass away at fifty-five or sixty. It is a question of potential longevity. In the cases before us, sudden or unusual brain work or strain terminated the careers of those destined apparently to live from five to twenty years longer.

Vice-President Wilson died prematurely at sixty-three, without doubt the victim of extreme over-work. The death of a distinguished Senator at the same age was precipitated by overwork both inside and outside of the Senate, and by worry and excitement growing out of the opposition and censure of the Legislature of the State which he represented. A Representative in Congress, and one high in judicial position, subjected to special causes of work and worry, died at sixty-five of cerebral hæmorrhage. Another Representative, overwhelmed with labors and anxieties of a peculiarly harassing character, contracted diabetes, of which he eventually died between sixty and seventy.

Men may live for many years in comparative comfort, and able to do a reasonable amount of work, with organic disease of the kidneys, liver, heart, or other organs, as long as they are not subjected to any unusual physical or mental strain. One of America's most distinguished physicians died a few years ago at the age of eighty-two, and was found after death to have advanced disease of the kidneys which had not been suspected; but the last twenty years of his life were free from strain.

The history of many old hemiplegies is confirmative of this point. At the Philadelphia Hospital, I usually have under observation a score or more of hemiplegics, the victims of thrombosis, embolism, or hæmorrhage. These cases, some advanced in years, with brittle, atheromatous vessels, and (as numerous autopsies show) with disease in almost every organ of the body, live on year after year without

change, because their absolute pauperism has its compensation in that they are no longer subjected to the strain and friction of life.

The contrast to such cases is found in the histories of some of those who form the subject of the present study.

One died at the age of sixty-six, holding a position of high rank and responsibility at the time of his death. He was of good heredity and physique, and had been thoroughly educated. In early life his habits of eating and drinking had been irregular, and at one period he suffered from gouty symptoms. For fifteen or twenty years before his death, however, he had been careful and systematic in his habits, mental and physical, and enjoyed fair health, when suddenly he was subjected to unusual labor and anxiety, because of a great public catastrophe. He could not escape the suddenly-imposed strain. His health failed rapidly in a few months, and he died of Bright's disease.

Another, also in high official position, died at fifty-eight He also was of good heredity, used alcohol moderately, and tobacco freely. Mentally he had been through life a fair but not unusual worker. Twenty-seven years before his decease he had suffered severely from scurvy. After this he was not sick until his fatal illness. Mental work, and cares and anxieties, to which he was unaccustomed, crowded upon him during the last three years of his life. Worn out, he took a sudden cold, was attacked with a local inflammatory trouble, and died.

Some of those who have been lifted from the ordinary walks of life into high official position by appointment, find themselves entirely unfitted for the tasks before them, and yet from these tasks they are unable to escape. If too old, or without sufficient fundamental education to learn, and if unable to do their work by proxy, failure in health, as well as in reputation, is sometimes the result. In England, so severe are some of the competitive examinations for positions in the public service that many are injured in health by the strain which they undergo in preparing for these examinations. With us, it is often the other way;

the strain and pressure come afterward. The positions acquired solely by favor are themselves the hard examiners. In one case, reported to me by a Washington physician, temporary glycosuria was developed in a man fifty-nine years old, largely as the result of anxiety induced by the fact that he was mentally unfit to make the annual report called for by his high position.

If ambitious and conscientious, the real mental labor connected with the position of a man high in public position is often very great. Pressing and perplexing committee work, attention to a large correspondence, the preparation of reports, bills, speeches, and points for debate, make incessant demands upon the time and strength of the Senator or Representative. A well-educated lawyer, coming to Congress at thirty-four, rapidly rose to prominence. He did a prodigious amount of work on the floor of the House of Representatives, and by correspondence, but especially on committees. He was taken seriously ill at the close of a session during which his labors had been unusually great even for him, because of the excessive extra work thrown on him by the sickness of another member of one of the important committees on which he was serving. He died at the beginning of the next session of Congress. Another reached the speakership of the House of Representatives, only to succumb at forty-nine to the brain work and multifarious cares of his high position. An abstemious New England Senator, an indefatigable worker, after suffering for long time from dyspepsia, and from insomnia and other nervous symptoms, was suddenly taken down with enteritis, of which he died.

A special cause of sudden failure in health among public men is the mental over-work, physical fatigue, and excessive emotional excitement attendant upon our political campaigns. In recent years a presidential aspirant was suddenly and seriously stricken during the meeting of the nominating convention. Horace Greeley died insane from brain disease at the conclusion of his unsuccessful campaign. A successful candidate for a high public position de-

veloped pneumonia after a campaign of toil and excitement. Similar instances could be given were it worth while.

Although perhaps not as important a factor in the causation of disease in political as in commercial life, the emotional element plays a large part. It is, as has just been shown, often a life of clamor and excitement. It is one too often of uncertainty, disappointment, and vain longing. Even to the man who is comparatively well fitted for his work, the political vocation in this country is never assured. The new Representative, for instance, feels that it is imperative for him to speedily make a career. Others aspire to his place, which can only be held by hard work, and too often also by low arts. The faults and foibles of a public man are laid bare, his mistakes are magnified, and his best efforts are sometimes misinterpreted by a thoughtless or merciless press. The tremendous sense of responsibility which important positions impose is a constant strain, particularly upon the higher orders of mind. This burden of responsibility, conjoined with Herculean labors, mental and physical, destroyed some of our greatest statesmen during the Civil War.

To such causes as these must be added the lack of recreation, and the excesses, excitements, and irregularities of social life at the National Capital, although it does not come within my purpose to consider either these, or the abuse of alcohol and tobacco, in the present paper.

Leaving the political and official circles, let us next glance at some of the conditions which lead to mental over-work and its consequences among the professional classes.

Defects in our system both of medical and legal education are at the root of failure in health, no less than of professional failure, in many cases. The physician or lawyer, half-educated and half-trained in youth, and yet ambitious and naturally able, is compelled to put forth efforts doubly tasking and straining because his mind has not been systematically developed for his life-work.

Physicians ordinarily do not afford many illustrations of pre-

mature disease from mental overstrain and over-work. Some lose their health from broken rest, irregular meals, physical fatigue, and the continual incurrence of the responsibilities of life and death; but the variety in their lives, the alternation between in-door and out-door existence, and the knowledge of health and disease which they are able to apply for their own behoof, serve in some measure to counterbalance these injurious influences. The physicians who succumb to mental over-work are usually those who, not content with the ordinary labors and rewards of an arduous profession, strive, in addition, for literary, scientific, or professorial honors. In this country it is the rule, rather than the exception, to find the professorships and the subordinate teaching positions in our medical colleges filled by men actively engaged in practice. We have not here, as abroad, scientific physicians in well-endowed professorships, or comfortably quartered on the Government in positions, the routine duties of which can be performed by deputy. The young American physician who, without means, influence, or friends, sets out for the high places of his profession, has before him a prospect which only fails to appall, because it is veiled by his ambition. Intellectual labor must be prolonged, encroaching upon intervals which should be given to rest or recreation; special appointments must be kept, no matter what the cost; the brain must be forced to constantly augmenting and multiplying tasks. Besides all this, he has, to a greater or less extent, the responsibility, the physical fatigue, and the irregularity in eating and sleeping, which belong to medical practice. Science and literature may be made instruments of health and happiness to the working physician; but when turned to for the purposes of ambition by those already sufficiently taxed by practical work, great care must be taken, or they will assist in sowing the seeds of disease and death. Five of six physicians in my list were engaged both in teaching and in literary or scientific work, besides attending to private and hospital practice. In the case of two of them, valuable contributions, the result of much labor, appeared about the time their health gave way.

Many lawyers are among the cases collected from those in political and official life; but, in addition, three judges and four lawyers in active practice, and not in political careers, are included in my notes. The temporary but severe break-down of two judges was attributable to the habit long persisted in of examining papers, comparing authorities, and preparing opinions at night—a form of mental labor taxing to the highest powers. Successful lawyers are often subjected to sudden, prolonged, and severe mental work and strain. Cases must be prepared with great rapidity, important principles of law must be mastered in a short time, and exhausting efforts must be made in courts under conditions of excitement and bad hygiene. With lawyers, as with physicians, self-imposed tasks, in addition to their necessary labors, are sometimes the cause of their downfall in health. A young lawyer, with a decided taste for philosophical pursuits, wrote an able scientific monograph, and developed insanity directly as the result of continuous mental work, legal and scientific. Another succumbed while editing a legal work.

Before success is assured the mental effects of pecuniary pressure are often felt with great force and intensity by men in the professions of medicine and law. Such men, waiting for business until reputation is acquired, and, in the meantime, often doing unrequited work that calls for an immense output of mental energy, have both their intellectual and emotional natures under constant tension. Both work and worry do their parts.

In attributing impairment of health to worry rather than to work, it is sometimes forgotten that a man with an over-worked brain often worries about small matters which would otherwise be met with fortitude. Worry, in such cases, is begotten of over-work. "When," says Blaikie,[1] "a celebrated editor complained of being—

> Over-worked, over-worried,
> Over-Croker'd, over-Murray'd,

the first word of his lamentation explained all the rest." Worry,

[1] Macmillan's Magazine.

moreover, is in itself a form of brain work ; to worry means to cerebrate intensely.

Two clergymen, both of whom were compelled to do severe mental work, and at the same time sustain grave responsibilities, were the only representatives of theology in my list of cases. Chance may not have thrown a fair proportion of mentally over-worked clergymen within my reach, but this small number is probably not entirely accidental. Many clergymen suffer from the symptoms of a mild but annoying form of neurasthenia, but comparatively few succumb completely to mental over-work. Their unusual longevity is well known. Sherwood's statistics have already been quoted. Their variety of toil, their comparative freedom from financial anxiety, their superior mental endowments, and their temperance and morality are the reasons assigned by Beard, and I believe correctly, for the greater longevity of clergymen than of other brain workers.

Of the four journalists, three were engaged upon the highest order of journalistic work. The work done by editors and leader-writers often calls for the severest intellectual effort under pressure. The writer of leading articles will probably average two or more columns daily. His matter must be interesting and forcible ; facts must be rapidly obtained and marshalled; judgments on important topics must be formed instantaneously. Often the brain must be goaded to do work against the mental grain. The work must be done, and must be done on time; there is no putting off to a more convenient season. Editors, moreover, often do their work under bad hygienic conditions—at night, in the glare and heat of gas, sometimes in badly ventilated rooms. One of my patients suffered so much from insomnia, cervico-occipital pain, nervous dyspepsia, and other symptoms, distinctly traceable to his work and mode of life, that he finally left journalism entirely. Two others were forced temporarily to quit their labors.

Scientific work is, as a rule, conservative rather than destructive of health. The scientist, unlike the journalist, is not usually com-

pelled to do severe intellectual work under pressure for time. His danger, as noted in the six cases that have come under observation in the preparation of the present paper, is from sedentary habits, and from intense and prolonged activity of the mind in certain limited grooves. To some minds scientific work has a fascination which becomes a source of peril; the worker becomes a willing slave to tasks which are often of his own making. The six cases were all men who labored beyond the requirements of the positions held by them. Assiduous work with the microscope, steady concentration upon mathematical and engineering problems, and the laborious collection and comparison of data, produced, after a time, states of mental and nervous hyperæsthia and exhaustion, which led to albuminuria in one case, to insanity in two, and to temporary nervous collapse in three cases.

One of these cases, a man highly distinguished in the professional and scientific world, devoted himself with rare enthusiasm to scientific work under Governmental auspices. His method was simply one of intense and incessant application by day and by night, Sunday and week-day. Warnings in the shape of insomnia, great irritability, mental and physical weariness, oxaluria, and marked loss of weight came and went unheeded. Melancholia developed. Rest and travel twice restored him to mental health, but only to have the same history repeated, and to end with a third and complete mental collapse. Another, a young man who had educated himself scientifically, at the same time earning a living, frequently worked fourteen hours a day at tasks requiring close mental concentration. The tendency to over-work is greater among men who have, without the training of schools, raised themselves to honorable rank in science and literature than among those who have had the advantage of a systematic education.

The teachers that have fallen under my notice have been, with one exception (a college professor), principals of male grammar schools in Philadelphia. Five of them broke down completely from mental over-work, and the worry which went with and grew

out of this over-work. These cases were observed a few years ago when the system of competitive examinations prevailed in its worst form in Philadelphia public schools. The Boys' High School and Girls' Normal School had accommodations only for a fixed number of pupils. Any school of a certain grade could send pupils to the examinations; but those to be admitted were selected from the competitors absolutely in the order of the averages obtained. Twenty-five might be accepted from the grammar school of one section, and only five, or perhaps not one, from another school of an equal grade. Cramming was at the highest premium. A teacher's reputation, and even his position sometimes, depended upon the success of his pupils at these examinations. Teachers and pupils both frequently gave way under the terrible mental pressure to which they were subjected. One grammar school principal, just before his last illness, succeeded by extraordinary efforts in getting the highest general average of any school in the city, and also had in his successful class the boy who attained the highest average among all who competed. These were dearly bought honors. It was no uncommon thing for teacher and pupil to begin work at seven o'clock in the morning, to invade the dinner hour, and to continue their labors until ten or later in the evening. Happily, a quota system has taken the place of the murderous method here outlined—a method to which, I trust, Philadelphia will never return.

Not long since, in some of our newspapers, was noted the case of a colored girl who was in attendance at the same school with white children, and who died from "brain fever" brought on by over-work, in her efforts to compete with her more favored schoolmates. Scores of children whose skins are fair, differ as widely from each other in capacity and helpful surroundings as she differed from those with whom she vainly endeavored to compete.

Our children are too largely in the hands of those educationalists to whom Clouston[1] refers, who go on the theory that there is an un-

[1] Clinical Lectures on Mental Diseases.

limited capacity in every individual brain for education to any extent, in any direction, and that after you have strained the power of the mental medium to its utmost there is plenty of energy left for growth, nutrition, and reproduction, while nothing is more certain than that every brain has at starting just a certain potentiality of education in any one direction and of power generally, and that it is far better not to exhaust that potentiality.

Children varying in age and original capacity, in previous preparation and in home surroundings, are forced to the same molds and grooves. The slow must keep pace with the fleet, the frail with the sturdy, the children of toil and deprivation with the sons and daughters of wealth and luxury.

A child is always liable to suffer from mental over-work when the effort is made to force its education beyond its receptive powers. Education is not individualized enough. The mind of the child is often confused by a multitude of ill-assorted studies. Recreation is neglected and unhealthy emulation is too much cultivated. Some account has been given of the method in vogue at one time in Philadelphia, and which some are unwise enough to wish to revive. In many communities, outside of Philadelphia, admissions to the various grades of public schools are regulated entirely by the averages obtained at examinations, instead of on the general record of the pupils in connection with proper, but not too severe, examinations. In consequence, often after the campaign of over-work and confusion, called an examination, we see children developing serious disturbances of health, or even organic disease—paroxysmal fever, loss of appetite, headache or neckache, disturbed sleep, temporary albuminuria, chorea, hysteria, and hystero-epilepsy.

Premature disease, even in the medical profession, sometimes has its origin in student days. Such education as medical students receive is often obtained under the most trying circumstances. In some of our most celebrated medical schools many of the students are expected to attend lectures or do laboratory work for seven hours in the day-time, and in addition to dissect in the evening.

When to this is superadded attendance upon private examining associations and text-book cramming, the only wonder is that so many survive. Young men finish with credit and honor their medical course not unfrequently only to become invalids or to pass to their graves in a few months, victims of the mental over-work and bad hygiene of the colleges where they sought instruction in health and healing.

The symptom-groups and diseases represented by the series of sixty cases may be summarized as follows: Acute neurasthenia, 18 cases; insanity, 10; phthisis, 9; diabetes, 4; cerebral hæmorrhage, 4; Bright's disease, 3: posterior spinal sclerosis, 3; pneumonia, 3; bulbar paralysis, 1; angina pectoris, 1; erysipelas, 1; hepatitis, 1; enteritis, 1; glossitis, 1.

Beard[1] makes the sweeping assertion that neurasthenia "is at once the most frequent, most interesting, and most neglected nervous disease of modern times." He holds that it is a chronic functional disease of the nervous system, the basis of which is impoverishment of nervous force and waste of nerve-tissue in excess of repair. Professor Bartholow[2] denies that neurasthenia is a primary nervous affection or a substantive disease, holding that it is always symptomatic and secondary, and defining it as "a disease usually functional, situated in one or more organs, during the course of which reflex disturbances of the brain occur, and numerous subjective sensations in all parts of the body are realized by consciousness."

With reference to this difference of opinion, it may be said that, on the one hand, there is nothing either impossible or improbable in the assumption of a primary exhaustion of nerve-centres from over-work; but, on the other hand, cause and effect are doubtless often confounded by those who fall back on "neurasthenia" to clear up the mystery of every half-understood nervous case. A chronic condition, known properly as neurasthenia, is often met with among

[1] *Op. cit.*

[2] The Polyclinic, January 15, 1884. A paper read before the Philadelphia County Medical Society.

Americans in all walks of life, among women as often, or perhaps oftener, than men. These neurasthenics are commonly individuals who are careful rather than careless in their habits of living and working, although sometimes the reverse is the case. They may be, and frequently are, of the nervous diathesis. If men, they are not those who come to the front in politics and in the professions, and who form the subject of our study. They may be statesmen, or physicians, or lawyers, or journalists, but they do not represent the aggressive and successful elements in such careers. They are men who, having wet their feet in the ripples of endeavor, imagine that they have buffeted with great waves.

Those who form the subject of this lecture, on the other hand, are, as a rule, men of great natural vigor, who have worked with more energy than discretion. They are the swift and strong, the fit intended to survive. Sadly enough they often live only to teach the truth that the race is not always to the swift, nor the battle to the strong. While scarcely ever belonging to the class of chronic neurasthenics, these men are sometimes the victims of a nervous prostration, which comes on acutely, or so insidiously that its early warnings are overlooked or brushed aside. This condition may be followed quickly by some serious organic disease, or, under fortunate circumstances, it may be recovered from by rest and proper treatment. The cases classed as acute neurasthenia were of this character.

The condition of half-gout, or suppressed gout, named *lithæmia* by Murchison[1], and also sometimes spoken of as lithuria or lithiasis, has attracted much attention during the last few years. In addition to Murchison, the nervous symptoms of lithæmia have been ably discussed by Draper[2], Russell Reynolds[3], Dyce Duckworth[4],

[1] The Croonian Lectures on Functional Derangements of the Liver. By Charles Murchison, M. D., LL. D., etc.

[2] Series of American Clinical Lectures, edited by E. C. Seguin, M. D., 1875, and New York Medical Record, February 24, 1883

[3] British Medical Journal, December 15, 1877.

[4] Brain, April, 1880.

Da Costa[1], and Putnam[2], among others. A tendency is seen in some quarters to refer all the symptoms and disorders commonly classed as neurasthenic, and apparently resulting from mental strain and over-work, to a lithæmic state of the blood. Such symptoms are certainly sometimes thus best explained. For several years, and particularly since the appearance of Dr. Da Costa's paper, I have been on the alert for cases of lithæmia, and in a few instances I have had brilliant successes from anti-lithæmic treatment. Lithæmia and neurasthenia, however, are not interchangeable terms. Such symptoms as mental distress, insomnia, head and neck pains, neuralgias, etc., are present when neither gout nor half-gout can be demonstrated by examinations of the urine or blood, or by any other known means. When lithæmia is present and can be demonstrated by treatment or otherwise, the disorders of assimilation which have led to it are often of primary nervous origin. While it may be entirely the result of inheritance, or errors of diet, in other cases it would seem to be induced by nervous strain, and is, therefore, likely to occur in those with whom we are now concerned. My experience coincides with that of Da Costa, who says: "Lithæmia is much more common in men than in women. Its chief sufferers are men in the prime of life. It comes on in some who live luxuriously, eat largely, drink freely, take little exercise in the open air, and are indolent in their habits. But it is quite as often, or oftener, seen in the active brain workers of good habits, in the marked men in the community in which they live, and it is in them, too, that the nervous symptoms of lithæmia are most obvious. My list of lithæmic patients embraces many a name distinguished at the bar, in medicine, in the pulpit, in literature, and in the world of finance. And it is not only brain work and all the habits this implies, but strain and worry which induce it."

When vertigo is complained of by over-worked patients, I am particularly inclined to look into the question of lithæmia.

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1 American Journal of the Medical Sciences, October, 1881.

2 Boston Medical and Surgical Journal, December 13, 1883.

It is almost impossible to present in orderly array all the symptoms which may be regarded as the indications of nervous exhaustion, and the probable precursors of premature disease from brain strain and over-work. These symptoms, indeed, will vary somewhat with the individual—with his hereditary tendencies, his habits, and his surroundings. There are, however, certain common and positive evidences of existing or coming evil which are present in many cases. The most prominent of these early warnings, which are, at the same time, the symptoms of the affection or condition most conveniently termed acute neurasthenia, are as follows: 1. Certain psychical symptoms, such as excessive irritability of temper; depression of spirits; morbid impulses and fears; constantly recurring thoughts, phrases, or suspicions; sense of effort; impairment of memory and attention; and change in habits and methods of mental work. 2. Laxity or immobility of countenance. 3. A diminution or loss of physical resisting power. 4. Heart failure. 5. Sleeplessness. 6. Pain or distress in the back of the head and neck. 7. Nervous dyspepsia.

Excessive irritability of temper, a state of mental hyperæsthesia, is certainly one of the earliest indications of brain over-work. This irritability is apt to alternate with feelings of exhaustion and depression, and is occasionally the only marked precursor of serious disease. The account given by one of my patients, a professional man thirty-seven years old, is practically that of many. With a large amount of work on hand, with exacting literary and teaching engagements, with financial anxieties, he was the victim of mental over-work and worry to an extreme degree. The veriest trifles began to annoy him. He could scarcely endure the presence of his own children; their simple play and noise disquieted him and caused unwonted ebullitions of temper. He was abrupt and impatient in his business. He believed that his best friends were turning against him. He became unable attentively to follow a conversation. He would sit down at his desk to take up some professional or literary work, only before long to sink listlessly in his

chair unable to arouse either memory or attention. He had frequent spells of profound depression; and tinnitus aurium, a sense of weight in the head, disturbed sleep, and partial insomnia were symptoms that came and went. After this condition had lasted for three or four months his urine was tested, and, to his surprise, was found to contain sugar.

In this case, as in many, mental irritability was an evidence of impairment of power. Weakening of the inhibitory mechanism of the brain is likely to be one of the first effects of mental over-work. Perfect inhibition is the sign of perfect mental health. Owing to the sapping of inhibition, the man whose brain has been overworked or overstrained sometimes shows a tendency to morbid impulses and morbid fears. One of my staid but greatly over-worked patients felt himself moved by a strange impulse to shout on the streets, another was impelled to steal umbrellas from a rack, another to hurl his child over a stairway, another to commit suicide. Again, a man who has steadily over-worked himself may find himself developing a state of general timidity, and, along with this, a tendency to perform foolish and indiscreet acts. Special morbid fears sometimes arise; but Beard, I think, makes neurasthenia play too important a role in the causation of these fears—fears of open spaces and fears of closed spaces, of lightning, of disease, of defilement, and the like. These cases usually represent peculiar forms of inherited mental perversion rather than conditions of nervous exhaustion from brain work or strain. They are rather emotional monomanias, as held by Hammond and Ball. They are more likely to occur in those who never use their mental powers energetically than in the active brain-workers.

The inability in waking hours to banish some phrase, or thought, or suspicion, that has somehow gained a foothold in the mind, has been experienced by many who have suffered simply from temporary nervous depression. When mental conditions of this kind often recur, and increase in intensity, when associated with other morbid impressions and states, they are warning signs that ought

not to be overlooked. The unfortunate possessor of these feelings and emotions is already on the danger line.

A peculiar and unusual laxity or immobility of countenance is one of the minor and yet important early indications that a man's account of physical and mental vigor is being overdrawn. This takes the place of the firm lines and the quick and varied play of features so indicative of mental strength and acuteness. It is due to a loss of muscular tone which, in its turn, is dependent upon impairment of central nervous control. It is often present in the early stages of dementia of any form.

Brain-fag shows itself again in the want of zest and sense of effort which goes with every task. The desk-worker expects to accomplish some hours of useful labor; but, instead of his interest and enthusiasm awakening, as formerly happened, he becomes absent-minded, ideas fail to come to him, and he is unable to concentrate his attention. By persistence he may be able to arouse for a short time some of his former energy, but long, continued effort becomes impossible. The life has gone out of his work. His habits and methods of work change almost without his knowing it. He is obliged to get more time that he may, to some extent, compensate for lessened powers. Minutes are stolen from his meals, hours from his family and from sleep, and Sunday's rest is invaded and violated. He ceases to know the meaning of recreation, and becomes an abject slave to tasks which become every day more irksome and impossible of completion.

A diminution or loss of power to resist exposure, fatigue, or slight deprivation of food and rest is one of the surest evidences of nervous decline. The man in this case is showing prematurely that lessened power of resistance which comes on physiologically with advancing years. His nerve-centres are exhausted, they are wearing out, and are no longer capable of sending forth those nerve-stimuli which are necessary to assimilation and repair.

Heart failure was particularly observed in two cases—both physicians. In one, signs of a weak heart, with slow and sometimes

intermittent pulse, and anginal attacks preceded the development of phthisis. In the other, a similar condition of heart and pulse was present, with cardiac dyspnœa and vertiginous seizures.

The fact that sleeplessness is one of the effects and early indications of mental over-work is so well known that it is almost unnecessary to dwell upon it. It was present in some degree in almost every one of my cases in which the trouble from over-work did not come on suddenly. The progress towards complete insomnia is usually gradual; at first it is likely to be fitful slumber broken by dreams. What sleep is had is not refreshing. Soon the patient may not be able to sleep for hours. The brain is harassed by the thoughts of the day, which can neither be downed nor dismissed. They sometimes almost madden by their sameness. Finally, if not remedied, the insomnia becomes as absolute as that present in some forms of insanity, which perchance it presages, and which yields not to

——— "poppy, mandragora,
Nor all the drowsy syrups of the world."

Pain, or a feeling of intolerable distress in the back of the head or neck, was complained of in twelve cases. It is undoubtedly a frequent symptom of acute neurasthenia, and also not rarely a prodrome of coming organic trouble. A judge resigned his position on the bench because of this distress, coupled with insomnia; and because of it also an over-worked young physician seriously considered abandoning his profession. Pain in the back of the head, as well as other forms of headache, may be due to eye-strain; a cervico-occipital myalgia of rheumatic or lithæmic origin is often met with; and chronic neurasthenics suffer from nape-aches which their habits of self-inspection and self-analysis magnify to undue proportions; but over and above all these are cases in which this symptom can only be explained by extreme nervous exhaustion. Each patient complaining of this sensation should be carefully studied in order to determine whether it is a matter of trifling or serious import.

One form of occipito-cervical pain is indicative of serious disease of the kidneys. Seguin[1] has reported two cases of occipito-cervical pain of a severe type. Both patients were adults and had suffered from chronic headache more or less of the migraine type. At a certain period this headache became transformed into a localized occipital pain, very different from that of the former attacks. The pain in one case extended down the cervical spine, and was much aggravated by movement. The peculiar headache was distinctly paroxysmal and accompanied by nausea. In both cases evidences of chronic Bright's disease, in the form of urine of low specific gravity, and containing albumen and hyaline and granular casts. Convulsions were present in one case. One of the patients died, and the autopsy showed extensive disease of the kidney but none of the brain.

That disorders of digestion are sometimes early results of brain strain and over-work needs only to be recalled. A true nervous dyspepsia, associated with heart palpitations and coming and going diarrhœa, is often one of the first and most annoying evidences of nervous strain. A distinguished physician, when financially embarrassed and working with great energy for recognition, suffered so severely with dyspeptic symptoms that cancer of the stomach was suspected by himself and by some of his professional friends. With professional success came relief to his gastric symptoms.

Digestive disorders come early and late in the history of nervous break-down, but their true significance is often overlooked, and treatment is directed to the stomach when the over-worked brain is the organ really at fault.

In not a few cases which are supposed to be the result of over-work, and which are at first conveniently labelled as neurasthenia, the break-down in health is really due to some special physical conditions which may or may not be serious. Headache, vertigo, and mental distress may arise from the eye-strain which is caused by optical defect; and tinnitus and vertigo, which are regarded

[1]Archives of Medicine, August, 1880.

with alarm, may be dependent on some easily remediable ear-affection. Our discussion would not be complete without a reference to such cases, which have received the fuller attention which they deserve from Mitchell, Thompson, Risley, and others.

In five cases of cerebral syphilis, not included in the sixty cases, the symptoms were at first attributed to worry and mental over-work. Two were men engaged in scientific work, one was in official position, one was a physician, and one a merchant. All were actively engaged intellectually, and were under pressure. The brain symptoms were relieved in four cases by potassium iodide; the fifth, after three attacks of paralysis, died. Worry and brain work played an added part in this last case.

Insanity in some form was developed or precipitated, apparently as the result of mental strain and over-work, in ten cases. In those cases in which the patient's condition could be traced for some time prior to the outbreak of recognizable insanity, indications of coming evil were present, but went unheeded. Investigation revealed a family history of insanity in three cases, and some transmitted neurotic vice may have been in existence in other cases, but this was not ascertained. The forms of insanity were melancholia in six cases, paretic dementia in three, and acute mania in one case. From the cases studied in the present connection, as well as from general observation, I should say that melancholia is the type of mental disease most apt to result from pure intellectual over-work; that is, from tasking the highest cerebral centres beyond their inherited or acquired powers.

Paretic dementia is likely to occur among public and professional men when to intellectual labor are added emotional strain or excesses. It is undoubtedly seen more frequently among men in business careers. My note-books contain many cases fairly attributable to business worry and excitement. Dr. H. M. Hurd[1] believes that the disease has a direct relation to business reverses. He shows that since 1883, and the financial reverses which fol-

[1] Report of the Pontiac Michigan Hospital for the Insane, 1881–82.

lowed, there has been an increasing number admitted to the asylums until the present biennial period. He believes that the cases will decrease until a fresh financial revulsion occurs. Spitzka[1] holds that paretic dementia is primarily a disease of the medulla oblongata, ultimately due to overstrain of the encephalic vasomotor centre. The same author points out the striking analogies between this disease and posterio spinal sclerosis, of which latter affection my series furnishes three cases in terribly over-worked and greatly worried public men, without histories of syphilis, or of sexual or other excesses.

Of the nine cases of phthisis three were members of the lower house of Congress, three were teachers, two were physicians, and one was a lawyer. In each case the history of mental over-work was clear, and in each to a large amount of real intellectual labor was added more or less emotional strain. Next to the possessors of the neurotic or insane diathesis, men of superior brain power in whose families phthisis is hereditary would seem to be most likely to over-work themselves mentally.

The four cases of glycosuria furnish additional evidence of the correctness of the now generally conceded opinion that mental overwork and emotional strain are frequent causes of this disease. The influence of the nervous system in the production of saccharine urine has been shown by Bernard, Schiff, Pavy, Cyon and Aladoff, Eckhard, Brunton, and others.[2] In the four cases studied the disease came on insidiously, and not as the result of any sudden shock or emotion.

That either mental over-work or mental anxiety may lead to some forms of Bright's disease, by impairing vaso-motor control, is highly probable. In two of three instances of this disorder the habits of the patient with reference to alcohol and other abuses usu-

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[1] Insanity: Its Classifiation, Diagnosis, and Treatment. By E. C. Spitzka, M. D. New York. 1883.

[2] See Prof. James Tyson's Treatise on Bright's Disease and Diabetes for an admirable resumé of these researches.

ally assigned as causes were good, but both were hard intellectual workers. Temporary albuminuria was present in two other cases. Examinations of the urine were not made in many of the cases. In this connection, the report made by Dr. Andrew Clarke,[1] a physician connected with the Indian Civil Service, to Dr. Hack Tuke is interesting. He wrote that he was a witness to the grave, and sometimes irreparable, mischief done at schools and in working for competitive examinations. Of the young men passing the Civil Service examination for the Indian service, and afterwards sent to him for health certificates, ten per cent. had temporary albuminuria. Professor Tyson[2] says that it is certain that interstitial nephritis often exists for a long time undiscerned in business men who have lived under a state of constant mental tension. He quotes Dr. Clifford (from an article by Dr. Edes), who attributed twenty-four out of thirty-two cases in private practice to some long-continued anxiety or grief. When interstitial nephritis or some other form of chronic Bright's disease is in existence, but practically dormant, mental strain may spur it into dangerous activity and thus lead to premature death. One case of this kind has already been detailed, and reference has already been made to another in which long-standing disease of the kidneys was not discovered until after death.

In the case of bulbar paralysis and of angina pectoris post mortem examinations showed degenerative disease of the blood vessels; and in four cases—the diagnosis in two confirmed by autopsy—cerebral hæmorrhage resulted from a combination of excessive mental work and great responsibility. Two of these patients, however, were between sixty and seventy, and two between fifty and sixty, but in all, a life and usefulness might have been prolonged if mental strain had been avoided. On the one hand, arterial degeneration may occur as the result of continued cerebral over-work and emotional strain, and on the other, such strain and over-work

[1] The Journal of Mental Science, January, 1880.

[2] *Op. cit.*

are particularly dangerous in those whose vessels are atheromatous or otherwise diseased from age or special cause. Over-work of the brain, for a time, at least, flushes it with blood and distends its vessels. Even a pure intellectual act can be shown to notably influence the circulation, and change the temperature of the head.

Gley[1] has studied the influence of the intellectual act upon the circulation. He used a cardiographic tambour on his own carotid. A philosophical lecture, a geometric demonstration, and an arithmetical operation were used to excite the activity of the brain. He observed during the intellectual work, 1, augmentation of the number of beats of the heart, which appeared to be in direct ratio to the attention; 2, dilatation of the carotid artery and most marked dicrotism of the carotid pulse; 3, these characteristics persisted after cerebral activity had ceased. He concluded that these effects were neither cardiac nor respiratory, but vaso-motor changes.

The experiments of Lombard on the effects of mental activity in increasing cerebral temperature are now well known.

Frequent congestions of the brain cause peculiar kinkings and tortuosities of the arteries, even of those of large calibre. I have seen many remarkable examples of this condition in the post-mortem room of the Philadelphia Hospital. The fact that the perivascular spaces in the brain allow these kinkings to take place is, to a certain extent, conservative of the coats of the vessels; but, in process of time, the arterial tunics will degenerate as the result of the strain to which they are frequently subjected. We speak sometimes of cerebral centres and zones, referring to collections of nerve-cells which are supposed to have certain special functions; but centres and zones are vascular as well as nervous. Instead of innervation preceding circulation, or circulation innervation, the two practically go hand in hand in brain activity. An area of blood supply was regarded by Laycock[2] as indicative of an

[1] Revue des Sciences Médicales, quoted in the Journal of Nervous and Mental Disease for April, 1882.

[2] Medical Times and Gazette, August 19, 1871.

area of cells and tissues in functional relation with each other, and with a common source of blood, and of regulative *vis nervosa*, both vaso-motor and trophic. The correlation between the distribution of the arteries and the physiological regions of the brain has been demonstrated by Duret, Heubner, Charcot, and others.[1] The bearing of these and similar researches on the subject in hand is simply this, that it is certainly impossible to over-work any part of the brain without over-working the vessels going to that part; and it is equally impossible to subject vessels anywhere frequently and repeatedly to increased intravascular pressure without producing disease of these vessels, or exposing them to danger of rupture if already diseased.

The occurrence of such acute diseases as pneumonia, erysipelas, hepatitis, enteritis, and glossitis in those mentally over-worked, helps to emphasize still further the fact, which I wish to bring out, that overtaxing the nervous system may be the exciting cause of almost any serious disorder to which chance, accident, imprudence, or infection exposes the individual. One of the three cases of pneumonia occurred in a successful candidate after a campaign of mental and physical excitement and toil; the second came on after slight exposure in an overworked teacher; the third in one who had for a long time been engaged in laborious literary work. The case of erysipelas occurred in an official after a winter of toil and anxiety, in which his mental powers were strained to the utmost. The cases of hepatitis and enteritis were respectively an overdriven Representative and Senator; that of glossitis an overworked department official.

Just how mental over-work brings about its disastrous effects is not easily explained. We recognize the symptoms of brain tire and brain exhaustion; we see over-worked men falling by the wayside with this or with that well-known disease; but the exact process in the system

[1] Lectures on Localization of Diseases of the Brain. By J. M. Charcot. Edited by Bourneville, and translated by E. P. Fowler, M. D. W. Wood & Co. New York. 1878.

by which these results are brought about must remain largely a matter of speculation. The effect of emotion and intellectual action upon circulation and temperature can be, and have been, directly studied, but the intimate molecular changes which accompany such functioning cannot be directly determined. We know that an over-worked muscle will sometimes atrophy, and not only so, but also that the supplying nerves and their central nuclei will, in extreme cases, undergo degeneration. Every mental act is associated with some molecular change in the gray matter of the cerebral convolutions. When mentalization proceeds beyond the limits which are practically fixed for every individual, exhaustion, defective nutrition, and sometimes cell-atrophy result. Too prolonged drains upon energy will exhaust and sap the nutrition of nerve-cells anywhere. In cases of mental over-work tissues fail to regenerate as fast as they break down. "A thought," says Dr. H. C. Wood,[1] "is the index-hand that marks the death of a protoplasmic molecule, or rather of protoplasmic molecules, for the production of a thought is usually a complex process involving many molecules. Normally, this molecule, or these molecules, are removed and replaced by the processes of nutrition as fast as they are destroyed. If, however, thought follows thought with such instant rapidity that no time is allowed for the reproduction of protoplasmic molecules, by and by so many molecules or working units will have been used up as to produce a constantly growing scarcity of those normal particles which are capable of building up the new working units that shall replace those which have been lost by continuous mental efforts."

Many of the symptoms of nervous break-down, and many of the diseases induced by mental overstrain, are symptoms and diseases referable to the organic nervous system. These are doubtless precipitated by exhaustion or direct lesion of the centres of the organic functions situated in the medulla oblongata. It is to the im-

[1] Brain Work and Over-work.

pairment of the restraining and regulating influence of these centres that we must refer the origination of such serious organic diseases—not nervous in their manifestations, and yet evidently arising from primary nervous disturbance—as phthisis, diabetes, Bright's disease, pneumonia, erysipelas, etc. I have already spoken of the probable method of origin of paretic dementia and posterior spinal sclerosis. The occurrence of heart failure, cardiac palpitation, and digestive disorders through the involvement of the pneumogastric centres is readily explicable.

Even the cervico-occipital distress, which comes on as the result of over-work and overstrain, is probably to be attributed to exhaustion of the nerve-centres of the bulbar region. In some cases of organic disease with demonstrable involvement of these centres this symptom is present. It was prominent in the case of a man fifty years old, who was suddenly stricken in health as the result of overstrain in business, and died of acute bulbar paralysis. The succession of symptoms was facial paralysis, diplopia, difficulty in swallowing, muffled voice, laryngeal cough, oppressed breathing, nausea, vomiting, and fever with delirium. It was prominent also in the case of Vice-President Wilson, who had had a hemiplegic attack in 1874. When he last consulted Dr. Hammond,[1] in November, 1875, his marked symptoms were vertigo, thickness of speech, facial twitching, irregular respiration and heart action, slight difficulty of swallowing, extreme restlessness, sleeplessness, and intense pain in the back of the head and nape of the neck. His death was attributed by Hammond to plugging of one or more of the minute vessels of the nucleus of the pneumogastric.

The higher cerebral centres certainly exercise a certain amount of what might be termed unconscious control over the organic centres. Mental overstrain from excessive intellectual work weakens the inhibitory mechanism of the brain. The organic functions—respiration, cardiac and vaso-motor control, etc.—must be maintained

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[1] Boston Medical and Surgical Journal, December 16, 1875.

uniformly in order that the individual shall exist in good health. Their centres must be well nourished and their supply of blood must be even and regular in order that their tone shall be well-preserved. The initial lesion in cases of the kind considered in the present connection may sometimes be a molecular or protoplasmic alteration unrecognizable by our present means of research, or it may be a vascular disturbance or lesion.

Hyperæmias and even minute hæmorrhages into the pons Varolii and medulla oblongata doubtless sometimes occur after severe mental work greatly prolonged under pressure or excitement. Richardson[1] records the case of a well-known English statesman who had risen to fame by working early and late. At last his acquired reputation was at stake on a momentous question. While speaking in the great assembly of the nation he became faint, and was soon obliged to retire. From that moment he was stricken with diabetes, of which he died.

The cause of sudden disease in this instance, and the cause of sudden death in others, are to be looked for in extravasations, often minute, into vital regions of the medulla. In many cases of sclerosis, paretic dementia, and epilepsy, I have examined the medulla to discover the immediate cause of death, and have always found recent congestive and hæmorrhagic areas in the floor of the fourth ventricle. Widely dilated vessels are found containing freshly coagulated blood and surrounded sometimes by extravasated blood.

The most important conclusions to which our study has led may be summarized as follows:

1. Intellectual work does not of itself injure health or shorten life, but mental over-work, particularly when associated with emotional strain, is a frequent cause of nervous break-down and premature disease.

2. The average longevity of men in the higher walks of public

[1] The Diseases of Modern Life.

life is less in this country than in England. Politics here is not, as there, in the best sense a vocation; and our public men in many cases succumb in health, or fail to attain long life, because they go into careers unprepared by inheritance, education, and training for the severe demands to be made upon their powers.

3. Health and life are sometimes lost through forgetfulness of the fact that mental strain and over-work are particularly dangerous to those in middle life or advanced in years who attempt brain work and responsibilities to which they have not been accustomed. The effects of suddenly-imposed mental strain upon these classes are especially disastrous.

4. If not subjected to unusual mental or physical strain, public and professional men, as well as those in other walks of life, although afflicted with organic diseases, may live in comparative comfort, and be able to do a moderate amount of work for many years.

5. Among special causes of premature disease in public life are onerous and perplexing duties on Congressional Committees, the uncertainties and disappointments attendant upon public positions, the great strain to which candidates are subjected during political campaigns, lack of recreation, and social excesses and abuses at the National Capital.

6. Among physicians, lawyers, and journalists the performance of brain work under pressure for time, and under bad hygienic conditions, is a common cause of ill health. Defective education and pecuniary harassments are also special causes of nervous breakdown and premature disease among physicians and lawyers.

7. Comparatively few clergymen succumb completely to mental over-work, although many suffer from a mild but annoying form of neurasthenia.

8. The danger to the scientific worker usually arises from too intense and too prolonged activity of the mind in one direction. It is a danger which springs largely from the fascination which such work has for its votaries.

9. The system of severe competitive examinations in vogue in

many communities saps the health both of teachers and pupils. In our schools generally educational methods are bad, recreation is too much neglected, and unhealthy emulation too much encouraged. Education is not properly individualized.

10. Chronic neurasthenia is not common among men prominent in public affairs and in the professions. Such men are, however, sometimes the victims of a severe acute nervous prostration, which may result in serious organic disease.

11. Nervous strain is one of the causes of lithæmia, which is of not infrequent occurrence among public and professsional men, but lithæmia and neurasthenia are not interchangeable terms.

12. The warnings of mental over-work and over-strain vary with individuals and circumstances, but certain psychical symptoms, and such physical symptoms as laxity or immobility of countenance, diminished resisting power, heart failure, sleeplessness, cervico-occipital pain or distress, and dyspepsia, are of most frequent occurrence.

13. Insanity, particularly in the forms of melancholia and paretic dementia, is sometimes developed by brain strain and over-work. A family history of insanity is often present in such cases.

14. Phthisis, diabetes, and Bright's disease—next to insanity—are among the diseases most likely to be developed by mental over-work. Men in whose families phthisis is hereditary should carefully guard against such over-work.

15. Over-taxing the mind and nervous system may be the exciting cause of almost any serious disorder to which chance, accident, imprudence, or infection exposes the individual.

16. Many diseases, not nervous in their seat or manifestation, are developed directly or indirectly as the result of mental and nervous strain, through exhaustion, impairment, or lesion of the centres of the organic functions.

TRANSACTIONS

OF THE

ANTHROPOLOGICAL SOCIETY

OF WASHINGTON.

PUBLISHED WITH THE CO-OPERATION OF THE SMITHSONIAN INSTITUTION.

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PUBLICATIONS OF THE SOCIETY.

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TRANSACTIONS Vol. I, 142 pp., includes transactions down to January 17, 1882.

TRANSACTIONS Vol. II, 211 pp., includes transactions to and including May, 1883.

Communications for the Society should be addressed to Col. F. A. SEELY, U. S. Patent Office.

Exchanges and specimens should be sent to Dr. W. J. HOFFMAN, Bureau of Ethnology.

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OF THE

ANTHROPOLOGICAL SOCIETY OF WASHINGTON

FOR THE YEAR 1885.

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CONSTITUTION.

ARTICLE I.—*Name.*

The name of this Society shall be "THE ANTHROPOLOGICAL SOCIETY OF WASHINGTON."

ARTICLE II.—*Object.*

The object of this Society shall be to encourage the study of the Natural History of Man, especially with reference to America, and shall include Somatology, Sociology, Philology, Philosophy, Psychology, and Technology.

ARTICLE III.—*Members.*

The members of this Society shall be persons who are interested in Anthropology, and shall be divided into three classes: Active, Corresponding, and Honorary. The active members shall be those who reside in Washington, or in its vicinity, and who shall pay the dues required by Article XV. Failure to comply with this provision within two months after due notice of election, unless satisfactorily explained to the Council, shall render the election void. Corresponding members shall be those who are engaged in anthropological investigations in other localities; honorary members shall be those who have contributed by authorship or patronage to the Advancement of Anthropology. Corresponding or honorary members may become active members by paying the fee required by Article XV. Any corresponding member from whom no scientific contribution is received for two years after his election may be dropped from the list of members by a vote of the Council, but when so dropped shall be eligible to reinstatement.

All members shall be elected by the Council and by ballot, as follows: The name of the candidate shall be recommended to the Council, in writing, by two members of the Society, and eight affirmative ballots shall be necessary to an election.

No person shall be entitled to the privileges of active membership before paying the admission fee provided in Article XV.

ARTICLE IV.—*Officers.*

The officers of this Society shall be a President, four Vice-Presidents, a General Secretary, a Secretary to the Council, a Treasurer, and a Curator, all of whom, together with six other active members, shall constitute a Council, all to be elected by ballot at each annual meeting. The officers shall serve one year, or until their successors are elected.

ARTICLE V.—*The Council.*

All business of the Society, except the election of officers at the annual meeting, shall be transacted by the Council, five members of which shall constitute a quorum.

The Council shall meet one half-hour before the regular sessions of the Society, and at such other times as they may be called together by the President. They may call special meetings of the Society.

ARTICLE VI.—*The Sections.*

For active operations the Society shall be divided into four sections, as follows: Section A, Somatology; Section B, Sociology; Section C, Philology, Philosophy, and Psychology; Section D, Technology. The Vice-Presidents of the Society shall be *ex-officio* chairmen of these sections respectively, and shall be designated by the President to their sections after their election. It shall be the duty of these sections to keep the Society informed upon the progress of research in their respective fields, to make special investigations when requested by the Council, to announce interesting discoveries, to collect specimens, manuscripts, publications, newspaper clippings, etc., and in every way to foster their divisions of the work.

All papers presented to the sections shall be referred to the Council, and through it to the Society.

ARTICLE VII.—*The President.*

The President, or, in his absence, one of the Vice-Presidents, shall preside over the meetings of the Society and of the Council, and shall appoint all committees in the Council and in the Society.

At the first meeting in February the retiring President shall deliver an address to the Society.

ARTICLE VIII.—*The Vice-Presidents.*

The Vice-Presidents shall respectively preside over the sections to which they have been designated, and represent such sections in the Council and in the Society.

Each of the Vice-Presidents shall deliver an address during the year upon such subject within his department as he may select.

ARTICLE IX.—*The General Secretary.*

It shall be the duty of the General Secretary to record the transactions and conduct the general correspondence of the Society.

ARTICLE X.—*The Secretary to the Council.*

The Secretary to the Council shall keep the minutes of the Council, shall keep a list of active, corresponding, and honorary members, with their residences, shall notify members of the time and place of all meetings of the Society, and shall perform such other duties as the Council may direct.

ARTICLE XI.—*The Treasurer.*

The Treasurer shall receive and have charge of all moneys; he shall deposit the funds as directed by the Council, and shall not expend any money except as ordered by the Council. He shall notify members in writing when their dues have remained unpaid for six months.

ARTICLE XII.—*The Curator.*

The Curator shall receive, acknowledge, and have charge of all books, pamphlets, photographs, clippings, and other anthropological material, and shall dispose of them in accordance with Article XVI, keeping a record of them in a book provided by the Society.

ARTICLE XIII.—*Meetings.*

The regular meetings of the Society shall be held on the first and the third Tuesday of each month from November to May, inclusive. An annual meeting for the election of officers shall be held on the third Tuesday of January in each year, a quorum to consist of twenty active members who are not in arrears for dues; and visitors shall not be admitted. The Proceedings of the Society shall be conducted in accordance with the established rules of parliamentary

practice. Papers read shall be limited to twenty minutes, after which the subject shall be thrown open for discussion, remarks thereon to be limited to five minutes for each speaker.

Article XIV.—*Publications.*

The address of the President, provided in Article VII, and the transactions of the Society, shall be printed and published annually or at such periods and in such form as may be determined by the Council.

Article XV.—*Fees and Dues.*

The admission fee shall be five dollars, which shall exempt the member from the payment of dues during the year in which he is elected. The annual dues thereafter shall be three dollars, to be paid prior to the election in January. The names of members failing to pay their dues one month after written notice from the Treasurer, as provided in Article XI, shall be dropped from the roll, unless from absence of the member from Washington or other satisfactory explanation, the Council shall otherwise determine.

Article XVI.—*Gifts.*

It shall be the duty of all members to seek to increase and perfect the materials of anthropological study in the national collections at Washington. All gifts of specimens, books, pamphlets, maps, photographs, and newspaper clippings shall be received by the Curator, who shall exhibit them before the Society at the next regular meeting after their reception, and shall make such abstract or entry concerning them, in a book provided by the Society, as will secure their value as materials of research; after which all archæological and ethnological materials shall be deposited in the National Museum, in the name of the donor and of the Society; all crania and somatic specimens, in the Army Medical Museum; all books, pamphlets, photographs, clippings, and abstracts, in the archives of the Society.

Article XVII.—*Amendments.*

This constitution shall not be amended except by a three-fourths vote of the active members present at the annual meeting for the election of officers, and after notice of the proposed change shall

have been given in writing at a regular meeting of the Society, at least one month previously.

Article XVIII.—*Order of Business.*

The order of business at each regular meeting shall be:

1. Reading the minutes of the last meeting.
2. Report of the Council upon membership.
3. Report of the Curator.
4. Reading the papers and discussions.
5. Notes and queries.

LIST OF SOCIETIES

IN CORRESPONDENCE WITH THE ANTHROPOLOGICAL SOCIETY OF WASHINGTON.

Essex Institute, Salem, Mass.

Peabody Museum of American Archæology and Ethnology, Cambridge, Mass.

Archæological Institute of America, Boston, Mass.

Numismatic and Antiquarian Society, Philadelphia, Pa.

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Royal Bohemian Society of Sciences, Prague, Bohemia.

Anthropological Society of Vienna, Vienna, Austria.

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Geographical Society of Lyons, Lyons, France.

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Mr. JOHN D. SMITH, U. S. Pension Office.
Mr. THORVALD SOLBERG, Anacostia P. O., D. C.
Dr. Z. T. SOWERS, Physician, 1324 New York Avenue.
Gen. ELLIS SPEAR, Solicitor of Patents, Lock Box 1.
Dr. J. O. STANTON, Physician, 1344 G street N. W.
Mr. JAMES STEVENSON, U. S. Geological Survey.
Rev. BENJAMIN SWALLOW, Washington, D. C.
Prof. WILLIAM B. TAYLOR, Smithsonian Institution.
Prof. CYRUS THOMAS, Bureau of Ethnology.
Mr. A. H. THOMPSON, U. S. Geological Survey.
Mr. GILBERT THOMPSON, U. S. Geological Survey.
Dr. J. FORD THOMPSON, Surgeon, 1000 Ninth street N. W.
Dr. J. M. TONER, Physician, 615 Louisiana Avenue.
Mr. FREDERICK W. TRUE, Smithsonian Institution.
Mr. LUCIEN M. TURNER, Smithsonian Institution.
Mr. LESTER F. WARD, U. S. Geological Survey.
Dr. JAMES C. WELLING, Pres't of Columbian University, 1302 Conn. Ave.
Dr. J. H. YARNALL, 3028 P street N. W.
Dr. H. C. YARROW, U. S. A., 814 Seventeenth street N. W.

LIST OF PAPERS READ.

TRANSACTIONS.

SEVENTY-SECOND REGULAR MEETING, November 6, 1883.

Colonel GARRICK MALLERY, President, in the Chair.

The SECRETARY reported for the Curator the receipt of fifty-three gifts of publications since the last meeting in May.

On motion of Col. SEELY, the Society passed a vote of thanks to the gentlemen who had donated the publications above referred to.

The retiring President, Major J. W. POWELL, then read his address entitled "HUMAN EVOLUTION."*

SEVENTY-THIRD REGULAR MEETING, November 20, 1883.

Colonel GARRICK MALLERY, President, in the Chair.

The election of Dr. Charles Warren, of the Bureau of Education, and Mr. S. H. Kauffman, as active members, was announced.

In the absence of Mr. L. A. Kengla, his paper, entitled "STONE MOUNDS AND GRAVES IN HAMPSHIRE COUNTY, WEST VIRGINIA," † was read by Prof. O. T. MASON.

ABSTRACT.

The mounds or graves described in this paper are found on the eastern side of the South Branch Mountain, Hampshire Co., W. Va., about one mile and a half from the mouth of the South Branch, on the property of Charles French. This entire region was once held by the Massawomec Indians, and the locality under consideration was the hunting ground of the Tamenents.

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* Published in Vol. II, Transactions Anthropological Society, Washington, pp. 176–208.

† Published in the Annual Report of the Smithsonian Institution for 1883, pp. 868–872.

The graves or mounds were of a very peculiar construction reminding one of the stone graves of Tennessee and yet possessing some specific characteristics. The most noticeable feature is the presence of a rude stone cist completely covered with a huge pile of loose stones. In some cases these piles were of great extent.

DISCUSSION.

Major POWELL said that as many were not personally familiar with the stone graves and mounds of the upper Mississippi and its many great tributaries, he would remark that these forms of receptacles for the dead consisted of stones placed edgewise so as to form an oblong space, the stones presenting an almost continuous shoulder, upon which was placed a stone slab as a cover.

The discovery of articles of modern manufacture was not of rare occurrence, and the recent investigation by Mr. Carr, of the Peabody Museum at Cambridge, and the researches of the Bureau of Ethnology combined to show that the "Mound-Builders" could not be classed as a people distinct from the historic Indians occupying those localities where such remains are still found.

Prof. MASON stated that the paper just read was useful for the reason that the subject pertained to a region comparatively near to our city, which had not yet been investigated. Several years ago, a party consisting of Dr. Rau, Mr. Reynolds, and other gentlemen visited the Luray Cave for the purpose of investigation, and Mr. Reynolds subsequently opened some stone graves near that locality. These were really cairns.

Major POWELL said that in Kentucky and elsewhere stone graves are found by the hundred. He had opened great numbers of graves in the same mound, showing that people had buried bodies in diverse ways and at different times, the manner being that stone grave was added to stone grave until scores were erected.

Prof. MASON inquired whether single stone graves had been discovered over which large heaps of stones had been erected, to which Major POWELL replied that he had not, to his recollection, found single graves so covered, but where there were several together, many of the western tribes are said to cast stones upon the graves of their dead; but more definite information as to their actual practice was desirable.

Prof. GORE said that during a recent visit to southwestern Vir-

ginia he learned of quite a number of mounds, none of which had yet been opened, and suggesting that this would present a good field for future investigators. The large number of stones referred to in the paper seemed a curious coincidence with a discovery made in New Mexico, consisting of a large stone erected near one of the pueblos about which lie several wagon loads of stones, thrown there, it is said, by passers by for "good luck."

Dr. REYNOLDS presented some facts referring to his examinations in various portions of the Potomac valley, and concluded by saying that at the site of an "ancient" burial ground at Front Royal, which had been partly washed down by high water at various times, he had found, among other things, medals, &c., of perhaps colonial times.

Major POWELL said that while in Minnesota last summer he inquired of a Sioux Indian their reason why they buried upon scaffolds, and was informed that in ancient times the Sioux lived among the lakes of Minnesota, and buried their dead in mounds; that when they left that country they expected some time to return, and so buried their dead on scaffolds, that they might gather the bones and bring them back and bury them in the grave mounds of their ancesters.

Prof. MASON stated in conclusion that many stone graves have been found in localities which do not abound in stones, plainly indicating that a strong motive caused them to be brought from great distances. Probably the people had originally lived in a stony country, and in new fields had clung to old usages.

Rev. J. OWEN DORSEY then read a paper entitled "AN OSAGE SECRET SOCIETY," which was further illustrated by a chart, enlarged from an original pictographic representation obtained from an Osage Indian.

ABSTRACT.

The writer has found traces of secret societies among the Omahas and cognate tribes of the Siouan family. Such a society is still in existence among the Osages. It must not be confounded with the secret societies of the Indian doctors. Each gens in the Osage tribe has a place in the order, the latter being the depository of the

mythical accounts of the origin of the gentes. It takes four days to relate the tradition of any gens, making eighty-four days needed to hear all the traditions. The order consists of seven degrees: 1. Songs of the Giving of Life. 2. Songs of the Bird (dove). 3. Songs of the Sacred Thing (bag). 4. Songs of the Pack-strap. 5. Songs of the Round Rush. 6. Songs of Fasting. 7. Songs of the Return from the Fight. Women are admitted to the order; but none of the younger people are initiated. Extracts were made from the two versions of the tradition of the Tsi-shu wa-shta-ke or peace-making gens of the left side of the tribe. This tradition is entitled "What is told of the old time (U-nu^{N} U-dha-ke)."

DISCUSSION.

Major POWELL thought it probable that this society might be for the preparation of medicine, or for some mystic rite other than the perpetuation of mythic history.

Mr. DORSEY replied that there are other societies than the above mentioned, entirely distinct, and solely for the preparation of medicine, as he had been able to ascertain. From this society emanate the directions to heads of war parties, plans for erecting lodges, hanging the kettles, and laying the pieces of fire-wood; also to the makers of the war drum, the stand, moccasins, and war bows, certain individuals being selected for each of these duties. Women belong to this society, and these have two small circles tattooed upon the forehead, one above the other. The crease or parting of the hair is painted to represent the path of the sun. In prayer they face the east at sunrise, and the west at sunset. The doors of the lodges are placed at the eastern side, and the dead are buried with their heads toward the east; hence no one will ever sleep with his head pointing in that direction.

Major POWELL then stated that he had, during last winter, investigated the organization of medicine societies among the Muskoki. According to this tribe diseases are caused by mythical animals, such as the bear, elk, deer, owl, spider, &c., and for each disease there is a distinct medicine society, the head personage of which initiates each year young men to cure the various forms of disease belonging to his class. The traditions of the mythical origin of each disease is preserved by the different chiefs of the medicine societies.

The neophyte is instructed through four different nights, through

four different moons, and through four years to instruct him in the mythologic cause of disease.

There are certain medicines employed for the various complaints, composed in part of root decoctions. They are prepared by taking one root running from the trunk directly to the north, one to the east, one to the south, and one to the west. The preparation of the medicine require ceremonies which last during four nights each, of four moons, and of four years each.

Mr. DORSEY stated that part of the Osage ceremonies were strictly secret, though the latter portion was public.

Prof. MASON inquired whether these ceremonies had in any way been influenced by contact with the whites, or whether they were a crystallized custom.

Mr. DORSEY replied that he had found recurrences of these customs in other cognate tribes, and believed that this special ceremony was original.

Prof. MASON desired to know of Mr. Dorsey whether it was not unusual to admit him to the secret meetings, to which the latter replied that it was only after the Indians had discovered that he was familiar with the ceremonies, learned of the northern tribes, that they imparted to him the fact. The speaker further stated that the recitations are also in an archaic form of the language.

In general, all the points obtained from the Osages tally with the information obtained from other cognate tribes.

Major POWELL said that people on reservations may be classed in two divisions, those who are yet pagan and those who profess the Christian religion, but the latter take part in ancient religious rites.

The people of Jemez, although Catholics, still visit the mountains once a month to perform their mystic rites. Some of the Iroquois also adhere to their ancient mystic ceremonies and practise them at stated times.

The importance of a knowledge of Indian languages is illustrated by Mr. Dorsey's paper for the collection of myths and facts pertaining to secret ceremonies, as is also the knowledge of similar customs among other tribes so as to know the method of approach and extraction.

Mr. DORSEY replied that he usually gained the confidence of his hearers by first telling them the myths of other tribes.

SEVENTY-FOURTH REGULAR MEETING, December 4, 1883.

Col. GARRICK MALLERY, President, in the Chair.

The Council, through its Secretary, reported the election of Mr. Amos W. Hart and Dr. Horatio R. Bigelow as active members.

A letter was read from Mr. Gatschet giving information with respect to investigations in the folk-lore of the southern Sclavic peoples by Mr. Krause, one of the corresponding members of the Society.

The death of Sven Nilsson, of Lund, Sweden, an honorary member of the Society, was announced, whereupon the Secretary made brief reference to the labors of the deceased.

Mr. WILLIAM H. HOLMES then read a paper on "THE TEXTILE FABRICS OF THE MOUND-BUILDERS."*

ABSTRACT.

It was stated that very few specimens of these fabrics are preserved in our museums. They are subject to rapid decay and as a rule fall to pieces on exposure to the air.

Carbonization and contact with the salts of copper have been the most important means of perservation.

It has occasionally been noticed that fabrics of various kinds have been used in the manufacture of pottery and that impressions of these have often been preserved.

The writer conceived the idea of making casts in clay of these impressions and by this means restored many varieties of cloth heretofore unknown.

The restoration is so complete that the whole fabric can, in many cases, be analyzed.

It has been made of twisted cord and is seldom finer in texture than common coffee sacking.

The fibre used has probably been obtained from bark, weeds, and grasses.

* Published in the Third Annual Report of the Bureau of Ethnology with title "Prehistoric Textile Fabrics of the United States derived from impressions in Pottery."

The meshes are usually quite open, knotting and other methods of *fixing* the threads and spaces having been resorted to.

The combinations of threads are much varied and are of such a character as to make it quite certain that the weaving was done by hand, the threads of the web and woof being attached to or wound about pins fixed in a frame or upon the ground.

Specimens of the pottery and casts therefrom were shown and black board analyses of the fabrics were given.

DISCUSSION.

Prof. MASON inquired of Mr. Holmes whether he gave technical names to the various forms, to which Mr. Holmes replied that he found that impossible.

Major POWELL said the paper that had just been read by Mr. Holmes is of exceeding interest to all students of North American archæology; first, from the fact that his methods of research are unique; and, second, that the results of his investigations throw much light upon the status of culture reached by the people who constructed the mounds and other burial places found so widely distributed thoughout the eastern portion of the United States. The research sheds light both upon the textile and ceramic arts of these people, and in both departments they are shown to have been in no respect superior to the Indian tribes first discovered on the advent of the white man to this continent.

It is interesting to notice, in this connection, that the early publications in relation to the mounds and mound-builders of the valley of the Mississippi represent these people as having passed into a much higher culture than the North American Indians at large, and much has been written concerning a civilized people inhabiting this country anterior to its occupation by the Indians. In the light of the research which has been prosecuted during the past years in various quarters and by various persons, the manufactured evidence of the existence of such a people is rapidly vanishing, and this from many points of study. It is shown by a careful examination of the early travels in this country, and accounts of missionaries and various historic records, that some of the early tribes discovered were themselves mound-builders. This is clearly shown in the late publication of Mr. Lucien Carr, Peabody Museum, and by the researches of Professor Thomas, of the Bureau of Ethnology. The

researches of the Bureau of Ethnology also show that many of these mounds were constructed after the arrival of the white man on this continent, as works of art in iron, silver, rolled copper, &c., are found. Glass beads are also found, and many other articles manifestly manufactured during the last few centuries, these usually being such articles as are exchanged by traders to the Indians for their peltries.

Mr. HENSHAW, also of the Bureau of Ethnology, has made an interesting investigation of a subject which throws light upon this question. The early writers claimed that the stone carvings found in the mounds were often representations of birds, mammals, and other animals not now existing in the regions where these mounds were found, and that the mound-builders were thus shown to be familiar with the fauna of a tropical country. And they have even gone so far as to claim that they were familiar with the fauna of Asia, as it has been claimed that elephant carvings have been found. Now these carvings have all been carefully studied by Mr. Henshaw, and he discovers that it is only by the wildest imagination that they can be supposed to represent extra-limital animals; that, in fact, they are all rude carvings of birds, such as eagles and hawks, or of mammals, such as beavers and otters; and he has made new drawings of these various carvings; and will, in a publication which has gone to press, present them, together with the drawings originally published; and he makes a thorough discussion of the subject, being qualified thereto from the fact that he is himself a trained naturalist, familiar with these various forms by many years of field study.

It will thus be seen that many lines of research are converging in the conclusion that the mound-builders of this country were, at least to a large extent, the Indian tribes found inhabiting this country on the advent of the white man, and that in none of the mounds do we discover works of art in any way superior to those of the North American Indians.

I congratulate Mr. Holmes upon the skill with which he has prosecuted this work, and thank him for the clear exposition which he has given us this evening.

Prof. MASON stated that from the organization of the Society he had been more and more confirmed in the idea that the only way in which the truths of anthropology could be brought out was by specialists, artists, physicians, patent examiners, etc. The paper just read is an excellent illustration of this opinion.

Col. SEELY expressed his interest in the illustrations given by Mr. Holmes of research into the state of an art of which none of the products exist. Though absolutely extinct their vestiges remain in other arts; and to those able to read the record written in these vestiges they reveal facts as interesting as they are well ascertained. It takes the trained eye and skillful hand of an artist, supplemented by technical knowledge, to unravel these records. Without intimate acquaintance with the textile art and the structure of different fabrics, the impressions found by Mr. Holmes were hopelessly illegible. This indicates the true method of research into primitive arts, and there should be more of it.

Mr. JAMES A. BLODGETT, Special Agent of the U. S. Census, read a paper on "THE CENSUS OF BENGAL."

ABSTRACT.

The first attempt at a general census of British India was in 1871–2 and showed the population to be about 238,000,000.

The report for the census of Bengal in 1881 has been lately received in this country. It includes the northeast part of India north of the 20th parallel of latitude and west nearly to Benares. Here in an area of less than 200,000 square miles, a little above the joint area of Ohio, Indiana, Illinois, and Iowa, is concentrated a population of some 70,000,000 or two-fifths greater than that of the whole United States.

The authorities took no account of resources or of any but personal items. The preliminary arrangements were so completely adjusted as to take on a single night not only the fixed population but generally all travelers and all vagrants.

Almost two-thirds of the people are Hindoos, nearly one-third Mohammedans, about 158,000 Buddhists, and 128,000 Christians. The enumerated members of the Brahmo Somaj, the reform sect represented by the learned Hindoo who spoke in Washington a few weeks ago, were under 1,000, chiefly in the city of Calcutta.

Child marrriages prevail to a considerable extent, the ceremony in a considerable per cent. of cases occurring before the tenth year of age. Although the parties may not at once live together, the death of one after the ceremony leaves the other legally widowed. Hindoo widowers marry again, but Hindoo widows do not. The ratio of child marriage is lowest among the Buddhists.

There are 65 castes reported of 100,000 or more each, and 265 lesser castes or tribes. Hindooism gradually absorbs the aboriginal tribes, and occupations mark castes something like guilds in western countries, so that caste mingles questions of religion, race, and occupation.

About twenty languages are spoken. Over half the people speak Bengali as their mother tongue, over one-third speak Hindoostani, and only about 36,000 speak English as their mother tongue.

Education is low. The Hindoos are best educated of the great classes. In Calcutta the education of boys compares favorably with that in some western cities. The education of girls is scarcely secured at all, except among the Christians.

Admirable maps and diagrams aid the presentation of the facts in the census.

The digest of the census of Bombay has also been received here without the fullness of discussion or the maps of the Bengal report. The general relations of population and of customs are much the same as in Bengal. A new series of languages occurs, however, and 830 castes are reported, some of which are essentially identical with some of the Bengal castes, but many castes are intensely local in India.

The reports do not follow a uniform spelling in anglicizing even so common words as Hindustani, Mahomedan, and Brahman.

DISCUSSION.

Major POWELL said: I have been much interested in the paper read by our fellow-member, Mr. Blodgett, as a simple and lucid presentation of the more important facts presented in the Bengal census. One line of facts is of especial interest to me—namely, that relating to the census of the castes of Bengal.

Two great plans for the organization of mankind into states, as tribes and nations, are known: Tribal states are organized on the basis of kinship; national states, on the basis of property, which in its last form appears as territorial organization. Yet from time to time there spring up incipient methods of organization of another class. Men are interrelated in respect to their wants, and ultimately organized thereby through the organization of industries or callings—that is, organized on an operative basis through the division of labor. This method of organization appears in many ways, and in one form

its ultimate outgrowth results in the organization of aristocracies in various grades, with subordinate classes, as serfs and slaves. Again it appears in the organization of guilds. This form of organization was well represented not many generations ago in England, and relics of it still exist among the English people. It appears again in another form in India by the differentiation of people into castes, each caste having a distinct calling or group of callings.

In my studies of sociology it has often been a matter of surprise to me that the state has not oftener and to a larger extent been based upon an organization dependent upon callings, trades, or occupations—that is, that the state has not oftener been organized upon an operative or industrial basis. But when we accumulate the facts of history relating to castes, classes, guilds, &c., it appears that the method has been tried in many ways and it has never succeeded in securing justice to that extent as to commend its adoption.

A caste may be briefly described as a body of men constituting a unit or integral part in the state, and such a body of men are organized upon the basis of the industries or callings which they pursue. Around this organization are centered many other institutional characteristics. Marriage within the group is prescribed, marriage without the group prohibited; and many religious sanctions grow up around these institutions, and many social barriers to prevent escape from the body and entrance into another.

Much has been written about these castes of India, sometimes from the standpoint of religion, sometimes from the standpoint of conquest, and sometimes from the standpoint of McClennan, erroneous theories relating to exogamy and endogamy, names which he gave to correlative parts of the marriage institution found among most of the tribes of the world who are organized upon a kinship basis. It is true that the institution of caste exhibited in India may be profitably studied from each of these standpoints, but the essential characteristic of caste organization is this: That the people are thereby organized upon an operative basis, about which religious and social sanctions are gradually accumulated; that such an organization is in part the result of internal agencies arising from the differentiation of industries, or division of labor, as it is called in political economy, and in part by conquest, as the conquerors usually engage in those vocations deemed most honorable, and compel the conquered to engage in those considered least honorable. By such methods, *i. e.*, the division of labor through the inherit-

ance of callings from family to family, and through the further division, through the selection of callings of conquerors and the imposition of others upon the conquered, castes are primarily established. In the process of this establishment, and subsequently, moral and social sanctions gather about these institutions, and castes are firmly established only to be overthrown by great social convulsions, or, and chiefly, by the march of civilization and the concomitant establishment of justice and those institutions designed to secure justice.

All light thrown upon the institution of caste in India must be welcomed by every scientific student of sociology, and this census of Bengal, as set forth by Mr. Blodgett, is a valuable contribution to this subject.

Dr. Johnson inquired as to the effects of these early marriages upon the offspring; whether the children were well developed or deformed; the effects upon health of the crowding of many individuals; whether syphilis prevailed and its general effects.

Mr. Blodgett replied that the census officials were extremely careful not to push questions that might stir into opposition the prejudices of the people. Great difficulty arose as to the question of early cohabitation from the delicacy of the question and the great variance of English and other European customs; but as the legal ceremony took place at betrothal, betrothal became the point at which to count marriage.

Cohabitation was probably at an earlier average than among western nations, but statistics do not, in this census, help us beyond the general knowledge obtained by observant individuals.

There seems to be a high vitality up to advanced maturity; but after, say, forty-five years of age, the vitality seems to be in favor of the European.

No statistics are recorded on syphilis. The vital statistics have considerable value, however, indicating the predominance of pestilential diseases in districts badly drained, overcrowded, or with other adverse sanitary conditions, and special inquiry was made as to leprosy.

As to guilds and castes, a trace of such tendency may be seen in the perpetuation as a civil corporation in the city of London of more than one society originally founded on the occupation of its members, and now retaining privileges then granted, although no

longer constituted of persons following the employment for which they were founded.

Dr. FLETCHER said he inferred from Mr. Blodgett's remarks that cohabitation does not follow betrothal, and added that it is considered a disgrace if a child is not betrothed when she arrives at menstruation.

Prof. MASON referred to similar kinds of legislation in this country, prohibiting marriage, especially the laws, in many states, against miscegenation. He also said that caste originated at a time when the conquering Aryans were in a great minority, and to preserve the purity of their stock they made stringent laws against intermarriages. The laws of Menu prohibit intermarriages.

The PRESIDENT informed the members that the 2d volume of the Transactions was now ready for distribution, and copies could be obtained by calling upon the Secretary, at the May Building, 7th and E streets N. W.

SEVENTY-FIFTH REGULAR MEETING, December 19, 1883.

President Col. GARRICK MALLERY in the Chair.

The Council reported, through its Secretary, the election of Mr. Perry B. Pierce, of the U. S. Patent Office, as an active member.

The Secretary of the Council read a letter from Mr. Wilson, U. S. Consul at Nantes, France, relating to his antiquarian researches in that country.

Prof. CYRUS THOMAS then read a paper entitled "THE HOUSES OF THE MOUND-BUILDERS,"* illustrated by diagrams and specimens of clay plastering.

ABSTRACT.

Prof. THOMAS commenced by saying that while the ruins in Central America furnished abundant materials for judging the architectural skill of the ancient people of that region, no such opportunity was offered in regard to the mound-builders, all their buildings having crumbled to dust. Still we are not left wholly in the dark in regard to them. He then went on to show that they must have

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*Published in Magazine of Am. History, 1884, 110–116.

been of perishable materials, and that the little circular depressions from fifteen to fifty feet in diameter surrounded by earthen rings are the sites of ancient dwellings. From the fact that the hearth is found in the center he inferred that they were much like the conical wigwams of the modern Indians. Remains of this kind are common in middle and west Tennessee and in southeastern Missouri.

Farther south, during the explorations carried on under the Bureau of Ethnology, there have been found in many of the mounds layers of burnt clay broken up into fragments. From numerous facts ascertained in regard to these remains, which cannot be given in this abstract, and the descriptions given by early explorers of the houses of the Indians of this section, he argued that these were the remains of the houses of the mound-builders.

DISCUSSION.

Mr. Jas. H. Blodgett said: I hope Prof. Thomas will heed the suggestion of Mr. Carr, whose recent work was referred to, and not suppress part of his own work because Mr. Carr has anticipated him in his statements. The public has become so thoroughly trained into the idea of a mysterious lost race of mound-builders that it will be necessary for every one who knows of facts indicating the contrary to state them on all proper occasions. Lately seeing a reference to the mysterious lost mound-builders in the manuscript of a prominent writer, I suggested to him that it might expose him to criticism, and referred him to one or two eminent names that endorsed the view that our red Indians were competent to do like work. My suggestion was the first information received in this author's office that any such view was seriously held and I was referred to an article in a standard Cyclopeædia some years old to inform myself as to the true view. I trust Dr. Thomas will add his testimony in its due place.

Prof. Mason said he had always wished to see this subject discussed by gentlemen who had had as much experience in the matter as Major Powell and Prof. Thomas. It seems that doubts are thickening more rapidly than the proofs are forthcoming. In his own mind he had no doubts upon the subject, but took this antagonistic stand for the purpose of drawing out such facts to enlighten others who were adherents of the belief that the mound-builders

were a distinct race, and one of greater antiquity than is now known to be the case.

Major POWELL said the paper by Prof. Thomas is a valuable contribution to our knowledge of the North American Indians. It opportunely falls in with the present lines of research in two distinct ways: First, as identifying the mound-builders with various tribes found on the discovery of this country; second, as an addition to our knowledge of the dwellings of the ancient inhabitants of this country.

At our last meeting we had an interesting paper from Mr. Holmes, who, from his studies, concluded that the mound-builders were no other than the Indians inhabiting the country. Last year we had a paper from Mr. Henshaw arriving at the same conclusion from the facts discovered in another field of research. And now Prof. Thomas finds that some of the earth-works of this country are domiciliary mounds, as suggested long ago by Lewis H. Morgan, who was the great pioneer of anthropologic research in America; and, further, that the houses found in ruins on the mounds are such as were built by the Indians, as recorded in the early history of the settlement of this country.

Thus it is that from every hand we reach the conclusion that the Indians of North America, discovered at the advent of the white man to this continent, were mound-builders, and gradually the exaggerated accounts of the state of arts represented by the relics discovered in these mounds are being dissipated, and the ancient civilization which has hitherto been supposed to be represented by the mounds is disappearing in the light of modern investigation.

But Professor Thomas' paper is valuable from the fact that it gives us a clearer insight into the character of the habitations of these people. The Indians of North America made their dwellings in various forms and of various materials. The rudest dwellings found in the country are those made by some of the Indians of Utah and Nevada of the great Shoshonian family. These are simple shelters made of banks of brush and bark, especially the bark of the cedar, piled up so as to include a circular space, but open toward a fire. Boughs near the summit of the bark project over a portion of this space, and bark and boughs are piled indiscriminately on all. Such a shelter is good protection against wind, and, to some degree, against snow and rain. But these same people occasionally build larger habitations with small posts and cross-

pieces, upon which wattles of willow withes are made, and the whole is covered with willows. I have known such a communal house to be built large enough to accommodate from seventy-five to one hundred and twenty-five persons—all the members of a little tribe—while at other times the same tribes have been found occupying the rude dwellings above mentioned. Nor have I been able to discover their reasons for changing from one to the other. This has been observed: that the communal houses are but rarely used.

Many of the Indians of California build houses made of wind-riven slabs and poles inclined against a central ridge-pole and banked with earth, sometimes but part way up the sides of the inclined pole, sometimes quite over the top. At one end of such a dwelling an aperture is left for the escape of smoke. The Navajos often build similar lodges, except that they are conical in shape and have a peculiar entrance—a kind of booth like a *porte cochère*. In the eastern portion of the United States, as among the Iroquois, large oblong house were made of poles and slabs. Many of these houses were communal. Around Pyramid Lake and in many other portions of the country their dwellings were made of reeds, called in the West *tules*. Sometimes these houses were made somewhat symmetrically of poles, into which the tules were woven as a kind of wattle. At other times they made fascines of the reeds and used them in the construction of their houses, and I have had described to me houses made of fascines and wattled tules on the shores of Pyramid Lake and other lakes of the West, and ofttimes built out over the water. In a large portion of the United States the climate is arid, and naked sandstone rocks appear in great abundance, while forests are very rare. In all of these regions the Indians built of stone. Sometimes they walled up the front of a cave, or built a house under an overhanging cliff, using the wall of rock behind as a part of the dwelling. Sometimes, where rocks were friable, they excavated chambers in the sides of the cliffs. The cliff dwellings and cavate dwellings are found in great abundance in New Mexico, Arizona, and some portions of Utah. Other dwellings have been discovered in certain hills of Arizona that are natural truncated cones. In such a case the summit of the hill is a volcanic breccia, exceedingly friable, through which shafts were sunk into a more friable breccia below. In this more friable rock extensive chambers were excavated, and the entrance to these chambers was through a shaft from above by means of a ladder. With the

extensive pueblos of that region you are all quite familar. To a very large extent it is observed that the arrangement of dwellings in a village is significant. In very many cases they are arranged by clans and phratries. When such an arrangement does not exist there is usually some other device taking its place. For example, among Muskokis, or Creeks, near the centre of the village, there is a square laid out in a very systematic manner with seats, or rather spaces for sitting, on the ground relegated in a particular manner to phratries and clans, so that the tribe was arranged, in the council held from time to time in the square, in a systematic order. Usually over these sitting places booths were erected, and the posts that upheld the booths marked in a more specific way the seats of the officers of the village. In connection with these council squares a very interesting council lodge has been discovered. The booths of the square did not furnish ample protection at all seasons of the year, and in order to meet their wants on such occasions a huge conical lodge was constructed of the tall trees of that country. Slender trees 50 or 60 feet in height were cut down, trimmed, and inclined against a central, standing tree. Thus a huge conical lodge, 50 feet or more in height, was constructed, under which the whole village could take shelter. Under this they gathered in inclement weather to conduct their dances. And just here it should be remarked that the Creek Indians have yet a tradition of a time when they built their houses with wattled walls, the interiors of which were plastered—exactly such houses as have been described by Prof. Thomas.

The subject of house-building among the North American Indians is one of very great interest, as the various tribes exhibited much skill in utilizing the materials at hand, whatever they might be—bark, poles, slabs, tules, skins of animals, stone, etc.

Prof. MASON further stated that he had handled thousands of Indian weapons, utensils, &c., and found that many objects occurred in the mounds for which no particular use could be now assigned.

Major POWELL replied that it was very doubtful, at this time, if anything existed that could not be explained through the survival of similar articles now in use among some of the more isolated tribes of Indians.

Prof. SCUDDER referred to and reviewed some of Prof. Putnam's investigations and discoveries at Madisonville, and referred specially

to the exhumation of figurines, pearls, meteoric iron, and rude plating of hammered silver.

Prof. THOMAS, in reply to Prof. Scudder's statement of what had recently been found by Prof. Putnam in certain Ohio mounds, stated that all of the types mentioned, except one, had been obtained by the assistants of the Bureau of Ethnology.

Major POWELL said: The discussion this evening has brought out many interesting facts relating to the early inhabitants of this country, especially to the dwellings which they occupied and to the antiquity of the ruins which have been discovered.

In 1856 or '7 I was making exploration of mounds on the shore of Peoria Lake, in Illinois, and I discovered in a mound a copper plate—a thin sheet of copper, cut in the form to represent an eagle. At the time I supposed it gave evidence of the superior civilization of the mound-builders. Some months after, in more carefully examining this thin copper plate, I discovered that it had been rolled and cut by machinery, and this led me to believe that it was not the manufacture of Indians, but that it was probably manufactured by white men. If the supposition were true it is manifest that the mound had been erected subsequent to the association of these Indians with white people. This was the first suggestion to my mind that the age of the mounds had been misinterpreted, and that the general conclusion that the mound-builders were not tribes found in this country on its discovery was erroneous. Since that time one line of evidence after another has led to the same conclusion. Some years ago I published this conclusion in general terms, and every year it is strengthened, and it may be fairly said at the present time that it rests on a sound inductive basis.

But this conclusion does not overthrow the belief that many of the mounds are of great antiquity. Domiciliary mounds, burial mounds, and mounds for many other purposes are discovered everywhere throughout North America in vast numbers, and doubtless the inception of mound-building dates far back in remote antiquity. The numbers of the mounds themselves testify to this conclusion, and the conditions under which many of them are found lead to the same opinion. To account for the great numbers of the mounds it is not necessary, but is in fact illogical, to assume a dense population. Length of time will give the same result; and I think it has been clearly shown that the number of Indians inhabiting the country at the time of its discovery by Europeans has been by many writers

enormously exaggerated. It is probable that at the present time the number of Indians in the country does not equal that of the time of the landing of Columbus. On the other hand, the disparity between the numbers of the two periods is not great.

But here I must be permitted to remark that ofttimes the evidence adduced to prove the antiquity of the ancient works discovered throughout the country is unsound. There is abundant evidence of antiquity—good geologic evidence. Stone implements are found in geologic formations to such an extent as to leave no doubt that this continent was inhabited by man in early quaternary time; but sound evidence must be clearly discriminated from much of the evidence which is adduced. Travelers and scholars sometimes talk very loosely on this subject. Let me illustrate this.

In the southwestern portion of the United States we discover in vast numbers the ruins of ancient stone villages. Often these ruins are found at sites where water is not now accessible, and hence it has been averred again and again that all this arid portion of the United States was at some early period densely inhabited, and that the country has been depopulated by increasing aridity. And this secular change of climate has been adduced as evidence of the great antiquity of these works.

In 1870 I discovered ruins on the Kanab Creek in Utah and some of its tributaries elsewhere in Utah and Arizona, away from the neighborhood of water, and, like many other travelers, it at first seemed to me that I had discovered evidence of change of climate. But my work in that region was that of the geologist rather than of the anthropologist, and I early discovered that such evidence is valueless. In that arid country years—perhaps tens or scores of years—will pass without great rains. During such times the larger valleys are filled with the materials brought down by the wash of rains and minor streams, and such accumulation in the valleys of this arid region is very often found. But there come at greater or less intervals storms of such magnitude, precipitating waters in such volume that the valleys themselves are cleared of the accumulated sands. When this is done streams flow through them for miles or scores of miles where they did not run before, and the few springs along the water courses are unmasked and yield a constant supply. And I have in my mind at the present time a ruin which I supposed to be far away from water, and which was far away from known water ten years ago, but at the foot of which to-day a beau-

tiful stream is running, this valley having been cleared of its débris not more than eighteen months ago. Abundant instances of this kind can be brought up.

Savage people abandon their homes for reasons not fully or easily appreciated by civilized men. Some disease carries off a great man or a number of persons in a tribe, and panic seizes the people and they leave their homes, perhaps burn them, under the belief that evil beings or evil influences have taken possession thereof. And this occurs very often. I have myself more than once witnessed the effect on a tribe of an epidemic or the mysterious death of a noted personage. For this reason the sites of Indian villages, even though dwellings may be erected of stone, are not very permanent; they are constantly changing. In the southwestern portion of the United States there are other causes for change, namely, those mentioned above—physical causes. A tribe settling on a flowing stream at one time may have that stream buried by drifting sands and the springs all masked and be compelled thereby to change their habitation. And such changes doubtless were frequent.

Again, we know that a people living in a central village build small summer residences scattered about the country by the sites of springs, where they cultivate their little crops of grain and other vegetables; so that a large group of such dwellings may be ofund gathered about some central pueblo—not giving evidence of a dense population, but only of the habits and customs of a small body of people. In such manner it may be shown that the extensive population of the southwestern portion of the country, based upon the evidence of the ruins so abundantly found, does not hold. A few people moving here and there from spring to spring and from stream to stream as pestilence and superstition and physical changes demanded would in many recurring centuries leave behind all the ruins now discovered. The antiquity of man widely scattered throughout this continent is firmly established on good geologic evidence, and it is not necessary to resort to evidence of doubtful character.

SEVENTY-SIXTH REGULAR AND SIXTH ANNUAL MEETING, January 15, 1885.

Col. GARRICK MALLERY, President, in the Chair.

The Council reported, through its Secretary, the election of Mr. W J McGee, of the U. S. Geological Survey, as an active member, and Hon. Thomas Wilson, U. S. Consul at Nantes, France, as a corresponding member.

The annual report of the Treasurer was then read and submitted to the Society.

On motion of Major POWELL, a committee was appointed, consisting of Messrs. Thomas, Dorsey, and Flint, to audit the report.

The Society then proceeded to ballot for the officers of the ensuing year.

The following is the result of the balloting:

PRESIDENT	J. W. POWELL.
VICE-PRESIDENTS	GARRICK MALLERY. OTIS T. MASON. LESTER F. WARD. ROBERT FLETCHER.
GENERAL SECRETARY	DAVID HUTCHESON.
SECRETARY TO THE COUNCIL	F. A. SEELY.
TREASURER	J. HOWARD GORE.
CURATOR	W. J. HOFFMAN.
COUNCIL AT LARGE	J. OWEN DORSEY. EDWARD ALLEN FAY. H. W. HENSHAW. CYRUS THOMAS. WESTON FLINT. C. C. ROYCE.

The amendment which had been duly proposed to the Constitution was then taken up and adopted as additional to Section I, Art. III, viz.:

"Corresponding members from whom no scientific contribution is received for two years after their election may be dropped from the list of members by a vote of the Council, but when so dropped shall be eligible to reinstatement."

Seventy-Seventh Regular Meeting, February 5, 1884.

Major J. W. Powell, President, in the Chair.

The Council, through its Secretary, reported the election, as active members, of the following gentlemen:

John Jay Knox, Dorman B. Eaton, John M. Gregory, Edward T. Peters, Herbert H. Bates, Anton Carl.

The Curator read the following report of the publications received by the Society since the first meeting of the present session in November:

From the Society.—Bull. Buffalo Society Nat. History. Vol. IV. Nos. 1, 2, 3, for 1881, '82.

——— Wyoming Historical and Geological Society. Publication No. 7. 1883. Memorial. (Isaac Smith Osterhout)

——— Ymer. Bull. issued by the Swedish Anthropological and Geographical Soc'y. Stockholm. 1883. Parts 1—6.

——— Bull. Anthropological Society of Paris. 6th vol., 3d Sér· Part 3. May and July, 1883.

——— Archivio, etc., from the Italian Society of Anthropology, Ethnology and Comparative Psychology. XIII, 2nd fascicule, 1883.

——— Annual Report of the Frankfort (Germany) Society of Geography and Statistics. 1881–1883

——— Bull. of the Library Co., of Philada. Jan., 1884.

From the Publishers.—Science and Nature. An International Illustrated Review of the Progress of Science and Industry. Paris. Ballière et Fils. Dec. 29, 1883.

From the Author.—No. III. American Aboriginal Literature. Consisting of "The Güegüence; A Comedy Ballet in the Nahuatl-Spanish Dialect of Nicaragua. Edited by Dr. D. G. Brinton. Philada. 1883. 8vo. Pp. 94.

——— Aboriginal American Authors and their productions, especially those in the native languages. By Dr. D. G. Brinton. Philada. 1883. 8vo. Pp. 63. [This memoir is an enlargement of a paper laid before the last International Congress of Américanists, at Copenhagen, Aug., 1883.]

——— A Brief Account of the More Important Public Collections of American Archæology in the United States. By Henry Phillips, Jr. Philada. 1883. 8vo. Pp. 9.

From the AUTHOR.—Micrometry. By D. S. Kellicott. (Sec. Buff. Acad. Sci.) Chicago. 1883. 8vo. Pp. 23. Reprinted from the Proc. Am. Soc'y of Microscopists.

——— Der Bronze-Stier aus der Bijčí Kála-Höhle. By Dr. Heinrich Wankel. Wien. 1877. 8vo. Pp. 32. Map and plates.

——— Ueber einen prähistorischen Schädel mit einer Resection des Hinterhauptes. By Dr. Heinrich Wankel. Wien. 1882. 8vo. Pp. 19. 2 plates.

——— Ueber die angeblich trepanirten Cranien des Beinhauses zu Sedlec in Böhmen. By Dr. Heinrich Wankel. Wien. 1879. 8vo. Pp. 11.

——— Eine Opferstätte bei Raigern in Mähren. By Dr. Heinrich Wankel. Wien. 1873. Pp. 22.

——— Prähistorische Eisenschmelz-und Schmiedestätten in Mähren. By Dr. Heinrich Wankel. Wien. 1879. Pp. 40. 1 pl.

——— Wo bleibt die Analogie? By Dr. Heinrich Wankel. Wien. 14to page. [On rock inscriptions, found in Smolensk, Russia.] By Dr. Heinrich Wankel. Without date.

——— Urgeschichtliche Ansiedelung auf dem Misskögel in Mähren. By Dr. Heinrich Wankel. Wien. W. d.

——— Bilder aus der Mährischen Schweiz, und ihrer Vergangenheit. Wien. 1882. 8vo. Pp. 422. Ill.

From ERNEST CHANTRE.—Études Paléoethnologiques dans le Bassin du Rhône. Bronze Age. Paris. 1877. 8vo. Pp. 8. Ill. and chart.

——— The Burial Places of the First Age of Iron of the French Alps. Lyon. 1878. 8vo. Pp. 15. 60 fig. 3 pl.

——— Anthropologie. A Lecture. Lyon. 1881. Pp. 29.

——— Paleoethnologic Researches in Southern Russia, especially in the Caucasus and the Crimea. Lyon. 1881. 8vo. Pp. 27. Pl. 12.

——— Geologic Monograph on Ancient Glaciers, etc. MM. Fahan and Chantre. Lyon. 1880. 8vo. Vol. I. Pp. 622. Vol. II. 572. Ill. folio atlas. These volumes are replete with anthropologic material.

——— The Bronze Age. Researches on the Origin of Metallurgy in France. Paris. 1875. 3 vols. Folio. Profusely illustrated.

——— The First Age of Iron. Mounds and Burial Places. Lyon. 1880. Folio. Pp. 60, and 50 lith. plates

From Dr. HEINRICH FISCHER.—A Review of the II and III Parts of Trans. Royal Ethnographical Museum of Dresden; consisting of a work on objects of Jadite and Nephrite from various quarters of the globe. By Dr. A. B. Meyer. 4to. Pp. 9.

On motion of Col. SEELY a vote of thanks was passed to the donors of books and pamphlets mentioned in the Curator's report.

Mr. CYRUS THOMAS then read a paper entitled "CHEROKEES PROBABLY MOUND-BUILDERS." *

ABSTRACT.

The speaker commenced by referring to some discoveries made by Prof. Lucien Carr in 1876 in Lee County, Virginia, which, taken together with the historical data, led him to the conclusion that some, at least, of the mounds of this region were the works of the Cherokees. The evidence in this case consisted of the remains of a building of some kind found in a mound which must have corresponded very closely with the "Council House" observed by Bartram on a mound at the old Cheroke town of Cowe.

He next referred to some mounds recently opened by the assistants of the Bureau of Ethnology in western North Carolina and East Tennessee, the contents of which, together with the history of the Cherokees, induced him to believe they were also built by them.

Prof. THOMAS then entered upon the discussion of the early history of this people, the purport of which was to show that they had occupied this region at least as far back as 1540, the date of De Soto's expedition.

He then referred to the specimens found in the mounds alluded to, which he contended indicated contact with Europeans, exhibiting some of the specimens to the Society as evidence of the correctness of his conclusion, maintaining that if the mounds were built after the appearance of the Europeans they must be the works of the Cherokees, as they were the only people known to have inhabitated this particular section from the time of De Soto's expedition until its settlement by the whites.

As further proof of his position he referred to carved stone pipes, engraved shells, and copper ornaments found in these mounds precisely like those described by early writers as made by and in use among the people of this tribe; also to numerous articles of aborigi-

* Published in Magazine of American History. 1884. XI, 396–407.

nal and European manufacture dug up from the site of an old Cherokee town near the Hiawassee river, the former being precisely of the same character as those found in the mounds alluded to.

In order to show that these mounds could not have been built by the Creeks or more southern Indians he presented arguments to prove that the Etowah mounds in Bartow county, Georgia, were on the site of the town named by the chroniclers of De Soto's expedition Guaxule, which evidently from the narrative could not have been in the territory of the "Chelaques" (Cherokees). He then alluded to the construction of the mounds of this group, and to specimens found in one of them, (exhibiting some of the specimens), which showed clearly that they were built by a different people from those who erected the mounds of North Carolina and East Tennessee.

DISCUSSION.

Major POWELL said Prof. Thomas' paper furnished additional evidence that a number of our Indian tribes were primitive mound-builders. In relation to that part of the paper respecting the ancient habitat of the Cherokees, I have some curious evidence to offer. Some years ago I discovered that the Cherokees, Choctaws, Chickasaws, Muskokis, Natchez, Yuchis, and other tribes have among them the tradition of an ancient alliance for offensive and defensive purposes against the Indians to the west of the Mississippi river of the Siouan stock. In the grand council of the tribes mentioned the terms of an alliance were under consideration, and from day to day the subject was considered without arriving at a conclusion. The relation of the tribes to each other could not be adjusted satisfactorily to all, and it seemed likely that the council would break up without effecting an alliance. Now the savage state or body-politic is a kinship body; the ties are of consanguinity and affinity; and this is the only conception of a state possible to people in this grade of culture. So the disagreement arose about the terms of kinship by which the tribes should know one another, as this would establish their rank and authority in the alliance.

After many days had passed in fruitless discussion a Cherokee orator proposed a plan of alliance that has given him renown among all the tribes interested down to the present time. To those who have studied Indian oratory and the reasoning of Indian minds his plan and the reasons therefor are of great interest. He commenced

by describing the geography of the country inhabited by the several tribes in order from east, passing by the south to west, and passing by the north again to east. After describing all of this country—the mountains and valleys and rivers—he called attention to the fact that the rivers now known as the Savannah, the Altamaha, the Appalachicola, the Alabama, the Tombigbee, the Tennessee, and the Cumberland all head near one another in the mountain land occupied by the Cherokees; that the Cherokees, therefore, drank first of the waters of all the rivers, and that the rivers then passed from the land of the Cherokees into the lands of the other tribes to be used by them, and that, therefore, mother earth had signified their precedence to all the other tribes. He therefore proposed that the Cherokees should be the father tribe, and that the various other tribes should take rank as sons in the order in which the sun rose upon their lands—the tribe farthest to the east to be the first son or elder brother, the second tribe the second son, and so on. This geographical argument was at once recognized by all the tribes as being invincible, and the plan was immediately adopted.

Now this tradition serves us a double purpose. First, it exhibits the methods by which one tribe has called another, now here, now there, by terms of kinship, and that these terms of kinship do not signify that the people have traditions of formerly belonging to the same tribe, but that they give evidence of alliances having been formed by such tribes. The second point of interest, and that which bears upon the communication of Prof. Thomas, is this: That the traditions of all of these tribes place the Cherokees in the Southern Appalachian Mountains, about the sources of the rivers from the Savannah around to the Cumberland, this being the very territory which Prof. Thomas claims to have belonged to the Cherokees from historical evidence and evidence obtained from the mounds.

Mr. HOLMES exhibited and commented upon some delineations of the human figure in copper and on shell gorgets found in the mounds of Tennessee, remarking that the designs were not European but resembled the art of Yucatan, and if manufactured in Spain were made from designs furnished by those who had been in Yucatan, and if they were of European manufacture they were of no great value except to prove the intrusion of Europeans.

Col. SEELY remarked that the opinion that was gaining ground among American students, and particularly among the members of

this Society, as to the comparatively recent period in which mound-building was practiced, did not seem to be shared in Europe. He had just received from the Marquis de Nadaillac, one of our honorary members, and perhaps among Europeans the one person who kept himself best informed on all the developments of American archæology, the proof-sheets of an article in the *Revue d'Anthropologie*, in which he presented to European readers a *résumé* of Mr. Carr's recent work. While admitting the force of the facts set forth, the Marquis dissented from the conclusions, his particular reason for dissent being that the reversion to barbarism of tribes advanced in civilization was a thing unknown. He said a tribe or people partially civilized might be conquered by one more barbarous, and might become merged in it; but it had never been known that such a people, after once having fixed homes, agriculture, and arts of domestic life, had lost all these and fallen back to the barbarous condition of their conquerors. On the contrary, experience shows that the effect of such a mixture of races is to elevate the conquerors by imparting to them the arts and habits of the conquered people.

Col. SEELY read brief extracts from M. de Nadaillac's article, which concluded with very complimentary mention of the work of American explorers and an expression of belief that they would before long lead to a solution of the mystery of the mound-builders.

Major POWELL said: The criticism which Colonel Seely has read for us is interesting in various respects, but it fails to be valid by reason of a curious error. It is a great mistake to suppose that the Indians of North America were nomads. All of our Indian tribes had fixed habitations. It is true they moved their villages from time to time, because of their superstitions and for other reasons, but to all intents and purposes they were sedentary, living in fixed habitations from year to year, though from generation to generation they might change the sites of their towns. But of many of our Indian tribes because partly nomadic shortly after the advent of the white man, from whom they obtained horses and fire-arms. With horses they could easily move from point to point, and with fire-arms they could obtain a larger share of their sustentation by hunting than they had previously done, and many tribes gave up agriculture on this account. Instead of living in houses of wood and stone and earth they came to live more or less in skin tents.

If we attempt to mark off the progress of mankind in culture into stages, that which I shall call *savagery* is, in a general way,

well differentiated from higher stages. In this stage the state is organized by kinship. Tribes are kinship bodies. In the main, descent is in the female line—that is, mother-right prevails. In general, too, these people are in the stone age. They have not yet learned to use bronze; nor have they developed hieroglyphic writing. People in this stage of culture are called *savages*. When such tribes have changed their social structure so that father-right prevails, then the patriarchy is established. At about the same period of culture animals are domesticated, and doubtless the domestication of animals and the necessity for nomadic life which results therefrom is one of the most important agencies in breaking up mother-right and establishing father-right; and when father-right is established the patriarchy speedily follows. Such peoples we call *barbaric*, and the stage of culture in which they live *barbarism*. Barbaric people may be nomads; savage people are never nomadic. Some English anthropologists whose branch of investigation is confined chiefly to institutions, or, as we call it, "sociology," have traced back the history of Aryan civilization until they have discovered the patriarchy, until they find the early peoples from whom the present civilized States have descended in a state of nomadism—patriarchies with their great tribal families about them, together with their flocks and herds, all roaming from one district of country to another in search of pasturage and water. And they are accustomed to assume that this patriarchal condition, this nomadism, is the primitive form of society. Sir Henry Maine is one of the leading men of this school, and we are greatly indebted to his researches for the materials with which to trace the development of patriarchal institutions into national institutions. But there is abundant evidence to show that there are institutions more primitive than those of barbarism. The tribes of Australia and the tribes of North America and of South America are discovered to be in a state of culture lower and more primitive in structure than the peoples of early Aryan history. Herbert Spencer has in the same manner confounded tribal society, or savagery, with barbarism, and has entirely failed to understand the structure of the hundreds of tribal States of North America and of many others elsewhere throughout the world; and to him may be largely attributed the erroneous habit of calling the tribes of North America nomads. It should be distinctly understood that the North Americans are not nomads, that they have not the partriarchal form

of government, and that they have not domesticated animals. From this statement I must except certain tribes of Mexico and Central America, whose exact state of culture has not yet been clearly discovered. The criticism of the eminent author from whom our Secretary has read therefore falls to the ground.

Mr. WARD said he had looked up the exact meaning of nomadism under the impression that the term implied the state given by Major Powell. He had seen it used in the sense of a headless race, with no form of government, no arts, no domestic animals, therefore representing the lowest form of culture. The term was used in this sense by Mr. Herbert Spencer. There was some justification for the use of the term in this sense by European ethnologists. The meaning of the word does not involve domestic animals; it simply means to wander.

Prof. MASON said that the Cherokees might have been mound-builders, but the mound-builders were not all Cherokees. We cannot yet affirm that the ancestors of our modern Indians were the mound-builders of the Mississippi valley. He called attention to the fact that Dr. Brinton states that the mound-builders of the Missippi valley were Choctaws. He also spoke of the delicate and strange forms of objects in stone found in Ohio mounds and in immense stone graves compared with forms of articles made by modern Indians. There are many types of these mound-objects for which we have no names, because modern savages use nothing like them.

Major POWELL said there is no whit of evidence to show that the mounds were built by a pre-Indian people. For a long time it has been assumed that a great race of people inhabited the valley of the Mississippi anterior to its occupation by the tribes of Indians discovered by early European explorers, and it was claimed that these people had erected great earthworks of such magnitude that they could not be attributed to the Indian tribes, but that they must have been the work of people more highly organized. This error arose from the fact that early writers had no adequate conception of the character of tribal organization, and that kinship society is as thoroughly bound together, and perhaps more thoroughly, than that based upon any other plan. They also assumed that the works of art found in these mounds, or associated therewith, gave evidence of superior art. A careful examination of this theory has proved its fallacy. On the other hand, it has been discovered that the

works of art in the mounds are in no whit superior to the arts of the Indians discovered in this country. On the other hand, the Cherokees, Choctaws, Chickasaws, Muskokis, Shawnees, Mandans, Wintuns, and Siouans, and probably many other tribes, are known to have built mounds for domiciliary and burial purposes. The earlier explorers found tribes of Indians occupying and using mounds — the Natchez, Cherokees, and others; and the result of the last few years of investigation is this: That there is no sufficient reason, and in fact no whit of evidence, to show that this continent was occupied by a people anterior to its occupation by the Indian tribes, a people of a higher grade of culture. On the other hand, some tribes of Indians are known to have been mound-builders. We have not yet discovered what particular tribes built many of the mounds; nor is it possible to discover when they were built — that is, to fix with acccuracy the date of their erection. Some of them have been built within the historic period — doubtless but very few compared with the whole number — and some of them are doubtless of great antiquity. And during all the centuries of history when these mounds were erected some tribes may have been destroyed, and there may be mounds built by tribes whose history is lost. Some of the Indian tribes occupying the continent at the advent of the white man were mound-builders and a few mounds have been built since that time. The great number were erected prior to that time by these tribes, and perhaps by others still existing, but of whose mound-building we have yet no knowledge, and still others may have been built by tribes that are lost.

This seems to be the inevitable conclusion from the researches of the past few years, and the theory that a more highly cultured people inhabited this continent anterior to its occupation by the red Indian falls to the ground.

SEVENTY-EIGHTH REGULAR MEETING, February 19th, 1884.

Major J. W. POWELL, President, in the Chair.

Mr. DORSEY, in behalf of the committee appointed to audit the Treasurer's accounts, then reported that the accounts had been examined and found to be correct. The report was accepted by the Society.

The Secretary of the Council announced that the President had designated the Vice-Presidents to their several sections, as follows:

Dr. Fletcher, Section of Somatology; Mr. Ward, Section of Sociology; Col Mallery, Philology, Philosophy, and Psychology; Prof. Mason, Technology.

Mr. WARD then read a paper entitled "MIND AS A SOCIAL FACTOR." *

ABSTRACT.

It was maintained that, notwithstanding the general disposition to exalt and deify the mind, still this had thus far amounted to little more than lip-service, and that the real power of human intellect as the lever of civilization was not merely ignored but practically denied. Touching lightly upon the metaphysical school of philosophy, of which this had always been true, he directed his main argument against the now far more powerful influence in the same direction which the most advanced scientific thinkers are exerting. The tendency of the evolutionists to contemplate man solely from the biological standpoint, and to treat society as a simple continuation of the series of results accomplished by evolution in the lower departments of being, was strongly condemned. Himself a consistent evolutionist, and firm believer in the doctrine of man's descent from humbler forms of existence, Mr. Ward still cogently maintained that in studying development an entirely new set of canons must be adopted the moment the phenomena of the human intellect present themselves for consideration. Henceforth a new factor, wholly different from any before employed, enters into the problem, and correspondingly new and distinct methods of research must be adopted. Just as the biologist finds in the advent of life on the globe a new and enormous factor such as compels him to investigate the organic world with an entirely new set of principles and methods from those that are applicable to physics, chemistry, etc., so, Mr. Ward maintained, when the developed psychic faculty appeared a second change of base in science, equally thorough and complete, was imperatively demanded. The failure of modern philosophers, headed by Mr. Herbert Spencer, to recognize this patent truth had led to the let-alone doctrine, which possesses a certain fascination

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and justifies individual aggrandizement, and hence is making rapid inroads into the popular habit of thought. This *laissez faire* philosophy, which Mr. Ward characterized as the "gospel of inaction," is, in his opinion, distinctly negatived by the most advanced science, is contrary to the very law of evolution, and its legitmate workings almost justify Carlyle in denouncing the whole philosophy of science as the "gospel of dirt."

As against such sordid teachings Mr. Ward held: That without apotheosizing the mind, without denying its humble origin and slow development, it is still the greatest fact in the universe, produces the grandest results achieved on the globe, and in and of itself makes man the supreme arbiter of his own destiny, the great independent agency of the world and master of the planet.

DISCUSSION.

Prof. THOMAS remarked that for a clear comprehension of the problem presented in Mr. Ward's paper a definition of what he meant by mind was necessary. He cited illustrations to show that animals and even insects have memory and reasoning powers—in short, mind. What then, he asked, is the human as distinguished from the brute mind?

Prof. WARD, in reply, said that so far as the purposes of the present paper were concerned the only definition of mind necessary was the one given in the course of the paper, viz., that it was the inventive faculty of man.

Mr. WELLING expressed his general concurrence in so much of Mr. Ward's paper as might be said to convey the positive and affirmative propositions of the writer, but intimated the opinion that on a deeper analysis and closer inspection it would be found that the dissidence between Mr. Ward and the scientific expositors of the naturalistic school was not so great as might be inferred from the terms of his negative criticism. That dissidence was perhaps formal rather than real, being, as between him and his opponents, a question of nomenclature rather than of substance—or, to speak more definitely, a question as to the precise point in the evolutionary process where the logic and nomenclature of the naturalistic school might be held to apply to the facts of psychic activity in the figure of human society. In so far as mind might be said to have a physical basis, Mr. Welling said that he saw no reason why the human organism should be exempted from the law of a physical natural selection and

survival, but at the same time it was very clear that we were not to look to man's *physical* organism for the highest expressions of that natural selection which was peculiar to him in the animal world. Regarded apart from all disputes as to their genesis, and considered simply in their functions, it might be said that a *plant* is a machine for coördinating a certain number of natural forces, and thereby lifting them above the realm of the inorganic nature which is below it; that the *animal* organism is a machine for coördinating another bundle of natural forces and thereby lifting them above the level of the plant world, and that *man* is an organism in which the vegetable and animal constitution simply lays the basis of a higher series of activities, in proportion as the natural forces below him are coördinated and transmuted by that which in him is highest—his *mind.* It is, therefore, in the creations of the human mind that we would naturally look for the natural selections and survivals which are most distinctive of man and most descriptive of his place in nature. If the place of man in nature and the place of mind in man be so regarded, it does not seem necessary to assume that there is any reversal of the logic of evolution when we come to study the phenomena of human society. It is not a reversal of this logic and nomenclature, as Mr. Ward seems to think, but a transference of that logic and nomenclature to a higher sphere of action—the action of man in society under the forces of an expanding science and a growing morality. It is in *these*—that is, in the rational and moral forces, which are dynamic in society—that we must look for the natural selections which are relatively the fittest to survive at any given stage of human history. And in properly coördinating the rational and moral forces by which he is lifted above the brutes of the field, it is just as important that man should act *with* the forces of nature below him and in him as that he should in a measure act *above* those lower forces by virtue of his mind—his "faculty of execution," as Mr. Ward calls it. And in making the purely artificial regulations which belong to him as "a political animal" he is perpetually in danger of making civil, political, and economical adjustments which sin against the laws of nature and against the natural rights of man. Against all adjustments which unduly restrict the natural freedom of man in his mind, his body, or his labor, we may therefore justly hurl the doctrine of *laissez faire.*

Mr. WELLING then proceeded to illustrate this point of view by citing the phenomena of political economy as presented to us in

France during the reign of Louis IX, when every branch of industry in the kingdom was put under governmental regulation and restriction. These regulations and restrictions were imposed in the name of a state-craft which assumed to be wise *above* the laws of natural production. They were the expressions of an artificial selection working against the natural selections of supply and demand in the figure of political economy, and it was in opposition to the enormities of this system that the school of political economists known in France as the *physiocrats* rose at the close of the 18th century to make their indignant protest in the name of *laissez faire.* And in subsequent times as well as in other lands there had been abundant room to challenge the tariff regulations of any given epoch in the name of the same watchword.

Shall we say, then, that the maxim of *laissez faire* is final in political economy? By no means. It is final as against the pretension that man, by legislative artifice, however ingeniously devised, can make any industry profitable to the commonwealth against the forces of natural production. But in so far as man has higher ends in society than the creation of wealth the maxim is *not* final. If there be those who, in the name of a naturistic philosophy, would plead for the right to grind up the bodies and souls of men in the natural pursuit of wealth, it is easy to see that such a false and one-sided adjustment of economical relations would but call for a new evolution of public intelligence and public morality, as seen in the preventive justice which should be devised in order to guard the community from such excesses of the *laissez faire* doctrine. But neither the public intelligence nor the public morality can have a full field for the exercise of their natural prerogatives in the sphere of public economy until *laissez faire* has allowed the forces of natural production to exhibit the full measure of their strength, without let or hindrance, save such as may be neeeded to guard interests higher than the material wealth of a nation.

Major POWELL: The paper by Prof. Ward has been of great interest to me, as well as the discussion which it has elicited. In the progress of institutions it often becomes necessary that the old should be torn down in order that the new may be erected on the same ground; and in every great civilized land there are those who devote themselves to destruction, while others engage in construction. The theory of the destructionist has of late years obtained much vantage-ground from the doctrines of evolution, and the com-

munication of this evening very clearly sets forth the improper use of the established doctrines of evolution by a class of philosophers who fail to appreciate fully the necessity for construction *pari passu* with destruction and who have lost faith in human institutions and neglect the teachings of all human history.

The Lamarckian doctrine of evolution was that of adaptation by exercise. The hypothesis did not obtain wide acceptation until it was expanded more fully by Darwin and his contemporaries into the further doctrine of the survival of the fittest in the struggle for existence through competition in enormously overcrowded population. By this latter philosopher it was shown that competition performed an important part in evolution, and that the Lamarckian method gained its efficiency through the law established by Darwin. Among the lower animals species compete with species, and individuals of the same species compete with one another, and as the number of individuals produced is greatly in excess of those which can obtain sustentation some must necessarily succumb, and in the grand average it is the unfit that yield their places to those better fitted to the conditions. With mankind this competition does not perform the same office that it does with the lower animals, and this by reason of the organization of society and of other human activities, whereby men, to a greater or less extent, become interdependent, so that the survival of one depends upon the survival of others, and the welfare of one upon the welfare of many. But competition still plays an important part in the life history of the human race. Man in his competition with the lower animals has so outstripped them in skill and power that he utilizes them for his wants. He destroys some, and others he domesticates for his purposes. It cannot properly be said that he longer competes with the lower animals—in fact, he utilizes them.

But man competes with man, and this competition is expressed in warfare—public and private. In public warfare state competes with state, and the question arises, does this competition, this warfare, ultimately result, in the average, in human progress? So far as it is a competition between states do the higher and better people survive, and the lower people succumb? He would be a bold man, indeed, who would assert that the victor is always the superior man in culture, and who would divide and relegate the victories of the world to the good and the bad, the wise and the unwise, the just and the unjust. It is a task too delicate for any-

thing but omniscience. But we may look upon it in another light. In the grand average the individuals who engage in warfare are those who are physically strong, and, as judged by the standards obtaining among their own peoples, they are the patriotic and the noble, and it has usually happened that the flower of the state has been absorbed in its armies. This is less true in modern warfare, but is more true as we go farther backward in the history of mankind. The strong, the brave, and the patriotic have fallen in battle; the weak, the cowardly, and the selfish have survived; and thus warfare has been a constant drain upon the best of all lands; and it may be confidently asserted that human competition by warfare has in this manner failed to be an agency for human progress. Often warfare has been the means of overthrowing unjust and unwise institutions, and in this manner warfare has ofttimes resulted in good in human progress. On the other hand the period of warfare, the time in which peoples are engaged in warfare, is usually a time when the institutions of a people lapse from a higher to a lower condition. The necessities of war ofttimes furnish the excuse and justification for the establishment of institutions, or for modifications unjust and tyrannical in character. In the main war periods are times in which public morals lapse toward barbarism.

If we turn to consider the effect of private warfare on the progress of mankind we again fail to discover an efficient agency in human culture. He would indeed be a bold man who should assert that it results in the survival of the fittest, and who would relegate murderers to the class called the best, and the murdered to the class called the worst.

But mankind engage in another form of competition. They compete for welfare or happiness; and in so far as it is true competition, as distinguished from honorable rivalry—that is, in so far as one man succeeds at the expense of another—in just so far is injustice done; for, by the establishment of interdependence among men, the welfare of one properly depends upon the welfare of others, and the essential characteristic of justice, for which all mankind have striven, is this: that no man shall reap advantage to the injury of his neighbor. Competition for welfare, in the sense in which the term is here used, is the prosecution of injustice, and to the extent that justice is established competition is avoided.

There is yet competition of a third class. Arts compete with arts, and in the average the best are selected, and the choice is

made by men themselves. Men do not choose the best men but the best arts, and indirectly choose the men as best because they represent the best arts. So, institutions compete with institutions, and the best are chosen in the average. So, languages and methods of expressing thought compete with languages and methods of expressing thought, and in the average the best are chosen. In like manner opinions compete with opinions, and in the grand average the best and the true are chosen. Now, arts, institutions, languages, and opinions are human inventions, and in every department of human activity, as thus represented, inventions compete with inventions, and as in the grand average the fittest are chosen, so those who represent the best, the fittest, achieve success as compared with others who represent inventions of less worth. In this field there is legitimate competition, and it is by this competition that man progresses in civilization; but it is the objective invention or activity that survives, not the subject man. Now that class of sociologists who appeal to the established facts of science relating to competition, and use the laws of competition as they are exhibited in the lower animals, as if they properly applied to man, use them for destructive purposes, to destroy institutions, and they use them illegitimately, for human progress is not made by competing for existence, or by directly competing for welfare, but only by indirectly competing for welfare through the direct competition of arts, institutions, languages, and opinions; and in order that this indirect competition may be efficient all such competition must be in conformity with the principles of justice. Therefore, institutions designed to establish justice among mankind cannot properly be judged by the canons derived from the laws of competition, but only by the canons derived from the principles of justice, for the efficiency of competition itself in human progress depends primarily on pre-established justice.

The destructionists who thus illegitimately use the doctrines of evolution in their warfare against all human institutions to a large extent deny the efficiency of altruistic motives. They do not clearly see that wise egoism is wise altruism, because they do not clearly understand the interdependence of mankind; and in denying the extent and efficiency of altruism they neglect the best side of human history. Man inherited altruism from the beast. The she bear loves her cubs, the lioness her whelps, and the eagless her eaglets, and beast, bird, and insect alike exhibit altruistic motives. Among

the lower animals the group is very small indeed between the individuals of which such sentiments prevail; but steadily in their progress from savagery to the highest stage of civilization men have enlarged the group, as the small kinship group has expanded into larger, the clan into the tribe, the tribe into the confederacy, and confederacies and confederated tribes into nations; and altruism has expanded from smaller group to larger group, from family love to patriotism, and from patriotism to humanity; and in the light of the past we may safely prophesy of the future that this altruism will improve in quality and expand in scope until every man shall recognize in every other a brother in whose welfare he has an interest as deep as in his own, and when the doctrine of *laissez faire* shall be known no more forever.

SEVENTY-NINTH REGULAR MEETING, March 1, 1884.

Major J. W. POWELL, President, in the Chair.

The President announced the resignation of David Hutcheson, as General Secretary of the Society, and the election by the Council of S. V. Proudfit to fill the vacancy.

Ensign ALBERT NIBLACK, U. S. N., read the following paper on "THE SMITHSONIAN ANTHROPOLOGICAL COLLECTIONS FOR 1883."

With the exception of the year 1876, when the material was received from the Centenial Exposition, the accessions for 1883 exceed those of any other year both in number and value. As the annual appropriations are only made by the Government for the preservation of the collections in the National Museum, it is proper to refer most of the collections to the Smithsonian Institution, as the Museum is under the control of the latter. The sources of last year's receipts were as follows:

Donations; exchanges; collections by Government expeditions, required by law to be turned over to the Museum; purchases for the Fisheries Exhibition from a fund specially appropriated, and purchases from a fund of $3,000 or more, which the Secretary has been able to save from various sources for this purpose. The last named has been so judiciously applied and combined with other Government work as to have enabled the Museum to acquire most valuable collections, of which this sum spent represents but a fraction of their

real value. Various branches of the Government have contributed to this result by allowing their employés in the field to make collections for the Institution in connection with their regular work. It is to be hoped that the valuable results attained with such a small additional outlay will induce Congress to make some of the annual appropriation for the Museum also avaliable for the "*increase*" as well as the "preservation" of the collections. In fact, the Museum cannot grow in proportion to the demands of the public from the sources it now has to rely on. Those considerations which call for the existence of the Museum at all also call for a liberal fund with which to send out collectors and purchase valuable material.

The collections here considered are those entered in the catalogue during 1883. Some of the collections were actually made in previous years, but they have been stored and are now heard from for the first time.

In the organization of the National Museum, as outlined in the "Proceedings" for 1881, it is contemplated classifying the anthropological material under three departments: I, *Antiquities;* II, *Races of Men;* and III, *Arts and Industries.* The Assistant Director is Curator of the last named and Dr. Rau of the first; but otherwise the work embraced under the second department, "Races of Men," is really carried on under Arts and Industries under the general supervision of the Assistant Director.

The general routine work is as follows:

Collections, on receipt at the Museum, are acknowledged and given an accession number by the Registrar, who files under this number all manuscript accompanying the various collections. Each collection is classified or divided up and the proper portions sent to the various departments or sections, where each specimen or lot of similar specimens is entered in the ethnological catalogue and given a Museum number, which is painted on the specimen for its future identification. The entry in this catalogue is briefly made under the following heads:

Museum Number; Collector's Number; Name; Locality; When Collected; Nature of Object; Accession Number; Measurements; Received from or Collected by; Cost; When Entered; Number of Specimens; Remarks.

The descriptive cards to be printed to accompany each specimen are then written, access being had to the manuscript in the hands of the Registrar to get full data, and the collection is arranged and sent to the preparators for installation in the Museum.

ACCESSIONS FOR 1883, DEPARTMENT OF ANTIQUITIES.

Five thousand three hundred and thirty-nine specimens were received, making a total now on hand of 40,491. Three thousand five hundred and fourteen different specimens were placed on exhibition, making a total display of 24,731. The purely ethnological material is being gradually taken over to the Museum building, and soon the entire main hall of the Smithsonian building will be devoted entirely to antiquities. The great bulk of the collections in this department are in storage, and of this the material on hand for exchange is very large.

The greater part of the receipts this year are miscellaneous collections from all over the world (France, India, Alaska, Central America, and Mexico), but principally from our own country and presented by patrons of the Institution.

The principal foreign collections are as follows:

Two hundred specimens from Ometepec Island, Lake Nicaragua, by C. C. Nutting, who was sent out by the Institution. It embraces remains from graves, such as clay vessels, arrow-heads, and rude stone carvings. The collector only got these incidentally, as his principal collection was the birds of that region.

A collection from Los Novillos, Costa Rica, by M. C. Keith, embracing about 15 rude stone images or carvings of human figures. These are now mounted in the National Museum. A collection of casts from the paper moulds received from the Trocadero Museum, Paris, made by M. Charnay and presented by Mr. Lorillard to the National Museum. The collection is too familar to all to need any comment at my hands. There are about 82 reproductions of inscriptions, carvings, temples, altars, door-posts, etc., from *Palenque*, Mexico, *Merida*, Yucatan, *Chichenitza*, *Lorillard City*, and other less important places.

A small collection of about 15 specimens from Alaska, collected by McKay just before his death, which will be alluded to later. The collection embraces only a few Eskimo stone implements and carvings.

So far this year (1884) a collection has been received from J. J. McLean, of the Signal Service, from the shell heaps of Cape Mendocino, Cal., besides the usual number of miscellaneous articles donated to the Institution.

In the Department of Arts and Industries the various sections have

not as yet all been put in operation. The well-organized special sections are at present only two, *materia Medica* and *Foods and Textile Fabrics*. The fisheries section is well-organized as a sub-section, so to speak, but it will be some time yet before hunting can be taken up in connection with it.

Dr. FLINT has the *materia medica* collection well in hand. In a general way it is intended to illustrate the medicines in use in highly civilized countries at the present day, as well as the collections peculiar to certain countries. Of the latter the Museum has a small collection from Corea, one from China, and quite a complete one from India. (This India collection of course represents only native medicines.) To the collection in 1883 were added over 1,000 specimens, the addition to the general collections being supplemental —*i. e.*, intended to fill out the present exhibit of the medicines of civilized nations obtained from wholesale drug houses in this country. Quite a unique collection of mineral waters from all parts of the world is included in the latter. The additions to the special collections in 1883 may be summed up as follows:

1. About 275 specimens from the Kurrachee Museum, India.
2. Fifty specimens or more from the Madras Museum.
3. Ten specimens of Cinchona bark of different kinds from Ceylon, presented from the Government of India.
4. Seventeen specimens presented by the Corean Embassy.
5. 110 accessions from the Royal Botanical Gardens at Kew.

The Section of *Foods and Textile Fabrics* embraces more than the name implies—*i. e.*, food-stuffs, narcotics, distillations, drinks, furs and leathers, fibres, cordage, textile fabrics, needle-work, basket-work, paper, etc. Mr. Hitchcock has been in charge only since November, last. The collection of textiles now on exhibition is not a very large one, and consists mainly of the raw materials used, such as *wood*, *silk*, *cotton*, *jute*, *manilla*, *hemp*, *bark*, *grasses*, etc. In mats, cloths, etc., little has been as yet installed. The reserve collection is a large and valuable one. The Zuñians, Navajos, Indians of northwest coast, (particularly the Nootkas and Haidahs,) the South Sea Islanders, and the natives of the Phillipines, West Indies, Central America, and elsewhere are well represented, and when this collection is finally installed it will be a valuable addition to the collections on exhibition. Little attempt has as yet been made to illustrate the fabrics of civilized nations, but these are easily obtained when desired by purchase in this and other countries.

The collection of North American Indian foods, embracing over 250 specimens, is classified and on exhibition. The descriptive cards are in the hands of the printer. There are small classified collections of foods from China, India, and other countries, but the miscellaneous collection has not as yet been classified. In representing the foods of civilized nations, specimens can be obtained very readily when desired. At present the principal collection of such foods is one prepared for the Fisheries Exhibition. It will form a part of that exhibit in the Museum, as only a few representative specimens will be kept out to go with the food collection proper.

The large collections of the Bureau of Ethnology from Zuñi and the Moquis and New Mexican pueblos were, last November, turned over to the National Museum for installation. On the publication of notes by the Bureau, and on the return of Mr. Cushing from Zuñi, these collections will be written up. Not enough is known of the ceremonial material to attempt such a thing at present. The collection of pottery is simply exhaustive. It is now in the hands of Mr. Holmes, as is the entire pottery collection of North America. Incidentally, it may be mentioned here that a fine collection of pottery was also received from Chiriqui, and is now installed with the North American pottery.

The general Zuñi and Moqui collections comprise 6,370 entries for 1883, but as three or four specimens are sometimes entered under one number, this does not approximate to its real size. It embraces basket ware, pottery, gourds, grinding stones or mortars, weapons, and ceremonial, household, agricultural, and industrial implements.

A large portion of the archæological collections of the Bureau of Ethnology from the mounds of the United States was also turned over to the Department of Antiquities some months since. No mention of these specimens was made under that head. Prof. Cyrus Thomas has worked up these collections, and the results are published under the Bureau of Ethnology. Collections have been made under the Bureau, throughout all the important localities from Dakota Territory to Florida, and from Nevada to the New England States. These collections of aboriginal remains embrace skulls, bones, celts, fragments of pottery, and walls of dwellings, shells, copper and iron implements, flints, flakes, pipes, arrow-heads, perforated tablets, stone discs, ceremonial stones, etc.

The entries for 1883 comprise 3,544 numbers, which is much more

than the accessions of the Department of Antiquities itself, when we consider that several of specimens are entered sometimes under one number. Four specimens of quartz celts from near Madras, India, are among the accessions from the Bureau.

Among the most important collections made by employés of the Government, in connection with their regular work under other branches, and which were paid for out of the fund previously alluded to, may be mentioned:

A collection from Wm. J. Fisher, the Coast Survey tidal observer on Kadiak Island, Alaska, who made several trips on the peninsula and mainland. It embraces about 100 specimens, the most interesting being some heavy elaborate bead-work head-dresses, some of them weighing as much as 2¼ pounds.

The collections made by the United States Signal Service observers are as follows:

1. One, by C. L. McKay, from in and around Bristol Bay, north of the Alaska peninsula, from the Nushagag-mut and Ogulmut Eskimos of that region, about 45 specimens in all, including a full outfit for a Beluga whale-hunter. which was exhibited in London last year. This outfit includes harpoons, lines, buoys, extra heads, killing lances, etc. A second collection of about 50 or 60 specimens, consisting of household utensils and articles of personal adornment, were received after the death of McKay. He was drowned in April, 1883, while out in a *kyak* in Nushagak river in bad weather.

2. One, by J. J. McLean, from around Sitka, which had been pretty well worked up by other collectors. Besides the usual lot of wooden carvings, kantags, or wooden dishes, etc., there are some fine specimens of native wicker and basket work in the collection (made from a species of plant, *Iris tenax*).

3. A *kyak*, with complete fittings, from Greenland, deposited by the chief signal officer of the army. (It was exhibited in London.)

4. The Point Barrow collection, which was brought down when the expedition returned recently. The collection is a good one, and embraces over 700 specimens. Mr. Murdock is now working up the collection, and I will not anticipate his report. Part of the earlier collection which came down on the "Corwin" went to London to the Fisheries Exhibit.

5. Mr. Stejneger, of the Signal Service, made a small collection from the Aleuts on Behring Island, Commander group (off the coast

of Kamschatka). There are some interesting models of fox and bear traps and boats, some seal-skin costumes worn in their native dances, besides some accessories of costumes peculiar to the Aleuts.

6. A collection coming more properly under 1884 was received several weeks since from L. M. Turner, of the Signal Service, from the Eskimos of Ungava Bay, North Labrador. It is a fine one and embraces over 450 specimens. The articles have not the oily, used look that most Eskimo implements have, which indicates that other collectors have been among them recently, although a great many specimens are models of traps, snow-shoes, tobogans, and spears, and are necessarily new. There are some large tobogans and snow-shoes of a peculiar pattern that will be alluded to below. The costumes are peculiarly handsome, and show the effects of contact with civilization.

A second collection from Fisher, made in the Aleutian Archipelago and Alaska Peninsula, has just been received. It consists of about 120 specimens of costumes, peculiar Aleutian hats, household utensils, accessories of costume, etc.

Among the small purchased collections may be mentioned: A Zuñi sacred blanket, one hundred Peruvian water-bottles or huacas, and some shoes, hats, dishes, baskets, etc., (from the La Costa Indians of South California,) woven of mescal fibre and palm-leaves.

1. Among the principal donations are 40 musical instruments, supplemental to the set of American musical instruments, all presented by M. J. Howard Foote, of 31 Maiden Lane, New York.

2. The original Catlin collection of Indian portraits, etc., painted by him during his eight years amongst the 48 tribes, of which he has handed down to us these most valuable ethnological records. There are about 500 in the collection which Mrs. Harrison, of Philadelphia, has so generously presented to the Institution. One hundred and fifty have been selected and placed on exhibition in the lecture room of the National Museum, and arrangements are being made to increase the exhibit. The selection now exhibited is one from each small tribe, two or more from the important tribes, and a set illustrating hunting scenes, ceremonial dances, etc.

3. At the close of the Boston Exhibition recently some 50 musical instruments, numerous clay figures, and various other specimens were presented to the Institution by Surindro Mohun Tagore, Rajah of one of the provinces of India and president of the Bengal music school. The collection of musical instruments is accompanied by

full notes, and the Museum is taking steps to obtain a supplemental collection to complete the series. This collection was installed a few days since and is now on exhibition.

Among the principal *exchange* collections are:

1st. Some miscellaneous weapons from Polynesia and South America, obtained at the Fisheries Exhibition.

2d. Some 16 musical instruments and accessories from Tiflis, in the Caucasus, obtained through Mr. Engleman, of St. Louis.

3d. About 40 specimens from the Leipzig Museum, consisting of knives, bows, arrows, baskets, mats, etc., from Africa, particularly the Loango Coast and Gaboon river, on the west coast. The admirable native steel implements are well illustrated. This collection, combined with a few stray or miscellaneous articles and a small collection by Rev. Dr. Gurley, constitutes but a meagre African ethnological exhibit.

The Museum has just sent to the Trocadero, at Paris, an ethnological collection selected from the material in its possession, and doubtless their exchange will embrace some additions to the above.

Mr. J. G. SWAN, in addition to the regular collection which he sends in from time to time, made last summer a special trip for the Smithsonian Institution to the Queen Charlotte Islands, B. C., and the results have just been received.

In the early part of the year he sent in some photographs and about 100 specimens supplemental to his series of collections illustrating the fisheries of the Indians in and around Cape Flattery, W. I. (The complete collections went to London.)

In the trip referred to above he started from Masset Sound (N. Graham Island) and coasted around the west side, then through Skidegate Channel to the southeast coast; then home to Victoria. Now that he has partially carried out his long-cherished desire, it is to be hoped that his forthcoming notes will prove as valuable as his notes previously published. A better knowledge of the *Haidah* totems and totemic carvings is desired. The collection is rich in masks, wood-carvings, ladles, ancient stone implements, ropes, clubs, shaman's wands, ceremonial bows, whistles, rattles, fishing gear, etc., but particularly so in the slate carvings, of which he sends 30 specimens—dishes, boxes, and models of totem posts. There was already on hand a sufficient number of specimens to illustrate the *Haidah* wood carvings and working in silver, but the additions to the slate carvings have made it appear desirable to install the

latter as a monographic collection illustrating this art, which alone places the *Haidahs* at the head of the Indians of the northwest coast.

A comparison and study of all the carvings from the *Haidahs* is to be made, as it is difficult for the uninitiated to make out or distinguish between the conventional representation of animals. The *Haidah* totemism and mythology offer a most promising field to investigators.

Mr. Swan is anxious to make another trip, during the coming season, a attend to great celebration to be held in the fall. The Director has the matter now under consideration.

The Fisheries Exhibit, having returned from London, is now turned over to the Museum, and will form a monographic collection. The Makah Exhibit, collected by Swan, and the Eskimo, whale, seal, and walrus hunting outfits are peculiarly interesting to anthropologists.

In the matter of exchange, the Museum has recently sent to the Trocadero, at Paris, a small collection of models of ruins and cliff-dwellings, ethnological material from *Zuñi*, *Moqui*, and our Western Indians. The Museum has available for exchange a great deal of material from the collections of the Bureau of Ethnology and the northwest coast and Alaska collections.

In the matter of collecting every year increases the value of ethnological material. When Congress shall wake up to the necessities of making more liberal appropriations it will be found that it has been false economy to delay in the matter. A few thousand dollars now will represent a much greater outlay in future years.

The outlook for anthropological collections for 1884 is not so encouraging. Fisher, McLean, and Swan will be the main sources. No one has yet taken McKay's place, and Nelson has permanently withdrawn. Greely's party must have abandoned their collections North, and the present relief expedition can hardly accomplish much. Foulk and Bernadon may be heard from in Corea.

As stated originally the year 1883 has been a prosperous one for the Smithsonian and National Museum.

REMARKS ON THE CLASSIFICATION AND ORGANIZATION OF THE NATIONAL MUSEUM.

As a rule the earlier collections have lost much of their value, both from the want of care in preserving the accompanying data,

and from the absolute neglect of the collectors to forward any. A little preliminary experience of collectors in the Museum, before going into the field, would impress it forcibly on the minds of such that the descriptive cards should be practically written by the collectors in the field. Nelson and Swan have shown the best realization of this principle. The general form of the descriptive card adapted to the Museum, to accompany each specimen exhibited, is as follows:

> Object, (local or native name).------Materials of which made; brief description; use. *Tribe* or person by which used.
>
> Dimensions, length,------, breadth,------, etc.
>
> Exact locality, 18—, (date of collection). Museum number.
>
> How and through whom acquired.
>
> Fuller and more special notes in smaller type are appended as to origin, special variation in form and use in various localities, notes on the general series of which the specimen is a representative.

Each object or general series of objects is to be accompanied by such a label or card further supplemented by pictures or photographs when necessary to more clearly illustrate how the object is used or worn, or to show pattern where the object is folded or obscured. The cards are printed on herbarium board. Those on white paper are to send to other museums, preserve as records, and for use in making up the catalogues which will eventually be published.

(Ed.: Specimens were here exhibited of cards and photographs taken from specimens already on exhibition in the Museum.)

Cards are now being attached to the specimens already out, and a plan is under way to collect all the ethnological material not yet installed in one large store-room, where it is to be systematically classified. The incoming collections can be distributed according to the plan adopted, and duplicates can be selected before this temporary storage. This plan will greatly facilitate the routine work.

Greater progress has not been made in installing and describing the specimens and collections for many reasons, but principally on account of the various exhibits prepared at the Museum. which have diverted a large part of the force from the regular work, and besides this experiments are being made as to cases and styles of mounting

general and special exhibits. Moreover the force employed is not very large, but when the Fish Exhibit is permanently installed there will be more men available for the routine work.

Recently published criticisms on the classification and method of arrangement now provisionally adopted in the Museum have shown to a certain extent that there is a misunderstanding as to just how far the Museum is committed to any definite plan. The adopted unit box, in which specimens from the same locality are mounted for exhibition, enables a provisional classification to be adopted. The boxes slide in and out of the cases, and the whole character of the present arrangement can be altered and radically changed in a day. By putting only a few specimens in each box, room is left for future collections supplemental to those now installed.

Classification and method of installation depend upon various considerations. The material on hand determines the former, and experiment and trial the latter. Without going at all into the subject of Museum classifications in general, or into the *future* arrangement of the National Museum, it seems that every immediate consideration demands something like the present one, however much it may be understood or misunderstood from the published bulletins to that effect.

The broad aim of the present plan is a teleologic classification, one by *use* rather than by morphology. The comparative method has been adopted in preference to the ethnographic because it is demanded by the nature of the material on hand, and to a certain extent better suited to the American mental habit of analysis and comparison. I will try and illustrate these points by special examples.

For a few tribes and regions an ethnographical arrangement would answer admirably, viz., the *Eskimos*, *Zuñians*, *Moquis*, *Haidahs*, *Makahs*, and our *Western* and *Alaska* Indians, but such a general plan would be absurd and but show up the meagerness of our collections from every other region. Picture Corea with two small trays of stuff that can but be vaguely referred to it, Africa with three, and South America with only several cases! Even our Japanese, Chinese, and Indian collections would hardly admit of such an arrangement. Should Congress become suddenly liberal and place a fund at the disposition of the Museum to enable it to send out intelligent collectors well informed as to the Museum's wants it would doubtless occur in the course of time that an ethnographic

arrangement would be demanded as the only natural one (supplemented of course by occasional and separate comparative collections.) With the miscellaneous collections that are likely to come in, however, unless Congress does make special appropriations, the present arrangement is likely to be found the best one. A thorough and exhaustive ethnographic collection would show each product of a country's civilization in the different stages of its evolution and development, but with a miscellaneous and scattered collection we must draw on various countries to illustrate this development.

A recent article on museum classification says "The comparative method necessarily cuts across the natural order of things in their relations to time; and this is an obvious defect, which, when applied to anthropological collections, is destructive of all natural conceptions as to the way in which modifications and changes really arise or flow out of pre-existing, localized, or racial conditions." It seems to me, as far as I may express any opinion on the subject, that the question tends to settle itself thus.

With exhaustive collections from representative tribes and with sufficient funds to fill out or supplement the collections the ethnographic plan is the most desirable one.

With scattered and miscellaneous collections the comparative method makes the best use of the material.

The Museum plan is an improvement on each of the above, as it combines the advantages of both. The classification provisionally adopted is a teleologic one, subject to special modifications to suit special cases. To illustrate this:

In the Museum there is a collection of pipes from all parts of the world. The Haidah carved black slate pipe stands out as unique, and it might seem that the fault in this comparative method of arrangement is that it does not form a fair comparison of the intelligence or artistic tastes and abilities of the various tribes represented. It might be argued that possibly the pipe was the only thing they could carve or do carve. An ethnographic collection from this people would show that they carve equally surprisingly in wood, bone, etc., and have a great deal of artistic taste. The Museum, recognizing this, makes a separate monographic collection and exhibition of Haidah carvings, so we have one or two Haidah pipes in the pipe collection, and, besides this, one or two in a monographic collection of Haidah carvings.

It is aimed in all cases where such arrangement may seem to be

desired to thus draw off certain small ethnographic and monographic collections to call attention to any instructive peculiarities of any tribe or race. It also happens at times that large objects have to be left out of a comparative collection. In fact, any classification must be based on compromise and must yield to exceptions.

As an illustration of how we may show the development or evolution of any object with a widely scattered collection let us take the snow-shoe collection in the Museum. It is mounted on screens in the comparative style. If we had exhaustive collections from any one stock of Indians, say, we might show this development step by step (by the ethnographic method) from the time they borrowed or originated the idea up to its highest development, as shown. With the material at the Museum this evolution can only be suggested, as the steps are very wide, and intermediate ones are not at hand. We must in this adopt Mr. Spencer's plan of illustrating primitive man by our present savage tribe.

DISCUSSION.

Prof. MASON called attention to the advantages derived from a systematic classification and arrangement of the material in great collections like that of the Smithsonian Institution. He also said that an organized effort should be made looking toward a full utilization of the many resources afforded by the various departments of the Government for information valuable to the student of anthropology, and that the attention of the scientific world should be directed to the scope and character of these resources.

Mr. FLINT spoke of the manner in which aboriginal ideas had been followed up and finally developed, illustrating his remarks by showing how a study of the possibilities of the arrow as a projectile had resulted in its use for throwing explosives under a heavy air pressure, for which several patents have already issued.

EIGHTIETH REGULAR MEETING, March 15, 1884.

Major J. W. POWELL, President, in the Chair.

The Secretary of the Council reported the election of the following-named gentlemen as corresponding members of the Society:

CHARLES C. ABBOTT, Trenton, N. J.
HENRY B. ADAMS, Baltimore, Md.

Rev. JOSEPH ANDERSON, Waterbury, Conn.
Mr. H. H. BANCROFT, San Francisco, Cal.
Mr. AD. F. BANDELIER, San Francisco, Cal.
Dr. DANIEL G. BRINTON, Philadelphia, Pa.
Mr. LUCIEN CARR, Cambridge, Mass.
Mr. JOHN COLLETT, Indianapolis, Indiana.
Mr. A. J. CONANT, St. Louis, Mo.
Dr. GEORGE J. ENGELMANN, St. Louis, Mo.
Prof. BASIL GILDERSLEEVE, Baltimore, Md.
Mr. HORATIO HALE, Clinton, Ontario, Canada.
Prof. G. STANLEY HALL, Baltimore, Md.
Col. H. H. HILDER, St. Louis, Mo.
Dr. C. C. JONES, Augusta, Ga.
Rev. GEORGE A. LEAKIN, Baltimore, Md.
Prof. E. S. MORSE, Salem, Mass.
Prof. RAPHAEL PUMPELLY, Newport, R. I.
Prof. F. W. PUTNAM, Cambridge, Mass.
Col. CHARLES WHITTLESEY, Cleveland, O.
Dr. DANIEL WILSON, Toronto, Canada.

Mr. H. H. BATES read a paper entitled "DISCONTINUITIES IN NATURE'S METHODS," of which the following is a synopsis:

The ingenious analogy drawn by Mr. Babbage, in the ninth Bridgewater treatise, from the operations of his calculating machine, to enforce an argument in favor of the conceivability of miracle, by bringing it under the domain of law, was cited as illustrating some of the discontinuities of evolution, confessedly the result of similar complexities of natural law.

The great discontinuity involved in the passage from inorganic to organic life, which we infer to have taken place under law, but do not understand, was adverted to. Also such apparent discontinuities as the passage from invertebrate to vertebrate life, or the introduction of mammalian life, from lower forms, with the observation that wherever nature seems to have carried specialization to its full extent and to have exhausted the possibilities of structure by mere differentiation she is found to have laid the foundation for a new differentiation, and a new specialization, with higher possibilities, from a different stem low down in the scale, constituting an apparent discontinuity, on account of the obscurity and feebleness and instability of the first unspecialized departures, by which they

were either unobserved or early obliterated through the operation of competition.

Passing over the wide domain of biology, which affords so many instances of this complexity of natural action, illustrations of the same law were sought in the domain of anthropology. The advent of man, and his means of progress, affords such examples. The development of the inventive faculty, as the distinguishing characteristic of mind, caused a modification of the old plan of progress by natural selection. Instead of being himself modified by nature, as hitherto, man began to act upon nature, both organic and inorganic, and to modify it to his needs, as Mr. Ward has pointed out. Henceforth natural selection affected only mental and ethnic qualities, through modification of nervous structure. Physical modification ceased to any important extent. Instead of developing weapons, man constructed extraneous ones for his use. With these he conquered competition and removed the rivals most cognate to himself. Militarism ensued, and resulted in high specialization.

Differentiation, however, soon reaches its highest results in this direction, and obstructs further progress. An apparent discontinuity occurred in the rise of industrialism out of the humbler elements of society, through the germination of inventions, beginning with the rediscovery of gunpowder, which was the commencement of the downfall of militarism. The tool-making and tool-using faculty came into prominence. Peaceful arts began to flourish, man's condition became ameliorated, and a new progress supervened. The new direction of evolutionary development was adverted to. Man, having ceased to evolve by physical selection, evolves by extraneous organs. Weapons and tools were the beginning of these, but he has also now enormously developed his means of locomotion, as well as his organs of special sense and expression. His eye is reinforced by the telescope and microscope and any optical device he needs; his ear by the telephone. The products of artistic industry furnish him with means for unlimited gratification of the æsthetic faculty in decoration. The culinary art relieves him from some of the burdens of digestion and increases his range of nutriment. All these extraneous means constitute a departure from the old law of development of the individual by selection.

Moral and ethical development have not made a parallel advance

since the dawn of history, on account of the lack hitherto of any discovery in that field commensurate with the important discoveries which modified his intellectual progress. Such a discovery would afford, by its results, an instance of a true and beneficent discontinuity. The necessity has always been recognized, and many theories broached which accomplished great temporary results, but failed of permanent fruit for want of confirmation.

The operation of discontinuities in the complex law of evolution is not always or necessarily beneficent. Nature is not optimistic, and discontinuties are known to have occurred which were disastrous and retrograde, as geological history evinces. Dissolution is involved in evolution.

DISCUSSION.

Mr. Lester F. Ward said that he welcomed the term "discontinuity" in this new sense as supplying a need in biology. Its old use to denote actual breaks in the series and the special creation and fixity of species was no longer believed to express a scientific truth. But a special term was needed to designate certain apparent breaks which occur at irregular intervals both in the development of life and of society. Among these he enumerated the origin of life through the introduction of the substance protoplasm, the comparatively abrupt appearance of vertebrated animals which seem to have been developed from one of the lowest forms of invertebrate life, the equally radical change which resulted in the mammalian type, and the remarkable "short cut" by which man was reached through the lemurian and simian stem, leaving the other great branches, the *carnivora*, *ungulata*, etc., entirely out of his path. He had, in a paper read at a previous meeting, laid special stress upon the similarly sudden introduction of the developed brain of man, with its momentous consequences, as the first and greatest of this series of anthropic and sociologic strides to which Mr. Bates' paper was chiefly devoted.

In reply to remarks by Dr. Welling and Prof. Thomas inquiring how this kind of discontinuity was to be distinguished from the actual breaks postulated by the old school of biologists, Mr. Ward said that the reconciliation was effected through a recognition of the now well-established law of the ephemeral character of transition forms. The variations of structure which are destined to result in the dominant type take place at a point low down in the

scale. The first modified forms are few and feeble and leave no permanent record of their existence. The modifications required to give them a firm foothold take place with rapidity and the intermediate gradations are lost. The first evidence the investigator has that a new departure has taken place is the appearance of the more or less completely modified type, and it seems as though there had been a fresh act of creation, or *saltus*.

President WELLING said he would like to have Mr. Bates explain the precise sense in which he used the term "discontinuity" before conceding its necessity as an addition to scientific nomenclature. Without such explanation it would perhaps be held by many that the facts and principles recited in the essay were sufficiently covered by that law of succession, differentiation, and integration which the reflective mind of man had spelled out from the ongoings of nature. In these ongoings there had been constant *discontinuations* as well of processes as of products, but no *discontinuity*. If any actual discontinuity must be admitted then the whole doctrine of evolution, as commonly conceived, must fall to the ground, for that doctrine proceeds on the assumption of perpetual continuity amid perpetual discontinuations in natural processes. These perpetual discontinuations do but mark out the line of continuity along which nature has worked in the normal movement and projection of her processes and products. Discontinuations are matters of fact, but the principle which colligates them is *continuity*, not *discontinuity*.

In illustration of this point of view Mr. Welling then cited that latest and most stupendous evolution of man in society, known as international law. This law was built on the perpetual discontinuation of customs, practices, and institutions dating from the most primitive forms of social organization down to the present time, but none the less had it been built without the slightest lesion of continuity in the process of its evolution, for each successive differentiation in social and national relations had only paved the way for a new integration in thought and action.

Prof. THOMAS said that he agreed with Mr. Bates and Prof. Ward in believing that the term "discontinuity" was properly applied in speaking of some of the processes of nature. In following up the line of progress in the development of animal life we observed branches shooting out on either hand. For illustration, in passing from the higher Annuloida, Huxley's Scolecida, we are led by one

line, the Annulosa, to the Arthropoda, culminating in the higher insects. Here this branch appears to cease and is wholly separated from any of the higher forms of animal life. Here Prof. Thomas believed was a true discontinuity.

On the other hand, starting near the same point, was another branch embracing the mollusca.

The great vertebrate line, instead of originating from any of the higher forms of either of these branches, was supposed to arise directly or through a few transitional forms out of the Tunicata, the ascidian form.

There are many diverging branches, and as it appeared to be a law that no diverging line ever returned to the main stem or coalesced with another there must be discontinuities. No evolutionist can admit that there are any absolute gaps or breaks in the line of development, as this would be fatal to his theory. The line must be continous or the theory must fall to the ground.

Mr. MASON said that phenomena might be associated in such groups as to be habitually observed together. Now, the mind being turned for a while toward one part of a group, returns to find a great change. There has been a discontinuity. Let us further illustrate. If we were studying Indian pottery, we should want to investigate the material, the implements, the agent, the process, the finished product, and the design, or final cause. Here are six sets of entirely different observations, the discontinuance of any one of which would produce an apparent discontinuity in the final result. The material might give out; it might be replaced by other material; new tools might be invented or imparted. The change of social order might throw the industry into other hands, as for instance, potters might become men instead of women The introduction of varied processes, the multiplication of functions by the increase of wants would bring about the same result. The disconnections are apparent therefore, they are not real. In short, discontinuity anywhere either in natural or social phenomena is impossible.

EIGHTY-FIRST REGULAR MEETING, April 1, 1884.

Dr. ROBERT FLETCHER, Vice-President, in the Chair.

The Secretary of the Council announced the election of the following members:

Prince Roland Bonaparte, St. Cloud, France; Prof. A. Ponia-lovsky, Sec. Imperial Russian Archæol. Soc., St. Petersburg; Dr. Enrico Giglioli, V.-Pres., Anthropological Soc., Florence, Italy; Prof. Johannes Ranke, Editor Correspondenz-Blatt, German Anthropological Soc., and Sec. Anthropological Soc., Munich.

A paper entitled "RECENT INDIAN GRAVES IN KANSAS," prepared by Dr. ALTON H. THOMPSON, of Topeka, Kansas, was read by Colonel SEELY.

ABSTRACT.

The writer in 1879 assisted in the examination of four graves in an old burial ground connected with the mission to the Pottawotomies, six miles west of Topeka. The ground appears to have been the site of a former Indian village, believed by some to have been occupied by Crows. Careful inquiry, however, makes the identity of these people with that tribe very doubtful. Three of the graves were accurately oriented, the fourth being much inclined, as if made when the sun was at its northern limit. Besides the bones the first grave yielded quite a number of metal ornaments, consisting of disks of rolled silver with stamped perforations and incised ornamentation, small silver buckles, and pieces of chains like cheap brass watch-chains, all evidently of white manufacture. The traders say that it was formerly common to receive designs from the Indians, from which ornaments were made and furnished to those who had ordered them. Sometimes they also procured sheets of brass and silver, which they worked according to their fancy. Silver coins, particularly the old Spanish dollars, were often beaten out by the Indians into disks, and ornamented.

The condition of the remains in the first grave indicated it to be much more ancient than the others. No trace of clothing or of any enclosure for the body appeared. In the second, a fracture in the skull showed that the person had probably met death by violence.

The body had been enclosed in a hollow log or in bark. In this, and in the third and fourth graves, leather leggins, blankets of white manufacture, and a silk handkerchief were found, all much decomposed.

The skulls were all of true Indian type. The writer proposes to continue his researches in this interesting locality.

DISCUSSION.

Prof. THOMAS said that the paper was valuable as tending to throw light on the subject of intrusive burial and mentioned in connection therewith some recent finds in Wisconsin.

Mr. PROUDFET said that he had obtained from an Indian grave in Southwestern Iowa silver disks similar to those mentioned by Dr. Thompson.

Dr. FLETCHER, referring to the flattening noticed in certain skulls exhumed by Dr. Thompson, expressed the belief that such condition was probably not due to pressure in burial.

Colonel SEELY said that from what we now know it is evident that the savage was far more than a straggler in the wilderness. The remains of various ritualistic systems suggests a more elaborate conception in such matters than is consistent with notions previously entertained concerning the savage state. As illustrating this line of inquiry Col. Seely read an extract from the Gippsland Mercury, for January, 1884, giving an account of certain aboriginal ceremonies witnessed by A. W. Howitt on the occasion of admission of the youths of the Kurnai tribe to the dignity of manhood.

EIGHTY-SECOND REGULAR MEETING, April 15, 1884.

Major J. W. POWELL, President, in the Chair.

The Curator reported the following gift: Final report of the Anthropometric Committee of the British Association.

A vote of thanks was passed to the donors.

Dr. J. M. GREGORY read a paper on the "ELEMENTS OF MODERN CIVILIZATION."

Civilization is the supreme fact in sociology. It is the comprehensive name of all that marks progress and well-being in society and states. It is also the highest criterion by which to test the value of social institutions. Whatever promotes civilization we pronounce good and useful; whatever abases or destroys it is bad.

What is civilization? What are the essential elements of which it is composed, and by which it may be described? These are questions which confront the student of sociology at the outset of his studies.

To answer these questions properly drives us to a deeper analysis; it raises the profounder question, Is civilization external or internal? Is it in the man, or in his surroundings? In the general way, most of us will admit that it is in the man—in man and in society.

Settling down then upon the clear truth that civilization is essentially internal, that it is of the mental man, though working outwardly into necessary forms and movements, another question starts up to confront us. This question is as to the proper method and direction of our search. Shall we call to our aid our own conscious experience, and look to find what there is in man that impels him to outward action; or shall we neglect the mental forces and direct our study to external facts to ascertain their character, classes, and connections?

If we decide to confine our quest to the material and visible facts of social life, shall it be to the present or the past; shall we grope among the fossil remains of a paleozoic sociology, or shall we seek to analzye the phenomena of a living sociology?

No science can dispense with the study of the past, and all true students must acknowledge the usefulness as well as the curious interest which attaches to the discoveries of the archæologist and paleontologist, but Herbert Spencer says "it is hopeless to trace back the external factors of social phenomena to anything like their first forms."

We may without debate accept the doctrine of an evolution in civilization. All history implies development, or evolution, if the term is preferred. It exhibits the emergence of the new out of the old, the complex from the simple, the tribe from the family, the nation from the tribe, the civilized from the savage. But the evolution of society is not, as some represent it, a mere physical or biotic evolution. It is anthropic, and more, it is spiritual and volitional. Human passions, intellections, and volitions must be admitted as evolving forces.

The under estimate of the value of consciousness as a source of definite knowledge, and the over estimate of the value of the archaic and savage social forms are both serious mistakes of social science.

History rises out of the physical and the mechanical, and becomes human only by the introduction of the human intelligence among its causes and forces; and to refuse the aid of consciousness in the study and interpretation of history is to place it among the physical

sciences of geology and astronomy, or at best to rank it with the biological studies of botany and geology. Some have already taken this ground, driven, as they affirm, by the stern logic of observed facts. Sentient being appears to them as one of the phases of evolution of physical nature, and subject to the same laws as other physical phenomena. Such a theory may seem delightfully simple, but it is fearfully suicidal, since it hopelessly invalidates all the acts of thought and intelligence by which this or any other truth can be known.

Doubtless sociology and civilization have their laws of evolution as potential if not also as clear as those of the physical sciences; and these laws may be studied in the savage and archaic stages of society as well as in the more recent and more complex. Sometimes a law will be seen even more clearly in the earlier and simpler stages of evolution; but the higher evolution ordinarily involves forms and functions wholly unknown to the lower; and the complex modern civilization exhibits classes of phenomena of which the savage gives no hint or promise, or gives it only in so rudimental a form as to be unrecognizable, except in the light of fuller development.

If now, we accept the conclusions that civilization is essentially internal, that its external phenomena are the necessary outcome of the nature of man and of society; if we further agree that our study of civilization must begin with it as it exists, here and now; if we accept as a guiding truth that there is nothing in the essential nature and attributes of man which does not find its expression in history, and that there is nothing essential in history which does not find its root and explanation in the nature of man, then our search for the elements of civilization narrows its field to a study of those common and universal principles, or instinctive activities, in the human being which work outwardly into the facts and usages of society, meeting and modified as they must be by environment; or, to state the same thing objectively, it is to select, classify, and study all common universal social phenomena in the light of our conscious instincts, needs, and activities. In physics we ascend from effects to causes; from phenomena to forces; in sociology the cause is a conscious one and we may safely descend from force to phenomena.

Our method being explained and defended, we march to results.

I.

The commonest fact of human consciousness is the existence of the vital wants, hunger, thirst, and the desire of proper warmth. These act as a steady force compelling men to the efforts to secure their gratification. Out of these powerful and persistent appetites, spring through the slow round of the ages, what we call the useful arts, the food-producing, the cloth or clothes-making and the building arts; and ancillary to these, the arts of the tool-maker and machinist, and of those who collect, prepare or transport materials for the others. As the satisfaction of these wants is the vital condition of human existence, so these arts are the broadest fundamental element of external civilization. They uphold and help on all the others; and their advancement at once measures and promotes the social progress of which they are most prominent factors.

The vital wants of mankind are at first merely animal, and are as simple as they are savage; but they steadily multiply, diversify, and refine with every advance in man's intellectual and social development, till they mingle and interlock with all the higher desires and artistic tastes of civilized men. Keeping pace with these, the rude efforts, scarcely to be called arts, which supply the low needs of the wild man, divide and differentiate into all the innumerable industries of the highest sociologic condition. Thus the craving of a present hunger which drives the savage to the chase widens out into the prudent care for all future hungers, and the food-producing arts grow with the variations of soil and climate into the enormous reach of agricultural industries and the hundred commercial, manufacturing, chemic, and cooking arts till farms, forests, orchards, gardens, and breeding waters, with mills, and manufactories, cover the continents with their costly array to satisfy the needs of civilized society.

So also the shivering desire for shelter and clothing which the savage satisfies with the tanned skin of his game, and with the cave, hut, wigwam or tent, grows into that broad economy with builds houses, palaces, and cities, and evolves the great family of building arts which occupy and enrich so many thousands of mankind.

But however vast and varied these useful arts they all look back to the vital wants as their source and spring; and as these wants are persistent, and press always with resistless force, the resulting phenomena must constitute a universal and essential element in all civilizations.

II.

Next the vital wants, as a sociologic force, may be counted the group of social instincts. The sexual appetite which perpetuates the race and furnishes the basis of the family, the most natural and most persistent form of social organization, stands foremost of these, but it does not stand alone. Working with it is the love of offspring, and next to this comes that desire of companionship which we may call the social instinct proper.

To the student of modern civilization it matters little by what long evolutions these instincts gathered their present form and force; they impel men to live in communities and support the complex structure of society. Acting among men in the savage state, they gather them into tribes with scarcely more of organization than the cattle that feed in herds or the birds that fly in flocks. But developing with the advance of mankind in intelligence, by a process similar to that noticed in the useful arts, they finally produce highly organized society and states, with all their array of social and political interests and institutions.

The social instinct is strengthened as men find that society affords additional safety against enemies and widens the field of their arts and co-operations. Self-interest acts in the same direction as the social feelıng and doubles its effects; but we may doubt whether these selfish advantages of safety and profit sufficiently account for the existence and power of the social instinct.

I have grouped together the three facts of the sexual, the parental, and the proper social desires; but each of these gives also its own peculiar results in our civilization. Out of the sexual desire grow all marriage institutions, and as the human species seem naturally to associate in pairs, all abnormal institutions, like polygamy and polyandrya, must result not from natural instinct but from some necessities of savage society. The strong feeling in favor of the monogamous family shows that the native disposition of mankind is towards pairs and not towards herds.

The sexual instinct would give simply a married pair; the offspring instinct builds the permanent family. The love of offspring is a sort of extension of self-love—the widening and perpetuation of name and of personal power and possessions. It thus tends to the creation of aristocracies and dynasties.

The social instinct added causes the family to become persistent

and widens it out into the patriarchate and tribe—the earliest and simplest forms of political society.

Victor Cousin puts the sense of justice as the foundation principle of the state; but justice is simply regulative, and serves only for the organization and maintenance of a society already existent. It builds a government to protect those whom the social instincts have drawn together.

III.

Next to the vital wants, proceeding in the natural order, should come, perhaps, the æsthetic tastes—the love of the beautiful and of whatever inspires the higher emotions. The universality of the æsthetic feeling is proved by the fact that it is found in early childhood and among savages as well as among the mature and the civilized. Out of these tastes come the fine and decorative arts, sculpture, painting, architecture, landscape gardening, music and poetry, and all the ornamentation of dress or abode, with the graceful forms and bright coloring which men give to the commonest implements of life. Public amusements, in nearly all their forms, are but an appeal to some æsthetic principle, and what are known as the refinements of civilization are but applications of the same principle. As an element of civilization it is constant and often commanding, giving its chief coloring to some of the most noted civilizations of the world.

IV.

Advancing another step in our search we find in man, as a native instinct, the love of knowledge or love of truth. It is the intellectual appetite. It is shown in the tireless curiosity of childhood and savages, and in the universal tendency of mankind to seek the causes of phenomena.

Out of this intellectual appetite springs another group of facts in civilization—such as science, philosophy, literature, education, and language itself.

Whatever may have been the genesis of this power of thought, or the steps in its evolution, it is one of the largest forces in civilization, and it rises by a natural gravitation to the summit and dominates and directs all others. It is by the aid of his intelligence that man emerges from savagery, and achieves civilization. With the birth of science, all arts, useful and fine, and all institutions, social and political, take on new forms and rise to higher power?

V.

There remains in man another power or instinct which works out historical results, and is one of the elementary forces in civilization. It is the religious nature or faculty—that power within which pushes man to a recognition and worship of the divine. Efforts have been made to find the origin of this feeling in man in the reverence for great men, or in the superstitious fear of the powers of nature; but our inquiry is not with the origin of the faculty. We find it in its full grown state, and gathered around it we find the various institutions of religion, the schemes of faith and of morals, and coming from these, the most important and influential body of usages and opinions known to civilization. Whatever philosophers and men of science may think of this element in civilization, few have the audacity to propose its overthrow without an effort to replace it with some substitute which may give to society the moral support and regulation that religion affords.

This enumeration of the elements of modern civilization is exhaustive. Under one or another of these five fundamental facts all constant phenomena of civilization may be classed. In no civilization are they absent, though they enter into different civilizations not only in different forms but also in different degrees of strength and domination.

Some of the results of these five primal factors become in time prominent forces or factors in civilization. Thus the wealth which comes from the arts becomes in turn a great economic power; and the governments which arise out of the social needs end by becoming social forces of enormous strength. So to the external influences which press upon social growths—the physical environments and the political distributions and organizations to which they give rise, may easily be taken for new and independent factors, they are at most only secondary and modifying forces and not true original elements, at least in the restricted study of civilization as it presents itself in historic time.

DISCUSSION.

Mr. WARD remarked that he had long ago felt the need of a fresh method for the study of social science. The current method dealt with the facts objectively considered, whereas a truly scientific method must discover and recognize the *forces* by which social

phenomena are operated, just as all true physical science concerns itself with physical forces. Perceiving this, he had recognized in the physical desires of the human body the true social forces, and he had formulated the distinction between the true scientific method and that which is commonly pursued as the distinction between the study of society from the standpoint of feeling and its study from the standpoint of function. The current method of studying social science was to study the acts themselves which the desires prompt and their functional consequences; whereas the new and true method would study only the desires themselves as social forces and the direct results accomplished by the individuals thus actuated for the attainment of their satisfaction. The distinction is fundamental—the former method being properly designated as the statical, the latter as the dynamic method.

Mr. WARD had drawn up a system of classification of the social forces according to the dynamic method which he presented, with suitable explanatory remarks, to the Anthropological Section of the American Association for the Advancement of Science at its Boston meeting in 1880, only a brief abstract of which was then published.* The system thus sketched was more fully elaborated and in this form was presented to this Society in a paper read on May 2, and May 16, 1882, and illustrated by charts prepared by Dr. Frank Baker.† As it was then about to be published in permanent form it was not thought advisable to repeat it in the transactions of the Society.‡

Mr. WARD placed on the blackboard the outline of his classification of the social forces and showed that it coincided, with some slight exceptions, entirely with that which Prof. Gregory had presented.

EIGHTY-THIRD REGULAR MEETING, May 6, 1884.

Dr. ROBERT FLETCHER, Vice-President, in the Chair.

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* Feeling and Function as Factors in Human Development. "Boston Advertiser," Sept. 1, 1880, p. 1; The same more in detail with table of classification. "Science," Oct. 23, 1880, p. 210.

† Transactions of the Anthropological Society of Washington, Vol. II, pp. 11, 12.

‡ See "Dynamic Sociology," New York, 1883, chapters VII and VIII.

Rev. J. OWEN DORSEY read a paper entitled, "MIGRATIONS OF THE SIOUAN TRIBES."*

ABSTRACT.

Mr. DORSEY gave a classification of the Siouan tribes, including he Sioux proper, Assiniboin, Ponka, Omaha, Osages, Kansas, Iowas, Otos, Missouris, Winnebagoes, Mandans, Minntarees, Crows, and Tutelos. The general impression seems to have been that this stock moved from the northwest. Mr. Dorsey took an opposing view and traced the tribes from the southeast, up the streams, and from the region of the lakes westward.

DISCUSSION.

Major POWELL said that investigations like that of Mr. Dorsey were very valuable—serving to dispel popular myths as to the great number of tribes, and locating ancient villages so that the archæological material could be saved.

Prof. MASON said that he had commenced to work out a synonymy of all the tribes of North America, four years ago, under the patronage of Major Powell. Since then many others had participated in the work, and the whole body of American literature had been ransacked. It was quite possible that many tribal names and references have been overlooked. The members of the society, therefore, would confer a great favor by calling attention to such things occurring in out of the way places.

Dr. E. M. GALLAUDET read a paper on "INTERNATIONAL ETHICS."

There were in existence in Europe several societies whose object is to discuss the subject of international relations. The speaker took the ground that the proper basis of these relations should be ethical rather than legal. The law term for *jus gentium* was objected to and the phrase international rights or international ethics suggested. While nations would not listen to absolute commands of law, they have ever shown some willingness to listen to ethical arguments on the justification of their foulest acts by appealing to the verdict of humanity as to the justice of their cause. If publicists should insist that no act of nations should be justified that are not right between individuals, the subject of international law would be

* Printed in American Naturalist, Vol. XIX.

settled on a firm basis, and Mirabeau's words, "Le droit est le souverain du monde," would become a fact. The substitution of arbitration for war would advance the reign of right, relieve the burdens of taxation, make commerce free, and establish a brotherhood of nations.

DISCUSSION.

Major Powell referred to the origin of the term "*jus gentium*," and pointed out the fact that it meant the law found among all nations, rather then international law. While law and rights are nearly synonymous, the history of law developes the difficulty attending the determination of what is right. When that is so found by the majority it then finds expression in law. As the people in a nation find it difficult to ascertain what is justice, so the same obstacle is met in determining international rights. Referring to certain publicists who sought to control the disposition of property pending wars, he said that it was apparent that mankind was becoming more belligerent, and that wars were more destructive of life and property than formerly,

The result, however, of all this was to lessen the number of nations, and with fewer nations, organization with a view to permanent peace became more probable.

Mr. Otis Bigelow called attention to an extract taken from "Heber's Travels in India," (vol. 2, p. 28,) as follows:

"The Braijarrees, or carriers of grain, a singular wandering race who pass their whole time in transporting grain from one part of India to the other—seldom on their own account but as agents for more wealthy dealers. They move about in large bodies with their wives and children, dogs and loaded bullocks. The men are all armed as a protection against petty thieves. From the sovereign and armies of Hindustan they have no apprehensions. Even contending armies allow them to pass and repass safely, never taking their goods without purchase or even preventing them if they choose from victualling their enemy's camp. Both sides wisely agree to respect and encourage a branch of industry, the interruption of which might be attended with fatal consequence to either."

EIGHTY-FOURTH REGULAR MEETING, May 20, 1884.

Dr. ROBERT FLETCHER, Vice-President, in the Chair.

The Curator acknowledged the receipt of a series of photographs from Prince Roland Bonaparte, for which the thanks of the Society were voted.

Dr. SWAN M. BURNETT read a paper on "COMPARATIVE FREQUENCY OF CERTAIN EYE DISEASES IN THE WHITE AND THE COLORED RACE IN THE UNITED STATES."

ABSTRACT.

Dr. BURNETT related briefly the history of the manner in which the colored race was suddenly transported from its old to its new environment. Now, physicians have been earnest in the inquiry how much this race has been affected by contact with the superior race. Dr. Burnett himself has made extensive researches on this question at the eye and ear dispensary, and his address was a repetition of his experience, 2,341 cases having been examined—1,530 colored, 1,811 white. The statistics covered inquiries concerning constitutional diseases of the eye, as well as defects in the optical instrument itself. The most marked race difference is in the entire absence of granular lids in the blacks, while it forms quite a large per cent. of eye disease among the whites. In healing power the races are alike.

Dr. ELMER REYNOLDS read a paper on a "COLLECTION OF ANTIQUITIES FROM VENDOME, SENLIS, AND THE CAVE-DWELLINGS OF FRANCE."

Dr. REYNOLDS exhibited a beautiful collection of stone implements sent to him by correspondents in France, and his paper was a narration of his story, reaching through the archæolithic, the neolithic, and the bronze age. The objects were sent by the Count de Maricourt and his brother, the Baron de Maricourt, as types of all the characteristic stone implements in France. Dr. Reynolds reviewed the collection in the light of his own experiences, and showed the method of manufacture and the uses of each.

Mr. William H. Holmes read the following paper on

"Evidences of the Antiquity of Man on the Site of the City of Mexico."

Aboriginal art in Mexico seems, in a great measure, to have developed and flourished within her own borders, and the story of her culture is, therefore, quite fully recorded in the superficial deposits of the country. The volcanic and lacustrine formations of the elevated valleys and the rich soil of the *Tierra Caliente* teem with relics of many human periods, and the whole surface of the land is dotted with the ruins of temples and cities. Up to this time the efforts of investigators have been confined to the exploration of points of popular interest and in touching, somewhat superficially, upon the more glittering problems. Little attention has been given to classifying and describing the multitude of minor relics. The ceramic art, which was phenomenally developed, has received scarcely more than a passing notice. It is this condition of affairs that affords me an opportunity of presenting this paper, based as it is, upon a brief study of the contents of the soil within the limits of the City of Mexico.

Incomplete as my observations were, they afforded me a most welcome opportunity of beginning the study of the ceramic art of Mexico from the standpoint of actual observation of relics in place. Superb as are the collections within the Mexican Museum, their study is rendered extremely unsatisfactory by the absence of detailed information in regard to their origin and chronology. Fortunately the section of deposits here presented reads with the readiness of an open book, giving not only the proper sequence to its own treasures, but, I doubt not, making clear the relative position of many other relics that would, otherwise, go unclaimed and unclassified.

The site of the capital of the Montezumas is naturally a great repository of the ceramic remains of the pre-Columbian peoples. One has but to wander into almost any of the suburban villages, wherever excavations are going on, to witness the exhumation of multitudes of fragmentary utensils, many of which have been a second and a third time thrown up and rebuilt into the edifices and defences of successive cities.

During the spring of 1884 I spent a few weeks at the Central Railway station, which is located in the outskirts of the city. The old walls and fortifications of the city, dating back perhaps to early

Spanish times, lie just outside of the inclosure of the station, and the road has been cut through these leveled works, and through the accumulated refuse of a small suburban village, now represented by a dilapidated church and a few adobe hovels.

The section exposed by the railway cuttings exhibits a curious agglomeration of the deposits of all past human periods. The remains of previous times and peoples—pottery, stone, and skeletons—have recently been redistributed by the greatest of all innovators, the spade of the Yankee. To those, therefore, who halt only to examine the deposits along the immediate line of the railway there is nothing visible but utter confusion, although a glance is sufficient to show that, in every spadeful, there is evidence of many widely separated stages of art.

Just west of the line, however, and apparently outside of the old line of circumvallation is an area—an acre, more or less—on which an extensive manufactory of adobe bricks has been established. Here excavations have been made exposing the heretofore undisturbed accumulations of past ages to the depth often of eight or ten feet.

The general surface of this area is perhaps from three to four feet below the broad masses of ancient ramparts, and is, at the same time, perceptibly elevated above the level of the lacustrine plain about it. It has been stated by a recent writer, that there is probably no spot remaining about the city of Mexico that shows a trace of pre-Spanish structures, but I am convinced that here we have such a spot. The surface is humpy and uneven, the result probably of comparatively recent ditch-digging or house-building; but there is a gentle arching of the whole area which, taken in connection with the fact that the entire mass is composed very greatly of remnants of aboriginal art, seems to warrant my conclusion. Across one side of this area the old Spanish walls were built and the adobe diggers are now encroaching upon the other. So full is the soil of relics, chiefly of pottery, that the workmen are greatly embarrassed in their labors, even to the depth of many feet, and by the side of each pit is a great heap, composed of fragments too large to be worked into the brick. In one place a section is exposed in a continuous vertical wall nearly a hundred feet long and more than eight feet deep. The upper part bears evidence of more or less disturbance, but the greater part of the exposed deposits have remained absolutely undisturbed since the day of their deposition.

This is made apparent by the very distinctly stratified character of the soil, which consists of dark loam with more or less sand, impurities, and broken relics.

It is difficult to say to what extent the stratification is aqueous, or to what extent the result of periods of unequal artificial accumulation. The fact that the base of the exposed section is several feet lower than the present surface of the lake, suggests the possibility that its waters actually washed the walls of the ancient settlement. The level of the lake has, during historic times, undergone such diverse changes that it cannot be surmised what was its condition at any particular period of the remote past.

The accompanying section, figure 1, although representing but a small part of the horizontal exposure, shows all the important features in their proper relations to one another. It is the result of a number of visits to the spot, most of which were made with the purpose of assuring myself of the accuracy of preceding observations. The deposits of fragmentary pottery reach to the base of the

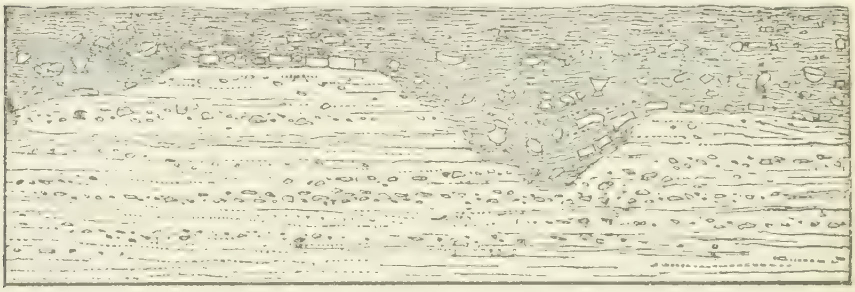

FIG. 1.—Section showing two periods of occupation.

section, and are so arranged as to show beyond a doubt, that they accumulated with the soil and are not subsequent intrusions. This is apparent, not only from their deposition in more or less continuous horizontal layers, as shown in the section, but from the identical character of fragments occurring at corresponding depths.

The prevailing type of ware, throughout the lower part of the section, is very archaic and is to all appearences quite distinct from the handsome pottery characteristic of the upper half of the section.

It was simple in form and rude in finish and little superior in any respect to the rudest products of the wild Indians of North America. At the base the fragments are small and much decayed; higher, they

are larger and better preserved, although I was unable to secure a complete, unbroken vessel.

The only form that came to my notice, although thousands of pieces were examined, is a kind of deep cup or bowl, not unlike our common flower pot, and having a flattish bottom and an extremely uneven and ragged rim. In all cases the exterior surface is covered with impressions of coarse woven fabrics, the single indication of advance toward better finish being a slight polishing of the interior surface, which was accomplished with a smooth implement, such as a pebble or shell. Where well preserved, the paste is generally hard and fine grained, but shows in all cases a rather rough granular fracture. The character of the tempering material cannot be made out, but, in a number of cases, the texture indicates the former presence of fibrous particles like finely pulverized grass, leaves, or straw. The surface is of a pale, yellowish red or terra cotta color, the result of the baking, while the interior of the mass is generally a dark gray.

In Fig. 2, I present an example of this pottery which is restored from fragments. These did not come from the wall of the section, but from a pit, a short distance away, where the pieces were larger

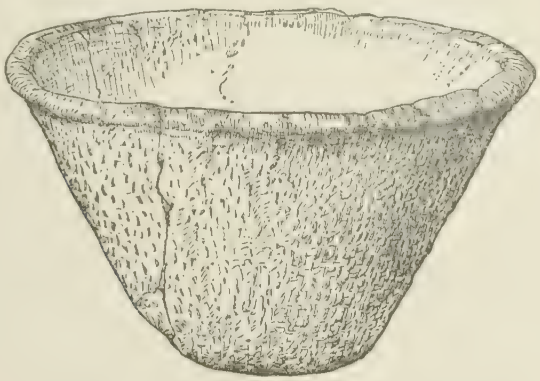

FIG. 2.—Vessel of the most primitive style.

and better preserved. In this example the rim is thick and slightly enlarged as if squeezed over the edge of a basket used as a mold. In most cases no attempt has been made to render the edge even or smooth, and the finger marks and the irregular partings of the margin, which came from squeezing the clay into or over molds and expanding the edges to secure greater size, are all visible.

It is difficult to find a well preserved and clearly defined impres-

sion of the fabric employed in the manufacture of these vessels. The clay was probably not of a character to take a clear impression and the cloth was apparently of a ragged, irregular kind. The mesh was open and the thread coarse and slightly twisted. The finer specimens show about eight intersections to the inch and the coarser probably six. In some cases one series of threads seem to have been large and the other small. These fabrics were applied to the entire exterior surface of the vessel, but not with much regularity. They may have served to facilitate the handling of the ware while in a plastic state.

This pottery is distributed in horizontal layers throughout a vertical series more than six feet in thickness, and represents an early epoch of the art of Anahuac.

In the upper portion of the lower group of beds we encounter two other varieties of ware. These may have been developed from the rude form in the natural course of progress but there are few indications of this growth here. They are much more nearly allied to the later than to the earlier stages of the art of the section. The transition is very abrupt.

As a matter of course I can only present this order of occurrence as characteristic of this locality and of this section. There may be very different combinations in other places, but the order of sequence here indicated is, in the light of history, very suggestive. If the Aztecs, as tradition has it, were the first to settle on this margin of the swampy shore of the lake, then this cord-marked ware is the product of their earliest or savage period, and the finer wares occuring at first so sparingly indicate trade with the more advanced peoples of neighboring settlements.

The variety of ware second to appear in the ascending scale is represented by fragments of large, round-bodied, symmetrical pots or casks, with gently constricted necks and thick rounded recurving rims. The paste is generally reddish upon the surface and gray in the mass, and there is a large percentage of silicious tempering material. The surface, exterior and interior, is painted a dark brownish red and has been evenly polished. Average specimens have been, perhaps, ten inches in diameter and a foot or more in height. The walls are always very thick. Fig. 3, is drawn from fragments sufficiently large to indicate the whole shape clearly. Pottery like this is found imbedded in the adobe bricks of the pyra-

mid of Cholula, and is common in the ancient graves of Costa Rica and New Granada. Large vases recently brought from the province of Chiriqui are identical with these in every respect.

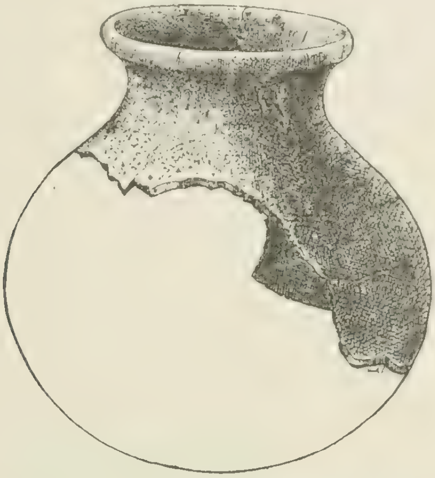

FIG. 3.—Earthern vessel from the lower series of deposits.

Associated with this ware and beginning apparently a little higher in the section, we find the remains of the third variety. The vessels are mostly cup-shaped. They are well made, are simple in treatment, and exhibit a fair degree of symmetry. The prevailing color is a light yellowish terra-cotta tending toward orange. The surfaces are moderately well polished but rarely show attempts at ornamentation. The forms are repeated in the more elaborate wares that succeed it. This ware is identical in most respects with much of that found in the adobe mass of the pyramids of San Juan Teotihuacan, Texcoco, and Cholula, and upon the slopes of the hill of Texcocingo. It is, apparently, the forerunner of some of the more elegant wares of the surface deposits of the section. In the upper part of the lower series of deposits this ware predominates greatly over both the heavy ware and the archaic pottery already described. By reference to the section it will be seen that the surface of the lower series of beds has been much disturbed by the more recent occupants of the site at the beginning of the second epoch. Excavations have been made and afterwards filled up with gradually accumulating refuse, so that a series of imperfect stratified deposits has been spread over all, at first following the curves of the disturbed surface. There is, however, no very

well defined line of separation between the older and newer formations. The distinction is rendered much clearer by the contents of the soil. There are occasional layers of stone and adobe bricks, representing the foundations of houses, as seen in the section. There are great quantities of fragmentary pottery, among which I find many of the artistic shapes and rich decorations characteristic of the surface deposits of Anahuac. Included I find also fragments of the two varieties last described. There are occasional stone implements and great quantities of obsidian knives, hundreds of which are as perfect as when first struck from the core. These are characteristic of the later Aztec period. Near the surface there are fragments of glazed ware indicating Spanish influence. It is not unusual to see in the shallow ditches of the suburban villages, fragments of vessels of aboriginal form and decoration, covered with Spanish glaze. Indeed such vessels can be seen in use by the Indians of to-day and are exposed for sale in the modern markets.

The pottery of the upper division of the section presents great variety of form and ornamentation, but in material and treatment it is extremely uniform. The paste is compact and heavy, and has a moderately even, finely granular fracture. In rare cases the fracture is smooth or conchoidal. The more common wares are lighter and more porous than those of finer finish. The whole mass is often of a pale brick-red color, the baking having been thorough; but more frequently the interior is of a dark blue gray, indicating imperfect firing. The paste is generally hard and the ware has in many cases a sonorous or metallic ring. The walls vary in thickness with the individual vessel. The tempering when distinguishable is always silicious.

The method of finishing the surface is quite uniform although carried to very different degrees of perfection. Occasionally we find a piece without polish; and figurines and elaborately modeled forms are generally quite plain. As a rule the vessels have been very carefully polished. In many examples the markings of the polishing implement are distinctly visible; indeed this is true of the unimportant parts of the majority of vessels of the most perfect finish. The polish of the finer examples is so perfect that it is difficult to believe it the result of purely mechanical processes. The polishing has generally been done after the application of the color and color-designs, but sometimes before. Unpolished surfaces show impressions of the potter's fingers.

There are no indications of the use of a wheel. The vessels are seldom absolutely true in outline, but in a general way are remarkable for symmetry and grace. The colors employed in finishing and decorating are pleasing and often extremely rich. The reds predominate, the whole surface of the simple forms being frequently finished with it. Upon this the designs are painted in black, white, and different tones of red. In the more common utensils the figures are drawn, often carelessly, upon the plain untinted surface. The brush has been handled with freedom and the designs are often quite elaborate. Occasionally we find incised figures and stamped patterns.

The various shapes of vessels obtained at this locality may be classified under a few heads.

First, there are many cups and bowls ranging from a few inches to a foot in diameter, and generally quite shallow. The bottoms are usually flat and the walls expand regularly to the rim. Two examples varying from the rule are given in Figs. 4 and 5. Fig. 4

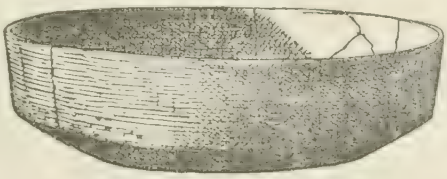

FIG. 4.—Vessel from the upper deposits.

shows a slightly polished, unpainted pan of dark, ochreous tint, with upright sides and flat bottom. The base, outside, is slightly convex next the circumference and concave at the center. It is

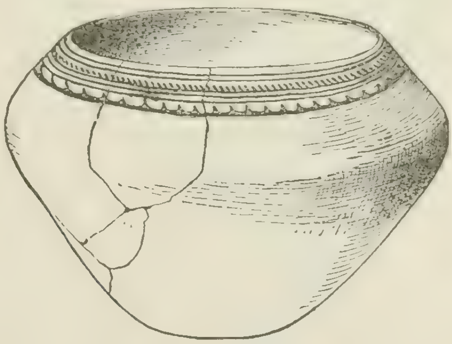

FIG. 5.—Vessel from the upper deposits.

eight inches in diameter. Fig. 5 illustrates a deep cup of similar color and finish; a painted design consisting of parallel encircling

lines occupies the exterior surface of the rim. The form is an unusual one in Mexico.

Most of the vessels obtained from the upper stratum are neatly finished and tastefully decorated. Some are polished like a mirror over the entire surface, exterior and interior. A favorite form is that of a shallow flat-bottomed cup of moderate size, Fig. 6. The designs are greatly varied and are painted in black or in black and white. The white pigment has been applied subsequently to the

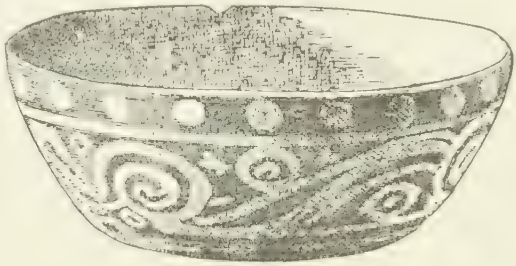

FIG. 6.—Vessel with figures in white upon a red ground, in U. S. National Museum.

polishing of the surface and can be removed with ease. Vessels of this and similar forms are often furnished with tripod supports. One example of the latter variety is given in Fig. 7. The bowls are often very shallow. The designs are simple and occupy the in-

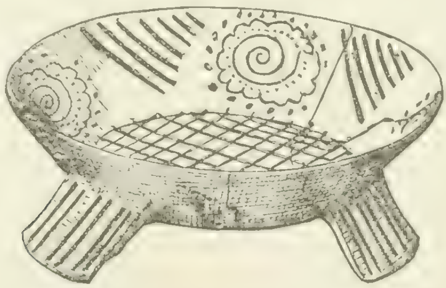

FIG. 7.—Tripod dish with designs in black.

terior surface. A curious device is shown in Fig. 8. The interior surface of the bottom is scoriated with deeply incised reticulated lines, a device probably intended for the grating of food or spices and one still employed by the present inhabitants. A few examples of this general class of ware show stamped decoration. In its manufacture molds were probably used in which intaglio designs had

been executed. Some fragments of cups exhibit figures formed of minute hemispherical nodes. They are further embellished by the

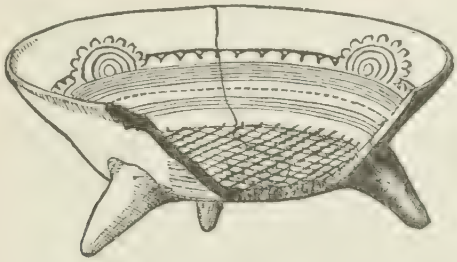

FIG. 8.—Tripod dish with scoriated bottom

addition of sharp conical nodes about the rim. A remarkable feature of these cups is the occurrence of groups of triangular perforations, cut with a sharp tool, and so arranged as to resemble a Maltese cross. These perforations are placed so low on the body as to make the vessel unfit for containing liquids. In the museum of Mexico there are a few examples extensively perforated, leaving about the middle zone of the body only a sort of lattice work of the original walls. The same style of work is elaborately practiced by Oriental peoples.

One large class of vessels resemble an hour-glass in shape. They are really double cups, one end being usually smaller than the other and serving as a foot, but both cups are equally well finished. The exterior surface is highly polished and colored a deep red, and painted with designs in black and white. The fragments are large and very numerous. Fig. 9 illustrates the prevailing form. The

FIG. 9.—Cup with designs in black and white upon a red ground, in Mexican National Museum.

diameter ranges from three to six inches or more. Some of the most beautiful vessels in the Museum are of this general shape.

It is but rarely that one comes upon fragments of the richly colored and highly finished wares characteristictic of the regions of Cholula and of the South. I was fortunate in securing a few small pieces. Two of these are shown in Figs. 10 and 11. Their rarity

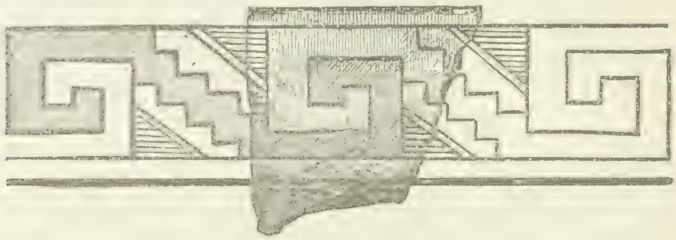

FIG. 10.—Meander design painted in rich colors.

makes it probable that they came to this spot by trade. The first shows a fine strong treatment of the fret and the other of the scroll.

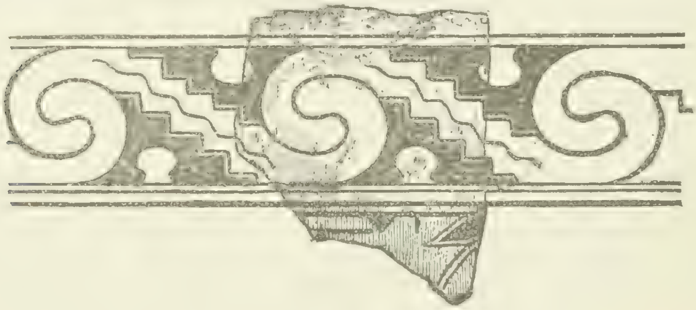

FIG. 11—Scroll ornament painted in rich colors.

These forms are characteristic of the best period of art in both North and South America. The chief charm of this ware is its rich color—an orange ground with figures in red and black, the whole surface being polished like glass.

I found no specimen exhibiting delicate green and pink decorations such as may be seen at San Juan Teotiahuacan, and such as are seen on some of the most beautiful vases in the Mexican Museum.

In the upper series of deposits indicated in the section, I found a fragment of a very remarkable form of vase. It is represented by a number of examples in the Mexican Museum, one of which is

shown in Fig. 12. It has been called a brasier and a censer, and is thought to have been employed in religious ceremonies, but its

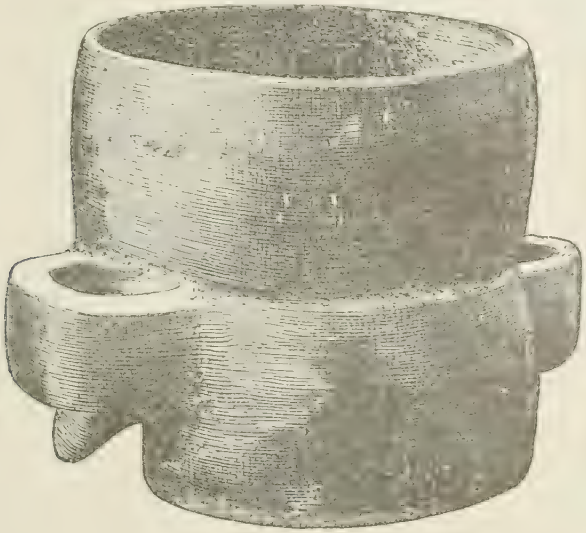

FIG. 12.—Ceremonial vase, in Mexican National Museum.

true use is probably unknown. The shape is, however, suggestive of some especial ceremonial office. It resembles a short, upright cylinder, encircled midway by a groove. There are two massive, horizontally-looped handles attached to the sides a little below the middle. The bowl is rather shallow. The lower third of the vessel consists of a hollow foot resembling the bowl above, but open at the sides beneath the handles. The conformation is such that a heavy cord could be passed through the handles and across the doubly cloven foot for suspension as a swinging censer. The exposed surfaces are usually highly polished and the colors embrace black and many rich tones of red.

It should be noted that no traces were found of the dark, highly ornate pottery so often seen in modern times and so frequently brought away by tourists. This ware may have a legitimate place in Aztec art, but does not occur among the ancient productions in any locality visited by me. It is absolutely certain that all the specimens now seen in the shops of Mexico and offered for sale by hawkers on the streets and at the stations—especially at San Juan—are modern products. They are, however, wonderfully well executed, and the appearance of antiquity given them is truly remarkable.

I have, from the pits at the railway station, a number of miscellaneous articles in clay, bits of images of men and animals, whistles, spindle-whorls, and the like. A portion of a curious head found is duplicated in a pipe preserved in the Museum and represented in

FIG. 13.—Pipe with grotesque heads on the bowl, in the Mexican National Museum.

Fig. 13. The whistles are generally of a very simple kind, and the spindle-whorls are not different from those of other parts of Anahuac.

In conclusion, I may recall in a very few words some of the more striking features of this section, calling attention to the order of events suggested by them.

It may be affirmed with certainty that the site of the City of Mexico was at one time occupied by a people in a very primitive stage of art, the remains of which art, so far as found, include nothing but fragments of an extremely rude pottery. There are no traces of tools and no indications of houses. This period of occupancy was a very long one, as it permitted the accumulation in nearly horizontal layers of at least eight feet of finely comminuted refuse.

It is further seen that far along in this period of occupancy new forms of art appeared that do not look like the work of the proper occupants of the site produced by gradual improvement, but rather like intrusive products acquired by exchange or otherwise from more cultured tribes. Again, at the end of this first period there is a horizon, pretty well marked, above which primitive forms of art do not appear.

Near the base of the deposits of the second period foundations of houses are discovered in which rubble, squared stones, and adobe bricks have been used. In this part of the section we find stone implements and ceramic products of a very high order of merit. With these, and especially near the surface, there is a layer abounding in obsidian implements. This marks the last and culminating stage of Aztec art, ending in the historic period proper.

Speculation upon the period of time represented by this section would be useless, and an attempt to correlate the events recorded with those shadowed forth in tradition would be equally vain. The earliest period is probably beyond the ken of tradition, and the last marks the historic period of Aztec occupation.

SPECIAL SESSION, October 11, 1884.

In accordance with a call of the Council, the Society met in special session at Columbian University Hall, for the purpose of listening to an address from Prof. E. B. TYLOR, of Oxford University, England.

Through invitation extended by order of the council there were also present members of the Philosophical and Biological Societies, of the Cosmos Club, as well as officers, professors and students of Columbian University.

The Society was called to order by President POWELL, who in a few words introduced the speaker, who delivered the following address on—

"HOW THE PROBLEMS OF AMERICAN ANTHROPOLOGY PRESENT THEMSELVES TO THE ENGLISH MIND."

I have seldom, ladies and gentlemen, felt myself in a more difficult position than I do at this moment. Yesterday morning, when we returned from an expedition out into the far west—an expedition which your President was to have joined, but which, to our, great regret, he was obliged to give up—I heard that at this meeting of the Anthropological Society of Washington I should be called upon to make, not merely a five-minutes' speech, but a subtantial address; and since that time my mind has been almost entirely full of the new things that I have been seeing and hearing in the domain of anthropology in this city. I have been seeing the working of that unexampled institution, the Bureau of Ethnology, and studying the collections which, in connection with the Smithsonian Institution, have been brought in from the most distant quarters of the continent; and, after that, in odd moments, I have turned it over in my mind, what can I possibly say to the Anthropological Society when

I am called upon to face them at thirty-six hours' notice? I will not apologize; I will do the best I can.

I quite understand that Major Powell, who is a man who generally has a good reason for everything that he does, had a good reason for desiring that an anthropologist from England should say something as to the present state of the new and growing science in England as compared with its condition in America—for believing that some communication would be acceptable between the old country and the new upon a subject where the inhabitants of both have so much interest in common, and can render to one another so much service in the direction of their work. And therefore I take it that I am to say before you this evening, without elaborate oratory and without even careful language, how the problems of American anthropology present themselves to the English mind.

Now, one of the things that has struck me most in America, from the anthropological point of view, is a certain element of old-fashionedness. I mean old-fashionedness in the strictest sense of the word—an old-fashionedness which goes back to the time of the colonization of America. Since the Stuart time, though America, on the whole, has become a country of most rapid progress in development, as compared with other districts of the world, there has prevailed in certain parts of it a conservatism of even an intense character. In districts of the older States, away from the centres of population, things that are old-fashioned to modern Europe have held their own with a tenacity somewhat surprising. If I ever become possessed of a spinning-wheel, an article of furniture now scarce in England, I can hardly get a specimen better than in Pennsylvania, where "my great-grandmother's spining-wheel" is shown—standing, perhaps, in the lumber-room, perhaps in an ornamental place in the drawing-room—oftener than in any other country that I ever visitied.

In another respect Pennsylvania has shown itself to me fruitful of old-fashioned products. I was brought up among the Quakers—like so many, I dare say, who are present; for the number of times in the week, or even in the day in which it occurs that those whom one meets prove to be at least of Quaker descent, represents a proportion which must be highly pleasant to the Quaker mind. In the history of the Society of the Friends there has recently come out a fact unknown, especially to the Friends themselves. Their opinion has always been that they came into existence in the neigh-

borhood of 1600, by spontaneous generation, in an outburst of spiritual development in England. It has now been shown, especially by the researches of Robert Barclay (not the old controversialist, but a modern historian,) that the Quakers were by no means the absolutely independent creation that they and others had supposed them to be; that they were derived from earlier existing denominations by a process which is strictly that of development. Their especial ancestors, so to speak, were a division of the early Dutch sect known as Mennonites. The Friends have undergone much modification as to theological doctrine; but some of their most pronounced characteristics, such as the objection to war an oaths, and even details of costume, and the silent grace before meals, remain as proofs of Mennonite derivation. To find the Mennonites least changed from their original condition is now less easy in their old homes in Europe than in their adopted homes in the United States and Canada, whither they have migrated from time to time up till quite recently in order to avoid being compelled to serve as soldiers. They have long been a large and prosperous body back in Pennsylvania. I went to see them; and they are a very striking instance of permanency of institutions, where an institution or a state of society can get into prosperous conditions in a secluded place, cut off from easy access of the world. Among them are those who dissent from modern alteration and changes by a fixed and unalterable resolution that they will not wear buttons, but will fasten their coats with hooks and eyes, as their forefathers did. And in this way they show with what tenacity custom holds when it has become matter of scrupple and religious sanction. Others have conformed more and more to the world; and most of these whom I have seen were gradually conforming in their dress and habits, and showing symptons of melting into the general population. But, in the mean time, America does offer the spectacle of a phase of religious life, which, though dwindling away in the old world region where it arose, is quite well preserved in this newer country, for the edification of students of culture. These people, who show such plain traces of connection with the historical Anabaptists that they may be taken as their living representatives, still commemorate in their hymns their martyrs who fell in Switzerland for the Anabaptist faith. There was given me only a few days ago a copy of an old, scarce hymn-book, anterior to 1600, but still in use, in which is a hymn commemorative of the martyr Haslibach, beheaded for refusing to

conform to the state religion, whose head laughed when it was cut off.

Now, to find thus, in a secluded district, an old state of society resisting for a time the modifying influences which have already changed the world around, is no exceptional state of things. It shows the very processes of resisted but eventually prevailing alteration which anthropologists have to study over larger regions of space and time in the general development of the world. In visiting my Mennonite friends in Pennsylvania, I sometimes noticed that while they thought it nothing strange that I should come to study them and their history, yet when I was asked where I was going next, and confessed with some modesty that I was going with Major Powell to the far west to see the Zuñis, this confession on my part was received with a look of amazement, not quite unmingled with kindly reproof; it seemed so strange to my friends that any person travelling about of his own will should deliberately go to look at Indians. I found it hard to refrain from pointing out that, after all, there is a community of purpose between studies of the course of civilization whether carried out among the colonists of Pennsylvania or among the Indians of New Mexico. Investigation of the lower races is made more obscure and difficult through the absence of the guidance of written history, but the principle is the same.

A glance at the tribes whom Professor Mosely and I have seen in the far west during the last few weeks has shown one or two results which may be worth stating; and one, merely parenthetical, I think I must take leave to mention, though it lies outside the main current of my subject.

Our look at North American Indians, of whom it has been my lot to write a good deal upon second-hand evidence, had, I am glad to say, a very encouraging effect; because it showed that on the whole much of the writings of old travelers and missionaries have to be criticised, yet if, when carefully compared, they agree in a statement, personal inspection will generally verify that statement. One result of our visit has been, not a diminution, but an increase of the confidence with which both of us in future will receive the statements of travelers among the Indians, allowing for their often being based upon superficial observation. So long as we confine ourselves to things which the traveler says he saw and heard, we are, I believe, upon very solid ground.

To turn to our actual experiences. The things that one sees

among the Indian tribes who have not become so "white" as the Algonkins and the Iroquois, but who present a more genuine picture of old American life, do often, and in the most vivid way, present traces of the same phenomena with which one is so familiar in old-world life. Imagine us sitting in a house just inside California, engaged in what appeared to be a fruitless endeavor on the part of Professor Mosely to obtain a lock of hair of a Mojave to add to his collection. The man objected utterly. He shook his head. When pressed, he gesticulated and talked. No; if he gave up that bit of hair, he would become deaf, dumb, grow mad; and, when the medicine man came to drive away the malady, it would be of no use, he would have to die. Now, all this represents a perfectly old-world group of ideas. If you tried to get a lock of hair in Italy or Spain, you might be met with precisely the same resistance; and you would find that the reason would be absolutely the same as that which the Mojave expressed,—that by means of that lock of hair one can be bewitched, the consequence being disease. And within the civilized world the old philosophy which accounts for disease in general as the intrusion of a malignant spirit still largely remains; and the exorcising such a demon is practised by white men as a religious rite, even including the act of exsufflating it, or blowing it away, which our Mojave Indian illustrated by the gesture of blowing away an imaginary spirit, and which is well known as forming a part of the religious rites of both the Greek and Roman church. How is it that such correspondence with old-world ceremonies should be found among a tribe like the Mojaves, apparently Mongolian people, though separated geographically from the Mongolians of Asia? Why does the civilization, the general state of culture, of the world, present throughout the whole range, in time and space, phenomena so wonderfully similar and uniform? This question is easy to ask; but it is the question which, in a few words, presents the problem which, to all anthropologists who occupy themselves with the history of culture, is a problem full of the most extreme difficulty, upon which they will have for years to work, collecting and classifying facts, in the hope that at some time the lucky touch will be made which will disclose the answer. At present there is none of an absolute character. There is no day in my life when I am able to occupy myself with anthropological work, in which my mind does not swing like a pendulum between the two great possible answers to this question. Have the descendants of a

small group of mankind gone on teaching their children the same set of ideas, carrying them on from generation to generation, from age to age, so that when they are found in distant regions, among tribes which have become different even in bodily formation, they represent the long-inherited traditions of a common ancestry? Or is it that all over the world, man, being substantially similar in mind, has again and again, under similar circumstances of life, developed similar groups of ideas and customs? I cannot, I think, use the opportunity of standing at this table more profitably then by insisting, in the strongest manner which I can find words to express, on the fundamental importance of directing attention to this great problem, the solution of which will alone bring the study of civilization into its full development as a science.

Let me put before you two or three cases, from examples which have been brought under my notice within the last few days, as illustrating the ways in which this problem comes before us in all its difficulty.

This morning, being in the museum with Major Powell, Professor Mosely, and Mr. Holmes, looking at the products of Indian life in the far west, my attention was called to certain curious instruments hanging together in a case in which musical instruments are contained. These consisted simply of flat, oblong, or oval pieces of wood, fastened at the end to a thong, so as to be whirled round and round, causing a whirring or roaring noise. The instruments in question came, one from the Ute Indians, and one from the Zuñis. Now, if an Australian, finding himself inspecting the National museum, happened to stand in front of the case in question, he would stop with feelings not only of surprise, but probably of horror; for this is an instrument which to him represents, more intensely than anything else, a sense of mystery attached to his own most important religious ceremonies, especially those of the initiation of youths to the privileges of manhood, where an instrument quite similar in nature is used for the purpose of warning off women and children. If this Australian was from the south, near Bass Strait, his native law is, that, if any woman sees these instruments, she ought immediately to be put to death; and the illustration which he would give is, that, in old times, Tasmania and Australia formed one continent, but that one unlucky day it so happened that certain boys found one of these instruments hidden in the bush, and showed it to their mothers, whereupon the sea burst up through the land in a deluge,

which never entirely subsided, but still remains to separate Van Dieman's Land from Astralia. And, even if a Caffre from South Africa were to visit the collection, his attention would be drawn to the same instruments, and he would be able to tell that in this country they were used for the purpose of making loud sounds, and warning the women from the ceremonies attending the initiation of boys. How different the races and languages of Australia and Africa ! yet we have the same use cropping out in connection with the same instrument; and to complete its history, it must be added that there are passages of Greek literature which show pretty plainly that an instrument quite similar was used in the mysteries of Bacchus. The last point is, that it is a toy well known to country-people, both in Germany and in England. Its English name is the "bull-roarer;" and, when the children play with it in the country villages, it is hardly possible (as I know by experience) to distinguish its sound from the bellowing of an angry bull.

In endeavoring to ascertain whether the occurrence of the "bull-roarer," in so many regions is to be explained by historical connection, or by independent development, we have to take into consideration, first, that it is an apparatus so simple as possibly to have been found out many times; next, that its power of emitting a sound audible at a great distance would suggest to Australians and Caffres alike its usefulness at religious ceremonies from which it was desired to exclude certain persons. Then we are led to another argument, into which I will not enter now, as to the question why women are excluded in the most rigid manner from certain ceremonies. But in any event, if we work it out as a mere question of probabilities, the hypothesis of repeated reinvention under like circumstances can hold its own against the hypothesis of historical connection; but which explanation is the true one, or whether both are partly true, I have no sufficient means to decide. Such questions as these being around us in every direction, there are only two or three ways known to me in which at preeent students can attack them with any reasonable prospect of success. May I briefly try to state, not so much by precept as by example, what the working of those methods is by which it is possible, at any rate, to make some encroachments upon the great unsolved problem of anthropology.

One of the ways in which it is possible to deal with such a group of facts may be called the argument from outlandishness. When a circumstance is so uncommon as to excite surprise, and to lead

one to think with wonder why it should have come into existence, and when that thing appears in two different districts, we have more ground for saying that there is a certain historical connection between the two cases of its appearance than in the comparison of more commonplace matters. Only this morning a case in point was brought rather strongly under my notice; not that the facts were unknown, for we have been seeing them for days past at Zuñi. The Indians of the north, and especially the Iroquois, were, as we know, apt to express their ideas by picture-writings, in the detailed study of which Col. Mallery is now engaged. One sign which habitually occurs is the picture of an animal in which a line is drawn from the throat, through the picture of the animal, terminating in the heart. Now, the North American Indians of the lake district have a distinct meaning attached to this peculiar heart-line, which does not attach to ordinary pictures of animals; they mean some animal which is living, and whose life is affected in some way by a charm of some kind.

It is expressly stated by Schoolcraft that a picture he gives of a wolf with such a heart-line means a wolf with a charmed heart. It is very remarkable to find, among the Zuñis, representations of deer and other animals drawn in the same manner; and the natural inference is, that the magic of the Iroquois and the Zuñis is connected, and of more or less common origin. I verified this supposition by asking Mr. Cushing, our authority on Zuñi language and ideas, what idea was generally attached to this well-known symbol; and his answer was, that it indicated a living animal on which magical influence was being exerted. May we not, then, consider—leaving out of the question the point whether the Pueblo people invented the heart-line as a piece of their magic and the nomad tribes of the north picked it up from them, or whether it came down from the northern tribes and was adopted by the southern, or whether both had it from a common source—that, at any rate, there is some ground, upon the score of mere outlandishness, for supposing that such an idea could not occur without there being some educational connection between the two groups of tribes possessing it, and who could hardly have taken it by independent development.

To mention an instance of the opposite kind; I bought a few days ago, amonge the Mojaves, a singular article of dress,—a native woman's girdle, with its long fringe of twisted bark. This or, rather two of these put on so as to form one complete skirt used to

be her only garment; and it is still worn from old custom, but now covered by a petticoat of cotton, generally made of several pocket-handkerchiefs in the piece, bought from the traders. Under these circumstances, it has become useless as a garment, only serving as what I understand is called in the civilized world "a dress-improver;" the effect of which, indeed, the Mojave women perfectly understand, and avail themselves of in the most comic manner. Suppose, now, that we had no record of how this fantastic fashion came into use among them: It has only to be compared with the actual wearing of bark garments in Further Asia and the Pacific Islands in order to tell its own history,—that it is a remnant of the phase of culture where bark is the ordinary material for clothing. But the anthropologist could not be justified in arguing from this bark-wearing that the ancestors of the Mojaves had learned it from Asiatics. Independent development, acting not only where men's minds, but their circumstances, are similar, must be credited with much of the similarity of customs. It is curious that the best illustrations of this do not come from customs which are alike in detail in two places, and so may be accounted for, like the last example, by emigration from one place to another. We find it much easier to deal with practices similar enough to show corresponding workings of the human mind, but also different enough to show separate formation. Only this morning I met with an excellent instance of this. Dr. Yarrow, your authority on the subject of funeral rites, described to me a custom of the Utes of disposing of the bodies of men they feared and hated by putting them under water in streams. After much inquiry, he found that the intention of this proceeding was to prevent their coming back to molest the survivors. Now, there is a passage in an old writer on West Africa where it is related, that, when a man died, his widow would have herself ducked in the river in order to get rid of his ghost, which would be hanging about her, especially if she were one of his most loved wives. Having thus drowned him of, she was free to marry again. Here, then, is the idea that water is impassable to spirits, worked out in different ways in Africa and America, but showing in both the same principle; which, indeed, is manifested by so many peoples in the idea of bridges for the dead to pass real or imaginary streams, from the threads stretched across brooks in Burmah for the souls of friends to cross by, to Catlin's slippery pine-log for the Choctaw dead to pass the dreadful river. In such correspondences of principle we

trace, more clearly than in mere repetition of a custom or belief, the community of human intellect.

But I must not turn these remarks into what, under ordinary circumstances, would be a lecture. I have been compelled to address myself, not so much to the statement in broad terms of general principles, as to points of detail of this kind, because it is almost impossible, in the present state of anthropology, to work by abstract terms; and the best way of elucidating a working-principle is to discuss some actual case. There are now two or three practical points on which I may be allowed to say a few words.

The principle of development in civilization, which represents one side of the great problem I have been speaking of, is now beginning to receive especial cultivation in England. While most museums have been at work, simply collecting objects and implements, the museum of Gen. Pitt-Rivers, now about to be removed from London to Oxford, is entirely devoted to the working out of the development theory on a scale hardly attempted hitherto. In this museum are collected specimens of weapons and implements, so as to ascertain by what steps they may be considered to have arisen among mankind, and to arrange them in consecutive series. Development, however, is not always progress, but may work itself out into lines of degeneration. There are certain states of society in which the going-down of arts and sciences is as inevitable a state of things as progress is in the more fortunate regions in which we live. Anthropologists will watch with the greatest interest what effect this museum of development will have upon their science. Gen. Pitt-Rivers was led into the formation of the remarkable collection in question in an interesting manner. He did not begin life either as an evolutionist or as an anthropologist. He was a soldier. His business, at a particular time of his life, was to serve on a committee on small-arms, appointed to reform the armament of the British army, which at that time was to a great extent only provided with the most untrustworthy of percussion-muskets. He then found that a rifle was an instrument of gradual growth; for the new rifles which it was his duty to inspect had not come into existence at once and independently. When he came to look carefully into the history of his subject, it appeared that some one had improved the lock, then some one the rifling, and then others had made further improvements; and this process had gone on until at last there came into existence a gun, which, thus perfected, was able to hold

its own in a permanent form. He collected the intermediate stages through which a good rifle arose out of a bad one; and the idea began to cross his mind that the course of change which happened to rifles was very much what ordinarily happens with other things.

So he set about collecting, and filled his house from the cellar to the attic, hanging on his walls series of all kinds of weapons and other instruments which seemed to him to form links in a great chain of development. The principle that thus became visible to him in weapon-development is not less true through the whole range of civilization; and we shall soon be able to show to every anthropologist who visits Oxford the results of that attempt. And when the development theory is seen in that way, explaining the nature and origin of our actual arts and customs and ideas, and their gradual growth from ruder and earlier states of culture, then anthropology will come before the public mind as a new means of practical instruction in life.

Speaking of this aspect of anthropology leads me to say a word on another hardly less important. On my first visit to this country, nearly thirty years ago, I made a journey in Mexico with the late Henry Christy, a man who impressed his personality very deeply on the science of man. He was led into this subject by his connection with Dr. Hodgkin; the two being at first interested, from the philanthropist's point of view, in the preservation of the less favored races of man, and taking part in a society for this purpose known as the Aborigines' protection society. The observation of the indigenous tribes for philanthropic reasons brought the fact into view that such peoples of low culture were in themselves of the highest interest as illustrating the whole problem of stages of civilization; and this brought about the establishment of the Ethnological Society in England, Henry Christy's connection with which originated his plan of forming an ethnological museum. The foundations of the now celebrated Christy collection were laid on our Mexican journey; and I was witness to his extraordinary power of knowing, untaught, what it was the business of an anthropologist to collect, and what to leave uncollected: how very useless for anthropologic purposes mere curiosities are, and how priceless are every-day things. The two principles which tend most to the successful work of anthropology—the systematic collection of the products of each stage of civilization, and the arrangement of their sequence in development—are thus the leading motives of our two great anthropological museum.

To my mind, one of the most remarkable things I have seen in this country is the working of the bureau of ethnology as part of the general working of the Government department to which it belongs. It is not for me, on this occasion, to describe the working of the Smithsonian Institution, with its research and publications extending almost through the whole realm of science; nor to speak of the services of that eminent investigator and organizer, Prof. Spencer F. Baird. It is the department occupied with the science of man of which I have experience; and I do not think that anywhere else in the world such an official body of skilled anthropologists, each knowing his own special work, and devoted to it, can be paralleled. The bureau of ethnology is at present devoting itself especially to the working-up of the United States, and to the American continent in general, but not neglecting other parts of the world. And I must say that I have seen with the utmost interest the manner in which the central organism of the bureau of ethnology is performing the functions of an amasser and collector of all that is worth knowing; how Major Powell is not only a great explorer and worker himself, but has the art of infusing his energy and enthusiastic spirit through the branches of an institution which stands almost alone, being, on the one hand, an institution doing the work of a scientific society, and, on the other hand, an institution doing that work with the power and leverage of a government department. If we talked of working a government institution in England for the progress of anthropology in the way in which it is being done here we should be met with—silence, or a civil answer, but with no practical result; and any one venturing to make the suggestion might run the risk of being classed with that large body described here as "cranks." The only way in which the question can be settled, how far a government may take up scientific research as a part of its legitimate functions, is by practical experiment; and somehow or other your president is engaged in getting that experiment tried, with an obvious success, which may have a great effect. If in future a proposition to ask for more government aid for anthropology is met with the reply that such ideas are fanatical, and that such schemes will produce no good results, we have a very good rejoinder in Washington. The energy with which the Bureau of Ethnology works throughout its distant ramifications has been a matter of great interest. It is something like what one used to hear of the organization of the Jesuits, with their central authority in a room in a Roman

palace, whence directions were sent out which there was some agent in every country town ready to carry out with skill and zeal. For instance, it was interesting at Zuñi to follow the way in which Colonel and Mrs. Stevenson were working the pueblo, trading for specimens, and bringing together all that was most valuable and interesting in tracing the history of that remarkable people. Both managed to identify themselves with the Indian life. And one thing I particularly noticed was this, that to get at the confidence of a tribe, the man of the house, though he can do a great deal, cannot do all. If his wife sympathizes with his work, and is able to do it, really half of the work of investigation seems to me to fall to her, so much is to be learned through the women of the tribe which the men will not readily disclose. The experience seemed to me a lesson to anthropologists not to sound the "bull-roarer," and warn the ladies off from their proceedings, but rather to avail themselves thankfully of their help.

Only one word more, and I will close. Years ago, when I first knew the position occupied by anthropology, this position was far inferior to that which it now holds. It was deemed, indeed, curious and amusing; and travelers had even, in an informal way, shown human nature as displayed among out-of-the-way tribes to be an instructive study. But one of the last things thought of in the early days of anthropology was that it should be of any practical use. The effect of a few years' work all over the world shows that it is not only to be an interesting theoretical science, but that it is to be an agent in altering the actual state of arts and beliefs and institutions in the world. For instance: look at the arguments on communism in the tenure of land in the hands of a writer who thinks how good it would be if every man always had his share of the land. The ideas and mental workings of such a philosopher are quite different from those of an anthropologist, who knows land-communism is an old and still existing institution of the world, and can see exactly how, after the experience of ages, its disadvantages have been found to outweigh its advantages, so that it tends to fall out of use. In any new legislation on land, the information thus to be given by anthropology must take its place as an important factor.

Again: when long ago I began to collect materials about old customs, nothing was farther from my thoughts than the idea that they would be useful. By and by it did become visible, that to show that a custom or institution which belonged to an early state

of civilization had lasted on by mere conservatism into a newer civilization, to which it is unsuited, would somehow affect the public mind as to the question whether this custom or institution should be kept up, or done away with. Nothing has for months past given me more unfeigned delight than when I saw in the *Times* newspaper the corporation of the city of London spoken of as a "survival." You have institutions even here which have outlived their original place and purpose; and indeed it is evident, that when the course of civilization is thoroughly worked out from beginning to end, the description of it from beginning to end will have a very practical effect upon the domain of practical politics. Politicians have, it is true, little idea of this as yet. But it already imposes upon bodies like this Anthropological Society a burden of responsibility which was not at first thought of. We may hope, however, that under such leaders as we have here, the science of anthropology will be worked purely for its own sake; for, the moment that anthropologists take to cultivating their science as a party-weapon in politics and religion, this will vitiate their reasonings and arguments, and spoil the scientific character of their work. I have seen in England bad results follow from a premature attempt to work anthropology on such controversial lines, and can say that such an attempt is not only in the long-run harmful to the effect of anthropology in the world, but disastrous to its immediate position. My recommendation to students is to go right forward, like a horse in blinkers, neither looking to the right hand nor to the left. Let us do our own work with a simple intention to find out what the principles and courses of events have been in the world, to collect all the facts, to work out all the inferences, to reduce the whole into a science; and then let practical life take it and make the best it can of it. In this way the science of man, accepted as an arbiter, not by a party only, but by the public judgment, will have soonest and most permanently its due effect on the habits and laws and thoughts of mankind.

I am afraid I have not used well, under such short and difficult conditions, the opportunity which you have done me the great pleasure and honor of giving me here. I have tried, as I said I would, to put in the simplest way before you some considerations which appear to me as of present importance in our science, both in the old world and in the new, and I thank you in the heartiest way possible for the opportunity you have given me to do this.

At the close of the address a vote of thanks was moved by Judge Arthur McArthur, of the Supreme bench of the District of Columbia, and passed unanimously.

The President announced that by direction of the Council there would be no regular meeting of the Society until the third Tuesday in November.

EIGHTY-FIFTH REGULAR MEETING, November 18, 1884.

Major J. W. POWELL, President, in the Chair.

The President stated that by action of the Council a place for the future meetings of the Society had been secured at the Columbian University.

The Secretary of the Council announced the election of Mr. M. D. Kerr, of the U. S. Geological Survey, as an active member of the Society.

A paper entitled "AUSTRALIAN GROUP RELATIONS," by Alfred W. Howitt of Gippsland, Australia, was then read by Col. Seely.*

EIGHTY-SIXTH REGULAR MEETING, December 2, 1884.

Major J. W. POWELL, President, in the Chair.

The Secretary of the Council announced the election as active members of Messrs. Victor Mindeleff, Cosmos Mindeleff, Wm. M. Poindexter, and Wm. H. Babcock.

Dr. FRANZ BOAS read a paper on "THE ESKIMO OF BAFFIN LAND."

Although the shores of Baffin Land have been visited by whalers for a very long time, there was still little known about the Eskimo tribes inhabiting this tract of land.

The southwesternmost region, the land about King's Cape, is called by the natives Sicosuilar, *i. e.*, a land which has no fixed ice floe during the winter. It is inhabited by the Sicosuilarmiut, who go deer hunting in the low land farther north. They have intercourse with the natives of the north shore of Labrador, the Iglu-

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* Printed in the Smithsonian Report for 1883.

miut, *i. e.*, the inhabitants of the other side, crossing Hudson Strait from King's Cape to Cape Wolstenholme.

The middle region of the north shore of Hudson Strait is inhabited by the Akudliarmiut who go deer hunting to the large lake Agmakdgua, where they meet with the Nugumiut, the inhabitants of the peninsula between Frobisher Bay and Cumberland Sound. The shore of Davis Strait is divided into three parts:—Oko, Akudnirn, and Aggo, *i. e.*, the lee side, the centre, and the weather side. Oko, the land of the Cumberland Sound, is inhabited by the Okomiut who in olden times were divided into the Tellirpingmiut on the west shore of Cumberland Sound; the Kinguamiut, at the head of it; the Kignaitmiut on the high Cumberland peninsula, and finally the Saumingmiut on Davis Strait, as far as Exeter Bay and Cape Dier. As the number of the Okomiut has been greatly diminished there scarcely exists any difference between these tribes now.

The inhabitants of Padli are nearer to the Akudnirmiut than to the Okomiut. The Aggomiut consist of two tribes: The Tudnumirmiut of Pond's Bay, and the Tudnunirossirmiut of Admiralty Inlet. Besides there are the Iglulingmiut of Fury and Hecla Strait, with whom we have been made acquainted by Parry and Hall.

I have visited the different tribes of Cumberland Sound and Davis Strait as far as Akudnirn, and no settlement in this country escaped my notice. As there are quite a number of natives of different tribes settled among these I was able to gather a good deal of information about all the Eskimos from Sicosuilar to Tudnunirn.

The most interesting tribe are the Tellirpingmiut, the inhabitants of the west shore of Cumberland Sound, more particularly speaking, of Nettilling fiord. This is one of the few Eskimo tribes living inland. From former reports we only learned that the Kinnepatu, the Eskimo of Chesterfield Inlet, on the west shore of Hudson Bay, live nearly all the year round on deer and musk oxen, which they hunt on the plains between Back River and Chesterfield Inlet, only coming down to the seaside during the winter.

At the present time the Tellirpingmiut have the same custom. In the month of May they leave their winter settlement and travel with their dogs and sledges inland to the large lake Nettilling, (Lake Kennedy, of the old charts) and get to the place of their settlement, Tikerakdjuak, on the south shore of the lake, long before the ice breaks up. They take with them one or more bags of blubber for their lamps; but sometimes they do not even carry

as much, as they are able to cook with the heather found in abundance on the vast plains of the lake, and burn deer marrow in their lamps.

Now and then they secure a seal in the lake, but they cannot rely on their hunt as these animals are too few in number. In the western part of the lake they seem to be more plentiful; but in the eastern portion their number has been greatly diminished. I suppose that this is principally the reason why the Tellirpingmiut do not any longer stay all the year round on the shores of the lake as many of them formerly did. They seem to have spent there the greater portion of their lives, occasionally visiting the seaside to provide themselves with skins of the young and old seals. It very seldom happens now that any men winter inland, as the number of seals is too small. In the spring of the year they live on deer and the innumerable birds which are caught while molting. The Eskimos return to the entrance of Neltilling fiord about the beginning of December, when the ice in the fiords is strong and well covered with snow.

The other Okomiut, who are settled in four places on the west shore, two on the east shore, and one between Cape Mercy and Cape Micklesham, never leave the coast for any length of time. Only a few go in their boats also to Lake Nettilling, as this is the best place for deer hunting. They leave after the breaking up of the ice in July and return during the first days of October.

By far the most of them spend the summer at the head of the fiords whence they start deer hunting inland, returning after a few days' absence. The old men and the women meanwhile live on salmon which are caught in abundance in the small rivers emptying into the fiords. In winter they settle on the islands nearest to the open sea. Throughout the cold months until the sun rises higher they go sealing with the harpoon, watching the seal at its breathing hole. In March, while the seal brings forth its young, all the natives are eager to secure as large a number as possible of young seal skins, which are highly valued for the under jackets and winter pants for men and women.

In the fall the inhabitants of Saumia and Padli secure a great number of walruses which supply them with food and blubber until late in the winter. They only go sealing in order to enjoy themselves, as they generally have sufficient walrus meat to last them the whole year.

Sometimes even there is some left in summer. In spring they go bear hunting. The skins of these animals are exchanged for guns and ammunition, when the whalers visit the coast returning from their hunting grounds off Lancaster Sound.

The Tudnunirmiut hunt the white whale and the narwhal whose ivory is highly valued.

Though the Eskimos shift their habitations according to the seasons from one place to another we must not consider them a people without stationary abodes, for at certain seasons they are always found at the same places.

There are some doubts about the origin of the old stone foundations met with in every part of Arctic America, even in countries not any longer inhabited by Eskimos, as the Parry Archipelago and the northern part of East Greenland. It was believed that the central Eskimos forgot the art of building stone houses and only lived in snow huts.

In Baffin Land I found a great number of stone, turf, and sod foundations, apparently of very ancient origin. If the Eskimos come to a place where they know that stone houses exist they build these up into a comfortable home, covering the old walls with a double seal-skin roof and heather. In the settlement Anarnitung, near the head of Cumberland Sound, and at Okkiadliving, on Davis Strait, they frequently live in these houses which they call Kagmong.

I found two different styles of construction, one with a very large floor and a remarkably short bed-place ; the other with both parts of about the same size. The former the Eskimos ascribe to the Tunnit, or as they are often called, Tudnikjuak, a people playing a great part in their tales and traditions. The latter are ascribed to their own ancestors, the ancient Eskimos.

Indeed they do not build any stone houses now, as they always find in the places of their winter settlements the old structures which are fully sufficient for the number of men inhabiting the country now, which is very small as compared with that of former times. From different reports I conclude that Cumberland Sound about fifty years ago was inhabited by 2,500 Eskimos who are now reduced to about 300 souls.

In winter time they mostly build snow houses consisting of a high dome with a few smaller vaults attached, used as entrances which keep the cold air out of the main room. The Okomiut and Akud-

nirmiut cover the inside of the same with seal-skins; while the Nugumiut and Akudlirmiut leave the walls bare. They cut the pieces of snow much thicker and bury the whole house in loose snow which they stamp down with their feet.

In summer they live in tents made of seal-skin. The back part is formed by six poles, arranged in a semicircle and lashed together at their converging points. Two poles run from this junction to the entrance, which is also formed of two poles. The Okomiut build the back part of the tents much less steep than the Akudnirmiut. The Aggomiut use a tent with only one pole in the center, and one for the entrance.

I have been informed that three different styles of clothing are used in Baffin Land, two o which I have seen myself. The Sicosuilarmiut are said to use jackets with a broad tail and a hood, which latter is not pointed. The Nugumiut and Okomiut are very well clad, having their garments neatly trimmed with skins of different color and adorned with skin straps. Their hoods are long pointed, and the tails of the women's jackets very narrow. The jackets of the men have either no tail whatever, or one that is very short. The women's pants consist of two parts, the leggins being fastened by a string to the short breechlets.

The Akudnirmiut and Aggomiut use very large hooded jackets with a small point at the top. Their clothing is much inferior to that of the Okomiut. I have seen scarcely any attempt to adorn it in any way. The women wear very large boots which reach up to the hips. In Pond's Bay they are sometimes kept up by whale bone, and they are in the habit of carrying the young children in them.

There exist only very slight differences in the dialects from Akudliak to Pond's Bay, and those I found refer only to the vocabulary. However, in the most common phrases, the way of greeting, etc., every tribe has its own style. Nor could I find any differences with reference to their traditions. It is possible that a number of the Oko stories are unknown in Tudnunirn, and *vice versa*, but I am not sufficiently acquainted with the Tudnunirmiut to positively decide the question.

There are some differences between the Okomiut and the Akudnirmuit in the arrangement of feasts, which are repeated every fall, during which some natives make their appearance disguised and masked as representatives of a fabulous tribe.

All the Eskimos of Baffin Land are fond of music and poetry. They sing the old songs of their people, and spend the long winter nights telling traditions and singing the old monotonous tunes of their songs or composing new ones. I made the acquaintance of a few poets whose songs were known in every place I visited.

All their tales and the themes of the old songs are closely connected with their religious ideas. · Though there is a strong resemblance between many of their own traditions and those of the Greenlanders, I found quite a number of new tales and religious ideas hitherto unknown. They are familiar with the Erkilik of the Greenlanders, whom they mostly call Adlet, and the Tudnik, who, however, do not inhabit the interior but are said to have lived formerly with the Eskimos on the same shores and in the same settlements. According to their tradition, which is only preserved in parts in Greenland, the Adlet, Kodlunarn, (white men) and Innuit are the children of one mother and her husband, a red dog, who lived at Igluling, in Fury and Hecla Strait. From there all the different tribes of Innuit are said to have spread over the country, now occupied by them.

It is worth noticing that the Labrador Eskimos know the Adlat and the Tudnik too. In Erdmann's Wörterbuch des Labrador Dialects, Adlat is explained as Indian of the Interior; Tudnik as a Greenlander. I believe, however, that these meanings were given to these words by the missionaries, while in reality they signify the same as in Baffin Land and Greenland. To learn whether there are any traditions relating to the Adlat or Erkillek would be of special interest.

The Eskimos of Baffin Land have no knowledge of the Supreme Being, Torngarsuk, whom the Greenlanders once considered to be superior to all the numerous lower spirits called the Torgnet. Of these there are a great many, but the most prominent ones appear in the shape of a bear, a man, or a woman, inhabiting the large boulders, which are found in great numbers scattered over the country.

These spirits act as genii of certain favored men who by their aid become great sorcerers. They are able to cure dieases, to detect offences, to give good luck in hunting, and they visit the spirits of the moon and of the stars.

The Eskimos entertain a great fear of the Tupilat, the Spirits of the Dead, who kill every one daring to offend them. This is the

reason why they are afraid to touch the corpse of the deceased, and why they destroy every object which once belonged to a dead Eskimo.

The soul of the dead Innung goes to the land Adlivum, beneath the earth of which an evil spirit, Sedna, is mistress. In olden times she was an Eskimo woman herself, married to a fulmar who used her very badly. She escaped in the boat of her father who flung her overboard to save his own life from the wrath of the bird, after having detected the loss of his wife. While Sedna clung to the edge of the boat the father cut off her fingers which were changed into seals and whales. To revenge herself she caused two dogs to gnaw off her father's feet and hands. Then the earth opened and they went down to the land Adlivum. As the Eskimos kill the seals and whales that have risen from Sedna's fingers she hates and pursues them. Only those who come to an unnatural death escape her and ascend to Heaven to the land Kudlivum where innumerable deer are found, and where they are never troubled by either ice or snow.

Sedna is feared by the Eskimos even more than the Tupilat and the traditions about her have the greatest influence on their habits, manifesting itself mostly in laws about food and interdiction of labor on certain days.

To compare the habits and traditions of the Eskimos of Baffin Land with those of the Smith Sound and Greenland will be of much interest, as these tribes connect the central with the eastern Eskimos.

Tribes which may easily be studied, and whose customs are of prime importance are the Sicosuilarmiut and Iglumiut, and their connections with the Labrador natives. It is a matter of regret that so little is known of the inhabitants of Southampton Island and of the west shore of Hudson's Bay, although Hall spent five winters in those regions. The researches of Mr. Turner in Ungava will fill a great gap in our knowledge of the central tribes.

Another tribe of great importance are the inhabitants of Admiralty Inlet, who seem to be very numerous up to the present time.

Even now it is possible to trace the connection between the tribes from King William's Land to Smith Sound and Labrador. The Netchillirmiut of Boothia Felix, who are now mixed with the Ugjulirmiut of King William's Land and Adelaide Peninsula most probably occupy part of the old country of the Ukusiksalingmiut of Back River. These natives, who live principally upon musk oxen, cross

the land in visiting the shores of Wager River. The Netchillik Eskimos travel through the land of the Sinimiut of Pelly Bay to Eivillik (Repulse Bay). The Eivillinmiut frequently have intercourse with the Igluling tribe, who formerly visited the Cumberland Sound Eskimos by the way of Majoraridjen, the country north of Lake Nettilling (Lake Kennedy). Three roads are used in traveling from Igluling to the west shore of Baffin Bay and to Lancaster Sound, the most western through the fiord Tessiujang, near Cape Kater, to Admiralty Inlet; the other to Ikalualuin (Arctic Sound) in Eclipse Bay and the third one to Anaulereelling (Dexterity Bay). The Tudnunirossirmiut sometimes cross Lancaster Sound, and were found on the western part of North Devon, which they call Tudjan. They cross this land and Jones Sound on sledges and have intercourse with a tribe on Ellesmere Land, which they call Umingmamnuna. From Bessels' researches we know that they cross Smith Sound, for he found amongst the Ita-Eskimos a man who had lived in former years amongst the Akudnimiut on the east coast of Baffin Land. I myself found a native near Cape Kater, north of Home Bay, who had lived somewhere near Cape Isabella at the entrance of Smith Sound for several years.

The questions which may be settled by a more thorough knowledge of the habits and traditions of all these and the more western tribes which have scarcely been seen by any white men, may prove of prime importance for the solution of the question relating to the origin and migrations of this people.

Mr. JOHN MURDOCH read the following paper on "SEAL CATCHING AT POINT BARROW."

The capture of seals is one of the most important of pursuits among the Eskimos of the two villages at Point Barrow. A failure of the seal harvest would be as disastrous to them as the failure of the potato crop to the Irish, or the rice crop in India. Not only does the flesh of the seal form the great staple of food, but its fat furnishes them with oil to light and warm their winter houses, to oil their water-proof boots and harpoon lines, and to keep the water out of their skin boats. The skin serves to make their water-proof boots and leggings, the soles of their winter boots, canteens, the covers of the kaiaks, or small skin canoes, and, rarely, their outer clothing; cut into thongs it furnishes a serviceable cord which they make into nets and harpoon lines, and employ for all the varied

purposes for which we use cord. In former times and occasionally at present, the skin served to cover the summer tent, or *tú pĕk.* No part of the animal is wasted. Even the entrails are saved, and dressed, and made into water-proof frocks to wear over the fur clothing in rainy and snowy weather. If their were no seals at Point Barrow there could be no Eskimos, barren as the country is of fish and reindeer.

The following species are pursued: First, and most important, the Ringed Seal or Nĕtyĭ (*Phoca foetida*). This is *the* seal par excellence, and the only one taken in any considerable numbers, by all the methods which will be described hereafter. Next in importance is the great Bearded Seal, úg′ru (*Erignathus barbatus*). This is comparatively rare, though a good many are taken much in the same manner as the walrus with the heavy harpoon and rifle from the umiak. The skins are especially valued for covering the large skin boats, and for making heavy harpoon lines. The other two species are of extremely rare occurrence. The Harbor Seal, kasĭgíä, (*Phoca vitulina*) is occasionally caught in summer in the nets at Elson Bay, and the rare and beautiful Ribbon Seal (*Histriophoca fasciata*), the kaixólĭñ, is now and then taken in the early winter.

When the ice-pack comes in in the autumn, and the sea is beginning to close, it may be about the middle of October, the natives who are now all back from their summer wanderings and settled for the winter, begin the pursuit of the ne′êtyê. At this season there are many open holes in the pack to which the seals resort. Here they are taken by shooting them with the rifle as they show their heads above water, and securing them with the retrieving harpoon or naúlĭgû. The line and harpoon-head belonging to this are generally carried attached to the gun-case which is slung across the shoulders, and the shaft serves as a staff for walking and climbing about the rough ice. A hunter is lucky if he secures more than one or two seals in this way in a day's tramp. He generally drags his game home by a line looped through a hole in the under jaw. Wherever the sea is sheltered by grounded ice, it will freeze on calm nights to the depth of three or four inches, and in these newly-formed fields of ice are soon to be found small round holes, which the seals have kept open for fresh air. The natives resort to these holes, provided with a rifle, a different form of harpoon, the úna, with a long, slender, loose-shaft, fitted for thrusting through the small hole, and a little three-legged stool, nĭgäwau′otĭn, just

large enough for a man to stand upon, to keep the feet from getting chilled by the ice. A little rod of ivory is sometimes thrust down through the hole to indicate the approach of the seal, and the hunter standing or squatting on the stool with his rifle and spear in readiness, waits patiently for the seal to come. As soon as he comes to the surface he is shot through the head and the úna is immediately thrust down through the hole to secure him. The ivory icepick, túu, serves to make the hole large enough to drag him through. Both these methods of hunting are pursued during the whole winter whenever there are open holes or fields of newly-formed ice, and natives are continually scouring the ice-field armed with rifle and naúlǐgû, in the hopes of finding open holes. The greatest catch of the year known takes place after Nov. 15th, when the sun has sunk below the horizon for his 72 days' absence, and the nights are long and dark, while the days are only a few hours' twilight. At this season, wide cracks frequently form in the pack, miles in length and a mile or two from the shore, and of course are a great resort for the seals. As soon as such a crack is discovered, and scouts are continually on the watch for them, the men turn out in force and skirt along the edge of the crack till they find a suitable place for setting their nets. A place is selected where the ice is level and not too thick for about 100 yards from the edge of the crack, and the nets are set as follows: The net is made of seal-thong in large meshes, and is about 15 or 16 feet long by 10 deep. Two small holes are dug through the ice, about the length of the net apart, in a line parallel to the edge of the crack, and between them is cut a hole large enough to admit the passage of a seal. A long line with a plummet on the end is let down through one of the small holes and grappled and drawn up through the middle hole by a long, slender pole with a hook on the end of it. This is made fast to one upper corner of the net, and a similar line drawn through the other small hole and made fast to the other upper corner. By hauling on these lines the net is drawn down through the middle hole and hangs like a curtain under the ice. A line is also attached to it by which it can again be drawn up through the middle hole. The end lines are loosely made fast to lumps of ice and as darkness sets in the hunter stations himself near the hole and begins rattling gently on the ice with the butt of his spear, scraping with a tool made of seals' claws mounted on a wooden handle, or making any gentle monotonous noise. This excites the curiosity of the seals who are cruising

around in the open water, and one will at last come swimming in under the ice towards the sound. Of course he strikes against the loose net, runs his head or flipper through it and his struggles to escape only serve to entangle him still more. The running out of the end lines informs the hunter that there is a seal in the net. He waits till he thinks that he is sufficiently entangled, and then hauls him up through the middle hole. If he is not already drowned, his neck is broken by bending the head back sharply, and he is disentangled from the net which is set again. Of course, he very soon freezes stiff, and if there is enough snow on the ice, he is stuck up on his tail, so as not to be covered up and lost should a drifting snowstorm come on. One man has been known to take as many as thirty seals in this way in a single night. This method of fishing can only be practiced in the darkest nights. A bright moonlight, or even a bright aurora seriously interferes with success. The dark nights in December, when the moon is in southern declination and does not rise, are generally the times of a great catch. The dead seals are stacked up and brought in when convenient by the women and dogsleds. Any small crack in the ice to which the seals resort is immediately surrounded by a cordon of nets which are visited every two or three days, and many seals are thus taken. About the end of February, when the sun is bright and the ice thick, the seals have formed permanent breathing-holes to which many resort. When such a hole is found, a net is set flat underneath it, by making four or five holes round it, drawing the net down through the main hole, and the corners out to these holes. One man, who has stayed at home from the spring deer-hunt, will generally have three or four nets set in this way, which he visits every few days. This method of netting is kept up during the spring till the ice begins to melt on the surface and the seals come out on it, where they are sometimes shot. Many seals are killed with rifle and naúlĭgû from the Miaks when whaling or hunting walrus in the spring and summer, and they are also caught in nets set along shore in Elson Bay.

There is still one more method of taking seals seldom practiced near the villages, and only in the summer. This is with the light darts, kúkigû, from the kaiak. These darts are so arranged that the little barbed head is detachable and attached to the shaft by a line forming a bridle, which always pulls the shaft transversely through the water. Three of these darts are carried in the kaiak and darted into the seal with a hand board. The resistance of all three shafts wearies the seal out until he can be approached and despatched.

DISCUSSION.

Mr. DALL gave a description of Norton Sound, which is a shallow estuary subject to sudden changes in depth due to direction of wind. Seal fishing in winter is practiced on the edge of the ice about ten to twenty miles from shore, but is attended with much danger owing to the liability of the floe to break up and go to sea with a strong eastwardly wind. The best seasons are early autumn and spring. In summer short nets supported by three stakes driven in the mud in about one to two fathoms water where thereis current are used and take many seal. The upper edge of the net is taut, the lower part hangs nearly free, and about five feet in height. The seal are usually drowned in the net, but if living are killed with a club. If a seal is shot and then secured, a pin like a large nail with a broad head is fastened in the wound to prevent loss of blood which is much esteemed in the Innuit cuisine.

A peculiar spear or lance is used by the Nunivak people, being a three-sided ivory point as large as the biggest walrus tusk will make, straight, mounted on a heavy wooden shaft. The head may be eighteen inches long, is drilled in the median line of each face to the center of the blade, and a slit is then sawed nearly the whole length. the three slits meet in the center which is entirely excavated, but without enlarging the slits which remain only as wide as the thickness of the saw. Pressure from behind springs out the thin walls of the lance head which has a sharp apex—on the removal of pressure the walls resume their position gripping firmly the tissues which have protruded into the slips. Pulling only tightens the grip. This style of lance has not as far as the speaker was aware been any where described, though the specimens which he saw in 1868 were afterwards sent to one of the museums in Germany.

Responding to a question, Mr. DALL said that he thought we were not at present in a position to adjudge whether the Eskimo were related to the cave dwellers as advocated by Dawkins, though their mode of life presents many similarities.

Prof. MASON spoke of the richness of information now at our command in Washington, Greenland being represented by Dr. Bessels; Cumberband Gulf by Dr. Boas; Ungava Bay by Lucien M. Turner; Point Barrow by Mr. Murdock; and the Western Eskimos by Mr. Dall. He also called the attention of the Society to the great amount of invention wrapped up in an Eskimo harpoon. Hitherto students had been satisfied with speaking of harpoons with-

out specifying the variety; but Mr. Murdoch's own collection contained three types: lances, darts, and harpoons. Of lances there were three kinds, the whale, the walrus, and the deer lance. Of darts there were several varieties, all carried by the throwing stick, among them the bird or pronged dart (with or without side prongs), the feather dart, the float dart, the bridle or martingale dart, and the harpoon dart. Of harpoons Mr. Murdoch could exhibit several varieties. The most interesting was the retriever. The Eskimo standing on the edge near thin ice shoots the seal in the water, and after breaking a channel with the ice-pick on one end, launches the whole implement at the animal, holding on to a line attached to the harpoon. By this means he could draw the dead body to the thick ice.

Mr. MURDOCH, in answer to a question of Dr. Bessels, said the seal-nets appear to have never been made from whalebone. Nets of this material with small mesh are used for taking whitefish, &c. The seal-net is a comparatively modern invention. Nĭkawáalu, an intelligent middle-aged native, full of tradition, says "Adráni (beyond the memory of man now living) there were no nets and they killed seals with the spear (únä) only." No work that requires hammering or pounding on wood must be done during the whaling season, and even rapping with the knuckles on wood is bad. They asked us to leave off work on our block-house in the spring of 1882, saying it would drive off the whales. The whaling was a failure that season.

Mr. MURDOCH also stated the following myths:

A′sĕlu, the mythical dog, was tied to a stake. He gnawed himself loose, and went into the house where he found an Eskimo women, with whom he had sexual intercourse. From this woman sprang the human race.

A "doctor" starting on a fishing trip in the fall gave tobacco to the dead man at the cemetery, breaking off tiny bits and throwing them into the air. When he arrived at the river he also gave tobacco in the same way to the demon *Tuûñ-a*, saying "Tuûña, Tuûña, I give you tobacco! Give me plenty of fish."

They said the aurora (kiólyä) was *bad*, that there was danger of its striking a man in the back of the neck and killing him. Consequently, in coming to and fro from the village after dark in twos or threes (they never dare go alone), one carries a drawn knife or dagger to thrust at the Aurora and drive it away. Frozen dogs' excrement thrown at the aurora will also drive it off.

During a bright aurora the children especially sing to it, sometimes nearly all night, performing a stamping dance, with the fists clenched. The song has many verses, with the same refrain. The first verse, as follows:

"Kiólyä ke! Kiólyä ke!
A yáñä, yáñä, ya!
Hwi, hwi, hwi, hwi!"

EIGHTY-SEVENTH REGULAR MEETING, Dec. 16, 1884.

Major J. W. POWELL, President, in the Chair.

The Secretary of the Council announced the election of Admiral Thornton A. Jenkins, U. S. N., Mr. John Murdock, and Mr. Lucien M. Turner as active members of the Society.

The Curator presented a report showing the receipt of seventy-three gifts, comprising books, papers, and pamphlets, as follows:

GIFTS.

From the DIRECTOR.—Second Annual Report of the Bureau of Ethnology. 1880–81. Major J. W. Powell. Washington. 1883. Pp. 487. 8°. Illustrations and plates.

From Mr. GEO. F. BLACK.—British Antiquities; their present treatment and their real claim. By A. Henry Rhind. Edinburgh. 1885. Pp. 47. 8°.

—— Notice of a collection of flint implements found in the neighborhood of Fordoun, Cincardineshire. Rev. James Brodie. Pp. 5.

—— On certain beliefs and phrases of Shetland Fishermen. Arthur Laurenson. Pp. 6.

—— Did the Northmen extirpate the Celtic inhabitants of the Hebrides in the 9th century? Capt. F. W. L. Thomas, R. N. Pp. 35.

—— Notice of a collection of flint arrow-heads and bronze and iron relics from the site of an ancient settlement, recently discovered in the Culbin Islands, near Findhorn, Morayshire. Hercules Linton. Pp. 4.

—— Notes respecting two bronze shields recently purchased for the museum of the Society, and other bronze shields. Wm. T. McCulloch. Pp. 4.

From the DIRECTOR.—Notes on Mediæval "Kitchen Middens" recently discovered in the monastery and nunnery on the Island of Iona. John Alexander Smith. Pp. 14.

—— Note of a fragment of a Rune-inscribed stone from Aith's Vol. Cummingsburgh, Shetland. George Stephens. Pp. 6.

—— Letter to the Schoolmasters of Scotland, from the Society of Antiquaries. Edinburgh. 1860. Pp. 13.

—— Note on a cist, with an urn, discovered at Parkhill, near Aberdeen, in Oct., 1881. Wm. Ferguson. Pp. 4.

—— Notes on some stone implements, &c., from Shetland. John Alexander Smith. Pp. 9.

—— Notice of the discovery of a massive silver chain of plain double rings or links at Hardwell, Berwickshire. By the Hon. Lord Douglas. With notes of similar silver chains found in Scotland. By John Alex. Smith. Pp. 7.

—— Notes on the Antiquities of the Island of Tiree. J. Sand. Pp. 5

—— Notice of a sculptured stone, bearing on one side an inscription in runes, from Kilbar, Island of Barra. Dr. Geo. Stephens. Pp. 4.

—— Notice of a Cranium found in a short cist near Silvermoor, Carstairs Lanarkshire. D. R. Rankine. Pp. 3.

—— Notice of an underground structure recently discovered on the farm of Mickle Kinord, Aberdeenshire. Rev. J. G. Michie. Pp. 3.

—— Notice of shell-mounds at Lossiemouth. E. G. Duff. Pp. 2.

—— Notice of urns in the museum that have been found with articles of use or ornament. Joseph Anderson. Pp. 16.

—— Notice of a hoard of bronze weapons and other articles found at Monadh-Mor, Killin. Charles Stewart. Pp. 5.

—— Notice of a flint arrow-head in the shaft, found in a moss at Fyvie, Aberdeenshire, with notes in illustration of the manufacture of arrow shafts with flint tools. Joseph Anderson. Pp. 6.

—— Notes on the character and contents of a large sepulchral cairn of the bronze age at Collessie, Fife, &c. Joseph Anderson. Pp. 23.

—— Notes on the contents of shell-heaps recently exposed in the Island of Coll. Donald Ross. Pp. 2.

—— Notice of ancient graves at Doudan, near Ballantrae, Ayrshire. John Carrick Moore. Pp. 3.

—— Donations to the museum. Francis Abbott. Pp. 3.

—— On the presentation of national antiquities and monuments in Denmark. J. J. A. Worsaae. Pp. 15.

From the DIRECTOR.—Notes of some recent excavations in the Island of Unst, Shetland, and of the collections of stone vessels, implements, etc. Thomas Edmonston. Pp. 5.

—— Note of a donation of four sculptured stones from Monifieth, Forfarshire. James Neish. Pp. 8.

—— Notes of the sculptured caves near Dysart, in Fife, &c. Miss C. Maclagan. Pp. 14.

—— Notice of the discovery of two sculptured stones, with symbols, at Rhynie, Aberdeenshire. Miss C. Maclagan. Pp. 3.

—— Notice of excavations in Cannis, in Strathnaver, Sutherlandshire, &c. John Stewart. Pp. 5.

From Prof. L. STIEDA.—Anthropologische Untersuchungen am Becken lebender Menschen. Paul Schröter. Dorpat. 1884. Pp. 83.

From the AUTHOR.—H. Fischer. On stone implements in Asia. Worcester, Mass. 1884.

From the AUTHOR.—Dr. H. F. C. Ten Kate. Quelques observations sur les Indiens Iroquois. Pp. 5. From *Revue d'Anthrop., de Paris.*

—— Sur la synonymie ethnique et la Toponymie chez les Indiens de l'Amérique du Nord. Amsterdam. 1884. Pp. 11.

[Reprinted from Trans. Roy. Acad. Sci. Amsterdam.]

—— Variétés. Notes sur l'ethnographie des Zuñi. Pp. 3.

—— Quelques observations ethnographiques recueillies dans la presqu'ile Californienne et en Sonora. Pp. 6.

—— Sur Quelques Crânes de l'Arizona et du Nouveau Mexique. Pp. 7.

(Extrait de la *Revue d'Anthropologie.*)

—— Matériaux pour servir à l'Anthropologie de la presqu'ile Californienne. Paris. 1884. Pp. 19.

[From Bull. Soc. d'Anthrop.]

From the AUTHOR.—Alph. de Candolle. Hérédité de la couleur des yeux dans l'espèce humaine. Geneva. 1884. Pp. 23.

[Ext. Arch. des Sciences Physiques et Naturelles.]

From the AUTHOR.—Baron Joseph De Baye. Sujets décoratifs au Règne Animal dans l'industrie Gauloise. Paris. 1884. Pp. 8.

[Ext. Mem. Nat. Soc. of Antiquaries of France.]

From the AUTHOR.—Adrian de Mortillet. Premier décade paléoethnologique. Paris. 1881. Pp. 11.

—— Deuxième décade paléoethnologique. Paris. 1882. Pp. 15.

From the AUTHOR.—Heinrich Fisher. Le Précurseur de l'Homme. 1884. (L'Homme, No. 13.)

—— Evolution des espèces, évolution des mots. (L'Homme, No. 20.) Further remarks on Nephrite. Verhandl. Berliner Anthrop. Gesellschaft. 1884. Pp. 2. Correspondenz-Blatt. June, 1884. Containing note on a Nephrite Axe, from Brazil.

From the AUTHOR.—Elmer R. Reynolds. Memoir on the Pre-Columbian shell-mounds at Newburg, Md., and the aboriginal shell-fields of the Potomac and the Wicomico rivers. Copenhagen. 1884. Pp. 22. From Proc. Cong. Amer. Copenhagen. 1883.]

From the AUTHOR.—Juan Ignacio de Armas. La Tabula de los Caribs. Estudios Americanistas, I. Habana. 1884. Pp. 31. [Read to the Soc. Anthrop. Havana.]

From the AUTHOR.—Protass Chandra Roy. The Mahabharata. Calcutta. Parts 9–11, inclusive.

From the AUTHOR.—A. B. Meyer. Ein Zweiter Rohnephritfund in Steiermark. Vienna. Pp. 12.

—— Uber Nephrite und ähnliches Material aus Alaska. Dresden. 1884. Pp. 21.

—— Ein neuer Fundort von Nephrit in Asien. Dresden. 1883. Pp. 10.

—— Ueber die namen Papúa, Dajak und Alfuren. Wien. 1882. Pp. 18.

—— Bemerkungen über Nephrit. Breslau. Dr. H. Traule. 1884. Pp. 1.

From the AUTHOR.—Henry Phillips. On a supposed Runic inscription at Farmouth, Nova Scotia. Philada. 1884. [From Proc. Am. Phil. Soc'y.]

From the AUTHOR.—Heinrich Fischer. Nephritfrage und submarginale (sub cutane) Durchbohrung von Steingeräthen. Berlin. 1884. Pp. 4. [Verhandl. Berliner Anthrop. Gesellschaft.]

From the AUTHOR.—C. C. Jones. The Life and Services of ex-Governor Charles Jones Jenkins. Memorial Address. Atlanta. 1884. Pp. 56.

From the AUTHOR.—G. A. Colini. Osservazioni etnografiche sui Givari. Rome. 1883. Pp. 47. [From Royal Lincean Acad.]

From the INSTITUTE.—Transactions of Vassar Brothers' Institute and its Scientific Section. Poughkeepsie, N. Y. 1883–84. Vol. 2. Pp. 166.

From the COMMISSION.—Bulletino della Commissione Archæologica Comunale di Roma. Rome. 1884. Pp. 138.

From the SOCIETY.—Boletino da Sociedade de Geographia de Lisboa. 1883. 4 ser. Nos. 8, 9.

From the COMMITTEE.—Mittheilungen des Komite der Geographischen Gesellschaft von Bern. Oct., 1883. Pp. 8.

From the SOCIETY.—VI. Jahresbericht der Geographischen Gesellschaft von Bern. 1883–84.

From the INSTITUTE.—Rep. of the Am. Archæol. Institute for 1884, at Boston. Cambridge. 1884.

From the COMPANY.—Bulletin of the Library Company of Philadelphia, for July, 1884.

From the SOCIETY.—Bulletins de la Société d'Anthropologie de Paris. Jan.–Mar., 1884.

—— Proc. and Coll. Wyoming Hist. and Geol. Soc'y, Wilkes-Barré, Pa. 1858–84.

—— The Manuscripts of the Earl of Ashburnham. (Remarks of American Newspapers.) 1884. Pp. 23.

From the INSTITUTE.—Bulletin of the Essex Institute. Vol. 15. Nos. 1–9, and Vol. 16, Nos. 1–6.

From the SOCIETY.—Bull. Société de Geographie de Paris. 1, 2, 3 Trimestre. 1884.

—— Compte rendu of the Society. Nos. 10–13, 15–17 of 1884.

—— Archivio per l'Anthropologia e la Etnologia. Firenze. 1884. XIV. Pt. 2.

—— Publications of the Imper. Russian Geograph. Soc. St. Petersburg. 1884. XX. Pts. 2, 4.

—— Report Imper. Russ. Geograph. Soc. for 1883. St. Petersburg, 1884.

—— Bollettino della Società Geografica Italiana. Roma. 1884. Pts. 1–7, 9–10, inclusive.

From the MUSEUM.—Sixteenth and Seventeenth Annual Report of the Peabody Museum. 1884. Vol. III. Nos. 3, 4.

On motion of Prof. WARD, the thanks of the Society was voted for these valuable documents.

Mr. W. H. HOLMES read a paper entitled "ORIGIN AND DEVELOPMENT OF FORM AND ORNAMENT IN CERAMIC ART."

ABSTRACT.

The material for this paper was derived chiefly from the native ceramic art of the United States. The advantages of this field, as compared with that of the classic Orient, is apparent when it is remembered that the dawn of that art lies hidden in impenetrable

shadow, while ours is in the light of the very present. The principles involved in this native art are applicable to all times and to all kinds of art, as they are based upon the laws of nature.

Ceramic art presents two classes of phenomena of importance in the study of the evolution of æsthetic culture. These relate, first, to form, and, second, to ornamentation.

Form in clay vessels embraces useful shapes, which may or may not be ornamental, and æsthetic shapes, which are ornamental and may be useful; also grotesque and fanciful shapes, that may or may not be either useful or ornamental. The shapes first assumed by vessels in clay depend upon the shape of the vessels employed at the time of the introduction of the art, and ornament is subject to similar laws.

Form may have three origins: First, adventition or accident; second, imitation of natural and artificial models; third, invention. In the early stages of art the suggestions of accident are often adopted by men, and are thus fruitful sources of improvements and progress. By such means the use of clay was discovered and the ceramic art came into existence. The accidental indentation of a mass of clay by the foot or hand, or by a fruit or stone, while serving as an auxiliary in some simple art, may have suggested the means of making a cup, the simplest form of a vessel.

In time the potter learned to copy both natural and artificial models with facility. The range of models is at first, however, very limited. The primitive artist does not proceed by methods identical with our own. He does not deliberately and freely examine all departments of nature or art and select for models those things most suitable to convenience or agreeable to fancy; neither does he experiment with the view of inventing new forms. What he attempts depends almost absolutely upon what happens to be suggested by preceding forms, and so narrow and so natural are the processes of his mind that, knowing his resources, it would be easy to closely predict his results.

The elements of ornamentation are derived chiefly from two sources—from the suggestions of incidents attending manufacture, and from objects, natural and artificial, associated with the arts. The first articles used by men in their simple arts have had in many cases decorative suggestions. Shells are exquisitely embellished with ribs, spines, nodes, and colors. The same is true to a somewhat limited extent of the hard cases of fruit, seeds, &c. These

decorative features, though not essential to the vessel, are nevertheless an inseparable part of it, and are cast or automatically copied by a very primitive people when similar articles are artificially produced. In this way a vessel acquires ornamental characters long before the workman learns to take pleasure in such details or conceives a desire beyond that of simple utility.

Artificial utensils have a still more decided influence upon ceramic decoration. The constructional features of textile vessels impress themselves upon the plastic clay in manufacture, and in time are repeated and copied for the pleasure they give. The simple ideas of embellishment thus acquired are constantly subject to modification. A single radical gives rise to a multitude of forms. The causes that tend to bring about these results are worthy of the closest study. They may be sought in the material, the form, and above all the constructional characters of the object decorated.

Prof. Mason followed Mr. Holmes with a short *résumé* of Prof. Hartt's theory of the rationale of ornament, published in the Popular Science Monthly, for January, 1884. Prof. Hartt maintains that the explanation of the shape and color of beautiful objects is to be found in the eye itself. We are pleased with certain lines because they bring the muscles of the eye into easy and healthful play.

Prof. Mason said that there was in his mind no conflict between the methods pursued in Mr. Holmes' paper and Hartt's theory—a little differently stated and expanded. Mr. Holmes traces the outline of that natural movement which aboriginal potters had followed. Hartt sought to show the subjective side and how it was that the primitive artist had chosen some forms and rejected others. If we will examine our own handwriting we shall find that the same two sets of facts present themselves. On the one hand we have books, papers, correspondence, copy-books, and many other printed and written things ever before our eyes. On the other hand there is the set of bones, muscles, and sinews, called the hand, with its great variety of lengths, thicknesses, flexibilities, so compounded in each as to give rise to a really individual hand. A man's handwriting is the movement of all these mobile parts in the lines of least resistance for each part, but always in the effort to conform to the pattern.

Now the natural world, with its shells, horns, gourds, carapaces, reeds; the mechanical world, with its shapes in hard material; the

curves and twists of spirals, cycloids, and circles innumerable, are all the patterns of things, the letters, the copy-book. The clay and the potters' tools are pen, ink, and paper. The lines of least resistance are partly in the hand of the potter, indeed, as Mr. Holmes has shown; they are partly in the muscles of the eye, as Mr. Hartt has said; but further back than all this is the force of usage and inherltance.

If we hang a hat intentionally on a peg eleven times, the twelfth time it will hang itself up. This is the universal and beneficent law of the passage of painful voluntariness into semi-automatism which follows the frequent repetition of any act whatsoever. We are pleased with certain muscular movements which have been oft repeated. There is no doubt, therefore, that the eye accustomed to certain outlines, the brain accustomed to certain consecutive impressions, are pleased with that which has become semi-automatic and habitual. We know that such tendencies are strengthened by inheritance, for we have here the application of a universal law of heredity.

Dr. FRANK BAKER said that Hartt seemed in some respects to ignore certain physiological laws in discussing the movements of the eye, and to have too little considered inventive geniuses. The source of art must be sought for in the brain that controls the eye; in the association of nerve cells that prompt the movement of muscles. Taste may follow and accept suggestions from natural forms, but art is not imitative, for, having its source in invention, it gives something nature does not.

Mr. FRANK H. CUSHING said that Hartt apparently did not try to ascertain what the eye might develop, but having certain forms at hand reasoned therefrom. The speaker had found in his studies of ceramic art in the southwest that decoration in basketry had long preceded that of pottery, and that the resulting forms might be generally attributed to adventition, and taste might have its principal source in the environment.

EIGHTY-EIGHTH REGULAR MEETING, January 6, 1884.

Major J. W. Powell, President, in the Chair.

The Secretary of the Council made the following announcements:

The election of Dr. J. H. Yarnall, as an active member of

the Society; and George H. Black, Edinboro', Scotland, and Hermann Ten Kate, The Hague, Holland, as corresponding members.

Mr. H. N. Bates read a "Memorandum concerning certain Mounds in Pontotoc county, Mississippi," visited by J. M. Pollard, Esq., of Louisiana. No abstract.

Mr. O. T. Mason read a paper prepared by Daniel G. Brinton,

"On the Probable Nationality of the Mound-Builders."

Dr. Brinton said: Further reading on the subject, and also the observations during a trip made to the principal monuments in Ohio, have confirmed me in the opinion that we need not go any farther than the Southern tribes to find the modern representatives of the mound-builders. Since I wrote the article on the mound-builders, Mr. Horatio Hale has published his suggestive paper, in which he adds strength to this position by linguistic evidence.

It would probably be hasty to point to any one of the Southern tribes as being specifically the descendants of the nation who constructed the great works in the Scioto and Miami Valleys. The evidence is ample that nearly all the tribes of the Gulf States and Lower Mississippi were accustomed to throw up works of similar character and often greater magnitude. They were of radically diverse languages, but nearly in the same plane of culture. The Natchez, the Taensas, the Choctaws, the Creeks, the Cherokees, and others might put in equal claims. The last mentioned asserted that they once lived in the Upper Ohio Valley, and that they built the Grave Creek and other mounds, and they are borne out in such claims by various historic data.

With regard to the Shawnees, it has not been sufficiently recognized by writers that their name in the Algonkin dialects is not a national appellation, but a geographical term. It means simply "Southerners," and in its earliest employment bore no special reference to the tribe whom we call Shawnees. It first appears in a map drawn in 1614, intended to show the Dutch colony around New Amsterdam. In this the "Sawannew" are located as inhabiting the whole of Southern New Jersey; whereas the Shawnees, as we understand the term first came to the notice of the New York colony in 1692. On this map it simply means "Southern rivers" with reference to the position of New York harbor.

By dialect, tradition, and political affiliation the Shawnees were a northern tribe who moved south at no very remote period. Their language, according to the Moravian missionaries, was closer to the Mohegan than to the Delaware, Nanticoke, or other Southern Algonkin dialects. By tradition they at one time were a branch of the Mohegans on the Hudson, and it was to them that they returned when driven from their towns in Carolina and on the Tennessee river. The name of their principal clan, the Pequa or Pick-e-weu, is said by Heckewelder to be the same as that of the Pequods, of Connecticut, and he relates that the Mohegans told him that the two were of the same family.

If we can depend upon this evidence, and there is no reason why it should be rejected, the "Pre-historic Shawnees" are to be looked for in New York and New England. I have no idea whether this will correspond with Professor Thomas' views, but I should be gratified to hear that we had reached identical conclusions from independent study of the subject.

The four clans of the Shawnees were assembled in Ohio, but in Pennsylvania I have not found evidence of any but the Pequas, who lived in the valley that still bears their name in Lancaster county. Their state of culture was nowise ahead of that of the Delawares. They had one clan named Chilicothe, and three of their settlements in Ohio bore this name, but while there they had not the slightest knowledge or tradition about the ancient earthworks, as we are assured by the Rev. David Jones, who went out to teach them Christianity in 1772, and who, I think, is the earliest writer who calls attention to the remarkable remains in Southern Ohio.

Prof. CYRUS THOMAS read a paper entitled "Prehistoric Shawnes, from Mound Testimony."

Before reading his paper, Prof. THOMAS said, referring to the preceding paper, that he had recently written a letter with a view to procuring an exploration of Pontotoc county, Miss., without any positive knowledge that ancient remains existed there, and that the paper of Mr. Pollard was in verification of the speaker's assumption that such remains would be found in that vicinity.

Mr. C. C. ROYCE, at the request of the Society, read an extract from a former paper of his on the origin of the "Shawnees."

President POWELL said that the papers read before the Society during the past two years seemed to establish the fact that the

mound-builders were Indians, and that many Indians built mounds. While small burial mounds were frequent and widely distributed, the larger mounds and earthworks with circumvallation—once probably crowned with palisades—were confined to narrower limits. The old theory that attributed these remains to an extinct high grade of civilization seemed to be well nigh abandoned.

Dr. GREGORY said that he had held to the old theory until he had become convinced of its error, and described a large mound, some fifty feet high, that he visited in Minnesota, which gave conclusive evidence of its comparatively recent structure. Depressions were still to be seen close about the foot of the mound, from whence material had apparently been taken to aid in forming the mound.

SEVENTH ANNUAL AND EIGHTY-NINTH REGULAR MEETING, January 20, 1885.

Major J. W. POWELL, President, in the Chair.

The Secretary of the Council announced the election of John Addison Porter and H. L. Reynolds as active members of the Society, and advised the Society of the death of Dr. Henri Martin, of Paris, France, and Dr. R. J. Farquharson, of Des Moines, Iowa, corresponding members of the Society.

The Treasurer then submitted his annual report.

On the motion of Col. MALLERY, the President appointed Messrs. Bates, Baker, and Holmes a committee (composed of members outside the Council) to audit the accounts of the Treasurer.

This session being the time for the annual election of officers, the balloting for officers resulted as follows:

PRESIDENT	J. W. POWELL.
VICE-PRESIDENTS	ROBERT FLETCHER. LESTER F. WARD. GARRICK MALLERY. OTIS T. MASON.
GENERAL SECRETARY	S. V. PROUDFIT.
SECRETARY TO THE COUNCIL	F. A. SEELY.

TREASURER	J. H. GORE.
CURATOR	W. J. HOFFMAN.
ADDITIONAL MEMBERS OF THE COUNCIL	CYRUS THOMAS. J. O. DORSEY. W. H. HOLMES. H. H. BATES. FRANK BAKER. DAVID HUTCHESON.

The President announced that the next meeting would be public, to which the members of the Biological and Philosophical Societies were specially invited for the purpose of listening to the annual address of the President.

NINETIETH REGULAR MEETING, February 3, 1885.

In accordance with previous announcement the Society assembled in public session to listen to the annual address of the President, there being present on special invitation the members of the Biological and Philosophical Societies and other friends of the Society.

Dr. J. C. WELLING introduced to the audience President J. W. POWELL, who delivered an address entitled "FROM SAVAGERY TO BARBARISM."

At the close of the address, on motion of Mr. MASON, a vote of thanks to the speaker was unanimously passed.

The Secretary of the Council announced that the Saturday course of lectures under the auspices of the Anthropological and Biological Societies had been arranged, and that programmes of the first part of the course were ready for distribution.

NINETY-FIRST REGULAR MEETING, February 17, 1885.

Prof. OTIS T. MASON, Vice President, in the Chair.

A report from the Curator was then read, including a list of publications received since his last report:

Bull. Library Co. Philada., No. 14. Jan., 1885.
Bol. Soc. Géog. Ital. Ser. II, Vol. IX, Fac. 12. Dec. '84, '85.
Mahabharata, Calcutta. Pt. XII, XIII.
Buł. Soc. Géog. de Paris. Vol. X, Tim. 4. 1884.
Compte Rendu, de la Soc. de Géog. de Paris. Nos. 18, 19.
Éléments d'Anthropologie. Par. Alphonse Cels. Bruxelles, vol. I, 1884. 8vo., pp. 202.
Les Habitans de Suriname. Prince Roland Bonaparte. Paris. 1884. Royal 4to , pp. 227, plt. 60.
Bull. Essex Institute. July–Dec., 1884.
Bull. Soc. d'Anthrop. Paris. Fasc. 2, 3. 1884.
Journal of Proc. of the Victoria Institute, London. XVIII, No. 70. 1884.
Grammaire Élémentaire. Quichée, L. Aleman. Pamph.
Quelques observations sur les ossements de notre musée. Maricourt et Vinet, Senlis, 1884.
Ymer. Parts 5, 6. 1884.
Bull. Soc. Géog. de Lyon. Sept.–Dec., 1884.
On the Cuspidiform Petroglyphs, etc. Dr. D. G. Brinton. Pamph.
Xinca Indians of Guatemala. " " "
Impressions of figures on a "Meday" stick. Dr. D. G. Brinton. Pamph.
Memoirs Soc. Antiq. de la Morinie, St. Omer. 1 Vol. 1883.
Bul. Russ. Géog. Soc. Also 2 pamphlets.
Mem. Soc. d'Hist., etc. Beaune. 1883.
Verein fur Erdkunde zu Halle a. S. Mittheilungen. 1884.
Imp. Soc. of the Friends of Nat. Hist. Anthrop. and Eth. Muscar. 3 vols. 1884.
Bull. Hungarian Geog. Soc. Budapest. Complete for 1884.

On motion of Prof. THOMAS, a vote of thanks was passed to the various authors and societies from which these gifts were received.

Mr BATES, from Auditing Committee, reported that the committee had duly examined the accounts of the Treasurer for the past year as reported at the annual meeting January 20, and had found the same correct.

Prof. WARD read a paper entitled "MORAL AND MATERIAL PROGRESS CONTRASTED."

One of the most obvious and frequently observed facts that lie upon the surface of modern society is the persistence of social evils in spite of the progress of discovery and invention brought about for the purpose of relieving them.

The actual removal of social evils constitutes moral progress; the discovery of principles and the invention of appliances calculated to remove them constitute material progress. It is these two forms of social progress which it is proposed to consider in this paper.

As to the degree to which moral progress has taken place and is taking place in society, there are wide differences of opinion. Some sanguine minds imagine it to be very rapid, but this is generally due to a confusion of unrelated phenomena. They either confound material with moral progress directly, or they confound the predominance of cherished religious beliefs with that of morality, or the establishment of favorite forms of government with that of justice and liberty. Others, and this is much the larger class, deny that any moral progress has ever taken place or is now taking place, and maintain, on the contrary, that there has been moral degeneracy, and that the world is growing constantly worse. In so far as these are merely influenced by the survival of a tradition very prevalent among early races they may, perhaps, be left out of the account. Many of them, however, disclaim such influence and base their convictions on the facts of history and the condition of society as it is. But such also must be set down as extremists, incapable of duly weighing the evidence from all sides of the question.

A highly respectable class, embracing many of the finest minds of the present period, see no hope except in the gradual change of the constitution of the human mind, to be brought about through hereditary influences and the slow developmental laws by which man has been at length raised above the brute. They deny the power of intelligence to improve the moral condition of society, and regard the ethical faculty as entirely distinct from the intellectual. "It is," said Mr. Herbert Spencer to an American reporter, "essentially a question of character, and only in a secondary degree a question of knowledge. But for the universal delusion about education as a panacea for political evils, this would

have been made sufficiently clear by the evidence daily disclosed in your papers." And in a private letter received after his return to England, relative to views which I had expressed, he re-asserts this doctrine, and says: "As you are probably aware, and as, in fact, I said very emphatically when in America, I regard social progress as mainly a question of character and not of knowledge or enlightenment."

In the light of all these somewhat conflicting opinions, if we were to rest the case altogether upon authority, we should at least be compelled to admit that the real moral progress of the world has been extremely slow, and that it is imperceptible even in the highest stages of enlightenment. Such, too, seems to be the lesson of history and of observation. It is only when we contemplate long periods of history and contrast the present or the recent past with the remote past that an advance can be perceived in the moral condition of mankind. Yet, when such an historic parallax is once secured, the fact that moral progress actually has taken place is distinctly seen. To read the history of England and compare the acts committed a few centuries ago by men of our own race, with what any one can see would be done now under like circumstances, is sufficient to demonstrate that improvement has been going on in both individual and public morals. Making every possible allowance for all that is bad in the present social system, no one could probably be found candidly to maintain that it is inferior, from the moral point of view, to that of the middle ages or even of the sixteenth century. Modern kings, bad as they are, no longer put their sons to death to prevent them from usurping their thrones, and the sons of kings, however profligate they may be, do not seek to dethrone their fathers. When Rome was at its zenith, it was no more than every one expected that the great armies of Cæsar and Pompey, on their triumphal return from victorious fields, would turn their arms upon each other for the mastery of the empire. And I have heard those familiar with Roman history predict, at the time when the vast armies of Grant and Sherman, far outnumbering the Roman legions, were marching victoriously through different parts of the South, that the last grand struggle of the war would be between the Army of the Cumberland and that of the Potomac—forgetting that since the age of the Cæsars there had been moral progress sufficient to render both the leaders and the soldiers incapable of such an act.

Political opponents are no longer beheaded on the accession of a new party to power; neither are they thrust into dungeons nor exiled, as formerly. Persecution for opinion's sake has practically ceased. Scientific men are no longer burned at the stake, like Bruno and Servetus, nor made to recant, like Galileo and Buffon. Witchcraft has dwindled into innocent palmistry, and heresy is only punished in a few backward communities by a mild form of social ostracism. Imprisonment for debt has been abolished, and the Fleet and the galleys are things of the past. Primogeniture and entail have disappeared from most codes of law, and trial by jury has been instituted in the most influential states. The slave trade has been suppressed wherever European powers have acquired supremacy, and slavery has been abolished in all the most enlightened countries. Vast public and private charities have been instituted, and societies for the prevention of cruelty to children and to animals receive the sanction of law. And finally a great moral crusade, with a display of far more zeal than knowledge, is being preached against the admitted evils of intemperance.

There has, then, been some moral progress within the historic period, but, considering the amount of moral agitation, it has been slight.

It is the characteristic of moral progress that it takes place rhythmically. In the achievement of moral reforms there are always experienced partial and temporary failures, prolonged interruptions, serious reverses, and constantly recurring waves of reaction, so that at no time has it been possible for the candid observer to perceive that any certain advance was being made. The ground continually being lost is never appreciably less then the ground gained, and none but the ignorant, the blinded, or the oversanguine see much cause for congratulation. In the great ocean of moral action so nearly equal are the tidal ebbs and flows that only the stoical philosopher whose vision ranges back into the remotest past or forward unto the remotest future, with utter contempt for the transient present, can perceive the minute increments of secular change—much as the geologist, provided with his vast time-measures, perceives the changes that are slowly taking place on the coasts of continents washed by the tides and waves ot the apparently changeless ocean of waters.

Such is moral progress in society. With it we may now compare, or rather contrast, the other form of social progress which we have distinguished as material.

Material progress results entirely from mental and manual labor laid out on invention and construction. Moral progress is a product of *feeling*, material progress one of *thought;* the action accompanying the former is called *conduct*, that accompanying the latter is called *labor*. Conduct is confined to the avoidance of interference with liberty of action in others. Labor is directed to the production and distribution of the objects of desire. Moral action aims at the restraint or control of the forces of society, of human desires, prejudices, and passions. Invention and labor aim at the control and utilization of physical and mechanical forces, and of such vital processes as underlie pastoral and agricultural pursuits.

The contrast in the essential nature of these two classes of social phenomena is thus seen to be very wide, but it is not greater than is the difference in their mode of operating. We have seen that moral progress always takes place by rhythmic action, and that its secular slowness is not due to its own inherent sluggishness, but to the fact that only the algebraic sum, of its many fluxes and refluxes can be counted. In material development nothing of the kind is found. Every step is a permanent gain. Every mechanical invention is an inalienable contribution to the material prosperity of society. If the particular device first produced becomes at length obsolete, as is usually the case, it is only because from it as a basis better devices, involving additional principles and doing more efficient service, have grown up. And such, in fact, is the nature of all inventions.

But the machine is only the material embodiment of intellectual conceptions, and it is these that lie at the foundation of all material progress. Indeed, much of this progress has consisted of such conceptions without any definite materialization. Of this class is all real knowledge of nature, only part of which can be directly applied to man's material amelioration. Every natural truth acquired proves advantageous, and the progress of pure science, like the progress of invention, has been steady though not uniform, never intermittent nor ryhthmical. The misguided forces of feeling which underlie the fluctuating moral activities of society have often resisted the progress of science, have seriously checked it, sometimes apparently arrested it during long periods, but they have never succeeded in forcing it backwards. The same is true of art, especially of practical or useful art. This fact is strikingly exemplified in the interest attaching to the few alleged "lost arts", as though

it were next to impossible for a single art to be wholly lost. And so it is. Every age has known all that was known by the age that preceded it and has added something to this. Every age has possessed all the arts of the age that preceded it, and has added something to them. And this in spite of the most prolonged moral reactions, such, for example, as that of the middle ages.

If we examine the arts, implements, utensils, and weapons of any of the lower tribes, as, for example, the Esquimaux of the extreme north, we shall find that they represent a high degree of skill, a large amount of inventive thought, and a considerable real knowledge of the laws of nature and of physical forces. A comparison of many such tribes also shows that these devices represent, like those of the most enlightened peoples, a series of steps in invention answering to our improvements. But a better implement is never abandoned for a poorer one, and here, as in the higher races, progress has been constant—always forward. We may therefore safely conclude that the present high state of material advancement in scientific nations is the result of a series of intellectual conceptions materially embodied in art, stretching back into that dim past when the club embodied the highest mechanical principles known to man.

Such is material progress, and such are the essential particulars in which it so widely differs in nature and method from moral progress. But, great as these differences seem and are, there is a point toward which they may be made, hypothetically at least, to converge. This point is where the human activities are conceived as natural phenomena, and their control through the normal inventive process is contemplated as a true art. If the power to do this shall ever be attained, there is no reason why morals may not progress in the same manner and at the same rate as material civilization. The true interpreters of human history now understand that it is to material progress, *i. e.*, to science and art, that what moral progress has actually taken place is indirectly due. It is knowledge of the universe enlarging the mental horizon that has dispelled the bigotry of pre-scientific ages and thrown the mantle of charity over individual conduct and opinion. And it is the arts of intercommunication that have really civilized the modern world, as compared with the world before their introduction.

But since morals, from the point of view of social science, are concerned exclusively with the welfare of men, and since material progress, both physical and intellectual, is also directed exclusively

toward this same end, the question naturally arises, why does not the welfare of men advance *pari passu* with the progress of science and art? As already remarked, no thoughtful person will maintain that it does so advance, some insisting that the two are wholly independent, and others claiming that the moral condition of society is degenerating in spite of the brilliant material civilization of these later times. After conceding all that is possible on the side of a real moral progress in society the case is bad enough, and the blunt comment of crude common sense naturally and properly is, of what use are science and art if they are incompetent to add anything to the general welfare of mankind? And to this question the response of the highest science is that if they cannot do this they are of *no* use. The welfare of mankind is the ultimate test of utility, and whatever fails to withstand that test stands condemned.

But admitting, as has already been done, that all the perceptible moral progress that has taken place has been due to that of intelligence in interaction with the practical arts which it necessarily creates, it may still be a question whether this trifling result is really worth the Titanic efforts which this teeming age puts forth. The attempt to answer this question would probably be attended with insuperable difficulties and need not be made. It will be more profitable to consider the far more important one whether, in the nature of things, this admitted slight influence of material upon moral progress could, even theoretically, be so far increased as to render them somewhat proportional in amount.

Moral progress may be defined as embracing all those changes in man's social condition which actually enhance his general well-being; material progress may be defined as embracing those changes which give him power, if judiciously employed, to improve his condition, without implying such employment. If these definitions are correct, it is evident that all that is needed to make moral progress depend quantitatively upon material progress is to secure the judicious employment of the modifications of crude nature which are produced by human thought and action. Knowledge, ingenuity, skill, and industry need to be applied to moral ends and directed to the attainment of the social well-being. At present science and art are only potential factors in civilization. The need is that they be converted into actual factors. They are well nigh omnipotent in the accomplishment of anything toward which they can be once

fairly directed. The difficulty is entirely that of securing for them the opportunity for free action. The power, for example, to produce a large quantity of a useful commodity may exist, but the conditions be wanting for placing the product in the hands of those who want it. This checks the production without affecting the producing power. That lies latent, and such latent power is simply wasted. Nor is it altogether a discrepancy between production and distribution. The power to distribute exists as well as the power to produce, but the conditions are wanting which are necessary to call that power into exercise. And this is the actual industrial state of society.

What is true of art is true of science. Intelligence, far more than necessity, is the mother of invention, and the influence of knowledge as a social factor, like that of wealth, is proportional to the extent of its distribution.

Society has always presented to the thoughtful student two great inequalities as the adequate explanation of nearly all its evils—inequality of knowledge and inequality of possession. Moral progress, in so far as it has taken place at all, has consisted in the slight diminution of one or both of these inequalities. This is always accomplished by the adoption of a better system of distribution. These two commodities, information and possession, differ in the essential particular that the latter is and the former is not destroyed in consumption. The existence of a supply of knowledge for distribution is therefore proved by the very fact of its inequality. But there is a sense in which the supply of wealth for distribution is also practically unlimited. Production never ceases from having reached a limit to the power to produce. It always ceases from having exceeded the power of the community to consume. But the limit of consumption is in turn never that of the desire to consume; it is always that of the power to obtain. The power of both production and consumption is limited only by that of distribution—not the mechanical means of distribution, for these, too, are unlimited, but the conditions to the performance of the sociological function of distribution. Could the distribution of knowledge and of physical necessities go on at a rate at all proportional to their possible creation, the moral progress of society, *i. e.*, the increase in its aggregate well-being or enjoyment, would not only be as rapid, but would also be as uniform and steady as its material progress. If the knowledge now in possession of the few were in the possession of all, its

benefits would be far more than proportional to its universality, since inequality itself often renders knowledge positively injurious. Although it be true that if the actual wealth of the world were equally distributed the share of each individual would be a very small fortune, yet if the limitations to possible distribution were removed production would so far increase that almost any desired portion might fall to each and all.

Wherein, then, consists this mysterious yet potent barrier to the distribution of wealth and wisdom: this practically prohibitory tariff upon the world's commerce in both thoughts and things?

The answer is rather deep than difficult. The two processes as they go on in society belong to antithetically opposite categories of social phenomena. We have in them the ultimate kernel of that broad contrast which has just been drawn between moral and material progress. It is the great distinction between natural and artificial processes, between genetic and teleologic activity, between growth and manufacture, between the method by which feeling works and that by which intellect works. The former is a method of direct effort, and fails in the great majority of cases to attain its end because of obstacles which are never taken into account. The latter is a method of indirect calculation by which the obstacles are foreseen, and in one way or another provided against before the advance is attempted. Hence it is always successful if the phenomena and laws to be dealt with are really understood. This is why science and art, as already stated, move ever forward, never backward. The discovery of truth on the one hand, and the invention of artificial appliances on the other, are always going on, multiplying the power of man to produce and distribute the objects of desire. Of the gain thus made nothing is ever lost. But when we come to the actual utilization of the products of discovery, invention, and handicraft, we find this under the control of the opposite class of forces. The power to produce either knowledge or wealth is controlled by man, exercised when it can serve his purposes, checked or arrested when it no longer does this. But the power to possess—the ability to obtain the truth discovered or the commodity wrought—is controlled by natural laws and depends upon the thousand accidents of life—the conflicting wills of men, the passions of avarice and ambition, the vicissitudes of fortune, the uncertainties of climate and seasons, the circumstances of birth and social station, the interests and caprices of nations and rulers. Of what use is discov-

ered truth to the millions whose minds it can never reach? Why produce useful commodities which those who need them are unable to obtain? For while all producers are also consumers, and nearly all consumers are at the same time producers, yet few can satisfy their wants, however capable they may be of producing an equivalent in value of other forms. Inventions in the practical arts by which the power is acquired to multiply the products of labor, instead of working the rapid amelioration of the laboring classes, actually injure their prospects by throwing skilled artisans out of employment; and instead of resulting in greatly increased production they do not appreciably affect production, but reduce the amount of labor to the disadvantage of the laborer. The plea of over-production in periods of financial depression is the sheerest mockery, since it is just at such times that the greatest want is felt. It may be true that more is produced than the consumers can obtain, but far less is produced at all times than they actually need and are able to render a full equivalent for. The eager manner in which every demand for laborers is responded to sufficiently proves this. It proves also that the industrial system is out of order, and that we live in a pathological state of society. The vast accumulations of goods at the mills avail nothing to the half-clad men and women who are shivering by thousands in the streets while vainly watching for an opportunity to earn the wherewithal to be clothed. The storehouse of grain held by the speculator against a rise in prices has no value to the famished communities who would gladly pay for it in value of some form.

Yet in all this the fault cannot fairly be said to lie with individuals nor with corporations, with manufacturer nor merchant, with producer nor consumer. These do but act the nature with which they are endowed. This defective circulation of industrial products is the result of the state of society. It is in one sense normal, since it is due to the operation of natural laws governing social phenomena. The enormous inequalities of both the classes named and the evils resulting, constituting the major part of the woes of mankind, are simply due to the fact that the agencies for distributing knowledge and wealth are *free* in the politico-economic sense, *i. e.*, not regulated nor controlled by intelligent foresight. The contrast between moral and material progress is the contrast between Nature and Art. Nature is free. Art is caged. The forces of Nature play unbridled among themselves, until choked by

their mutual friction, they are equilibrated and come to rest. Art commands them with tones of authority to pursue paths selected by intelligence and thus indefinitely to continue to exert their power. Under the dominion of Science, *i. e.*, under the intelligent control of physical forces, man's power to create the objects of desire and to send them where he will, is practically unlimited. But under the dominion of Nature, *i. e.*, under the free operation of the social forces, as yet beyond the reach of science, these objects of human necessity in seeking unaided their proper destination conflict perpetually in their passage, dashing against unseen obstructions, forcing themselves into inextricable entanglements, polarizing themselves around powerful centers of attraction, heaping themselves up in inaccessible "corners," or flying off on tangential lines to be lost forever.

This is what in modern phrase is very properly denominated the "waste of competition." But it is far more than the mere waste of the wealth produced. It is the paralysis of the strong hands of science and art as they co-operate with labor in the creation of value. It is the stubborn, the protracted resistance which the moral forces of society offer to its material as well as to its moral progress.

The statement of the problem is its theoretical solution, which can be nothing less than the conquest by science of the domain of the social as it has conquered that of the physical forces.

But alas! how wide is the difference between the theoretical and the practical solution of a problem to the bare statement of which the foremost thinkers of the age are as yet unwilling to listen.

DISCUSSION.

The paper was discussed at length by Messrs. Powell, Welling, Thomas, Baker, Peters, Hart, and Ward.

Major POWELL maintained that there had been much moral progress, and gave numerous illustrations of this among uncivilized races. He said that some of these races had elaborate codes of morals often worthy of imitation by civilized races, and that the work of devising means of preventing and terminating controversy and securing justice had engrossed the energies of all people from time immemorial, that it had been largely successful, and had resulted in great moral progress, as great as, or even greater, than the material progress achieved by such races..

Mr. WELLING, after paying a high tribute to Mr. Ward's paper, expressed the opinion that the complaint which it formulated, based on the assumed failure of moral progress to keep pace with material progress, was in itself the mark and the expression of growing moral aspirations, seeking more and more to realize themselves in the figure of society. It is a sign of intellectual growth when an age is working vehemently on unsolved problems along the converging lines of scientific inquiry; and it is an augury of moral progress when an age has become impatient of existing social adjustments in their relation to public well-being, and is longing for a better co-ordination of social relations and a better distribution of social advantages. The unrest of such an epoch, he said, is the unrest incidental to all transition periods, and is a ground of congratulation rather than a source of lamentation. It is necessary that social wants and moral aspirations shall be distinctly articulated before they can be properly embodied in institutions or in regulations; and this embodiment must needs be a slow process under the formula of social evolution, because social experiments are experiments made on the grandest of all living organisms—the body politic—and not *in corpore vili*.

Nor is it enough that the co-ordination of society should be directed by the highest intelligence of the community, if that intelligence be congested in the head of the social organism. It is so in China to-day, and has there resulted in a stationary civilization. It had been so in the feudal system of the middle ages, and had there resulted in a cast-iron polity destructive of moral progress and of social well-being, until that cast-iron polity had been broken by the expansive force of a larger and more complex social life permeating the lower members of the body politic. True moral progress can take place only in a social organism which is "vital in every part," for here the actions, reactions, and interactions of public opinions give the widest possible distribution to social thoughts, feelings, purposes, and aspirations. It is in such an organism that "discussion becomes the mould of measures," to use the fine phrase of Thucydides, and that the lines of safe social change can be soonest discovered and soonest followed. In such a community there will be a growing complexity and a growing difficulty in the problems to be solved by each generation, but the problems will not increase in difficulty or number beyond the growing resources of civilization for coping with them. He illustrated

this point by citing the new and difficult social problems created by the abolition of slavery, and by the removal of governmental restrictions on the freedom of industry.

Dr. BAKER said that in estimating progress in the domain of morals we should be careful to consider the average state throughout a sufficiently wide area. Comparing the present state of the civilized world with that of ancient Greece and Rome, we do not at first see such a marked advance, but it should be remembered that at the time of Socrates and Seneca the greater part of Europe was living in a state of low barbarism, comparable to that of nomadic savage tribes, preying on each other like hawks and falcons, and it was not until after the Norman Conquest that life and property in the northern part of Europe were safe from ruthless marauders and sea-robbers. Respect for abstract right and justice were matters of late growth, clearly recognized, it is true, by the Greeks and Romans, especially by the latter.

We may be in error in estimating the state of morals in any ancient nation, for we know that it is extremely difficult to correctly estimate our contemporaries. Thousands of Englishmen suppose to day that our late civil war was a mere struggle for supremacy, a conflict for territory, and it seems hopeless for an American to understand French politics or French morality. According to the average French novel, infidelity to marital relations is the rule, yet all who have had access to French households agree that in no country are the family habits more sweet, affectionate, and fixed. I am sure that we would err grievously to take our view of French morals from Zola, Balzac, or Sue. In reading Plato I have been startled at the mention of certain habits and practices in such a connection as to show that they were not regarded by the author as at all objectionable, practices which would to-day be considered infamous. The collection at the Museo Borbonico at Naples, contains many articles of personal adornment and public exhibition from Pompeii which are so shocking to our ideas that they are not shown to the general public, and Terence, Plautus, Juvenal, and Rabelais abound in passages which show that they addressed an audience to whom gross and lascivious ideas gave a pleasure which to-day is usually replaced by disgust. Indeed this attitude of mind was so common that even the purest Greek and Roman authors are now read in our schools with expurgated editions.

It seems to me clear that a certain unwritten code of morals not

always easily defined has been growing throughout the historical period with a steady progress on the whole. I refer to that code which has for its basis the criticism of our fellows, and which we call the morals and manners of a gentleman. Obscured by many absurd and trivial details as to what clothing we shall wear and what corner of our cards we shall turn down, it has yet a very substantial moral basis, and there are evident signs of its advance. Time was when it was not considered necessary to adhere closely to the truth, and when the seduction of young girls was considered an accomplishment. Our grandfathers reverenced a five-bottle man while we look rather askance at one who "tarrieth long at the wine." I believe that never in the world has the standard of clean, healthy morality been as high as to-day, although I am aware that the eager scramble for money perverts and injures many features of the fair ideal.

We do not always completely realize the Titanic task which this wonderful teeming nineteenth century has before it. The civilization of the past had for its object the training and enlightenment of the few; we are apt to judge of it by its results upon that few, and forget the countless miserable hordes of slaves and plebes that were little above cattle, and whose morals no one noted. These formed the armies that sacked and burned conquered cities, a proceeding that was once a matter of course, performing deeds of lust and rapine that are almost impossible to realize. The task to-day is to civilize *all,* to give to all the opportunity to live healthful, active, lives of usefulness and enjoyment. It will take long, and we are in the throes of the conflict. Of all biological processes those that bring the passions under control are the slowest. The African whose grandfather was a cannibal will not at once conform to the moral attitude of the descendants of a long line of civilized ancestry, however he may seem to do so.

On the other hand, I cannot but note that any stride in material progress must ameliorate the general condition, and so foster moral progress. That morality has something to do with food supply is evident to us all, and it is a matter of daily observation that one is more ready to do a good deed after breakfast. The poor half-starved Irish peasant ready to shoot his landlord on trifling provocation is transformed in the course of a generation to a jovial, hard-working, and tolerably law-abiding citizen when transferred to a more genial environment.

Mr. E. T. Peters said he had been deeply interested in listening to the paper read by Prof. Ward. He thought that in some of the comments made in the course of the discussion it had been assumed that the term moral progress, as used in the paper, referred to improvement in public morals; but, as the essayist had defined it, it embraced not only this but everything else which advanced the happiness of man. The lack of progress which had been chiefly dwelt upon in the paper just read seemed to him to consist mainly in the tardy advance of political and social science. Between this and the marvelous advances which in modern times had been made in the physical sciences and in their application to the arts of life there was indeed, a striking contrast. Referring to a remark which Major Powell had made as to the necessity for new adjustments in social organization arising from changes in the material conditions under which a society existed, the speaker said, that was a pregnant thought. The changes of condition brought about within the last one hundred years through the introduction of labor-saving devices into the industries of the civilized world had alone amounted to an economic revolution, and a need had thus been created for changes correspondingly great in the social adjustments which relate to the production and distribution of wealth. The knowledge essential to the making of such changes as the best interests of society required had, however, not been in existence, and although vast social changes had occurred, they had come about not in pursuance of any wise and comprehensive plan, but through the blindly exerted pressure of changing circumstances, and in a large part they had been productive of great social misery and discontent. To take a single illustration, the introduction of the new industrial methods had given a powerful impetus to the growth of towns and cities, causing them to spread over large areas of suburban land, or to rise up on land where none had stood before. This had operated to the great enrichment of a few land-owners, at the expense of crushing rents and ruinous over-crowding to the poorer portions of the urban population. Society had no interest in this enrichment of a few land-owners, because it had occurred independent of the exercise on their part of any of those economic or social virtues which it is the policy of society to encourage; while on the other hand the most imperious considerations of public policy had demanded that the correlative over-crowding of the poor—unwholesome no less from a moral than a physical

point of view, and tending to rapid social deterioration—should if possible have been prevented. A social adjustment adapted to that purpose might have been found in a land tax like that suggested by a very eminent English economist, the late John Stuart Mill, namely, a tax which as nearly as practicable should appropriate for public purposes the whole unearned increase in the rental value of land. But Mr. Mill's suggestion had not been made until about fifteen years ago, and the advanced public opinion necessary to the adoption of a plan involving some such principle did not exist even yet. That the situation created by the want of social and political adjustments adapted to modern industrial conditions was a very serious one was apparent from indications that might be seen on every hand. To close the great gap between social and physical science—between moral progress as defined in the paper just read and material progress as illustrated in the stupendous achievements of modern industrial art—was in the speaker's opinion the crying need of the time, and unless this need were supplied there would be imminent danger of a social catastrophe. In order that it might be supplied it was necessary that social questions should receive attention to a vastly increased extent. In particular should the most serious and unprejudiced consideration be given to the manifestations of discontent that came from the working people of every civilized nation. If they were not proposing the best remedies for the evils they complained of, so much the greater was the need that the deep sociological problems involved should be taken up in earnest by those who had more time and a better intellectual equipment for their study; and they must be taken up, not as it was to be feared they had been by some men rated high as political economists, namely, in the spirit of an advocate retained for the defense of the existing state of things—but in the pure spirit of the man of science, ready to follow where the truth should lead, however great and radical the social changes which might be involved in doing so.

There were very influential writers who would have us believe that the discontent of the poorer classes had no foundation unless it were in the mischievous meddling of governments with the natural course of affairs. The speaker believed that we should come much nearer the truth if we accepted the views advanced in the paper under discussion, which were directly the reverse of that just indicated, recognizing the necessity of social coördinations to which

only governmental agencies could be adequate. There was doubtless a field for legislative action in the repeal of bad existing laws, but there was a still wider one in the enactment of good ones adapted to the needs of soeiety.

Mr. WARD, in reply to numerous inquiries and objections made during the discussion of the paper, explained that for the sake of brevity he had omitted any precise definition of the term Moral Progress as used in the paper. He said that the term was often employed in two quite distinct senses, and that much of the discussion had considered it in the other sense from that clearly implied in the paper. There is a subjective sense which relates to individual character and an objective sense which relates to collective well-being. The paper did not pretend to discuss the question whether human character had advanced, or how much it had advanced. It aimed only to consider the relation of material civilization to social well-being, the sole test of moral progress in this objective sense being the condition attained with respect to the enjoyment of life. This progress might be either positive, consisting in an increase in the pleasures of life; or it might be negative, and consist in the reduction of the pains of life. In fact this negative progress has been by far the most observable, the chief improvement in man's condition thus far being some slight mitigation of the evils of existence. In view of this criterion of moral progress as measured by the degree of collective happiness, all that had been said respecting higher standards of taste in literature and social life was irrelevant to the discussion, since it simply confounded refinement with enjoyment, which are two entirely distinct things. Admitting that finer sensibilities are capable of higher enjoyment, this is far from proving that they necessarily enjoy more, for they are also capable of more acute suffering, and the whole question originally was whether material civilization prevents more of the latter than it occasions.

Mr. WARD in conclusion expressed surprise that Dr. Welling should have seemed to regard his paper at all in the light of a jeremiad. On the contrary, he tried to take such a view of the future as should be philosophic rather than either pessimistic or optimistic, but had sometimes been accused of expecting results that were not likely to be soon realized.

NINETY-SECOND REGULAR MEETING, March 3, 1885.

Major J. W. POWELL, the President, in the Chair.

The Secretary being absent the minutes were not read. The President announced that on account of the small attendance the Council had thought best to defer the regular program till another meeting, and that a portion of the time would be occupied by himself. He then addressed the Society upon Patriarchy, and the conditions of savage society which preceded and led to it.

He was followed by Mr. Cushing in some remarks upon artificial age and parentage among the Zuñis, illustrated by his own experience.

NINETY-THIRD REGULAR MEETING, March 17th, 1885.

Major J. W. POWELL, President, in the Chair.

The Secretary of the Council announced the election of Prof. W. C. Kerr, of Raleigh, N. C., as a corresponding member, and Mr. E. R. L. Gould, of Washington, D. C., as an active member of the Society.

The following papers were then read:

"STUDY OF THE CIRCULAR ROOMS IN THE ANCIENT PUEBLOS," by Mr. VICTOR MINDELEFF.

"CIRCULAR ARCHITECTURE AMONG THE ANCIENT PERUVIANS," by Mr. W. H. HOLMES.

DISCUSSION.

Prof. MASON. A very interesting separation has been made by the speakers of the evening without design. The subject for discussion is "Circular Architecture of the American Aborigines." Now in discus ing this theme we may have regard either to structure or function. If Mr. Turner had not been called away he would have told us of the Eskimo *igloo*, or winter temporary hut of ice or snow; Mr. Mindeleff described at length the circular rooms in the pueblo structures of our southwest territory, and Mr. Holmes has dwelt upon the chulpas. Structurally we have the material at hand

wrought into the most natural shape for a cist or cell, the most simple being that of the Eskimo, the most complex, the chulpa of dressed stone. Now as to function, they differ very curiously, the igloo teems with daily life, the estufa is open to ceremony and conventions, the chulpa is a sealed tomb. The Eskimo has a council chamber, a place of public meeting in the permanent undergound dwelling. The Chibchas and Peruvians had both dwelling and meeting places apart. Descending the continent from north to south it is curious to notice the transfer of function in circular architecture from dwelling place to meeting place, from meeting place to tomb.

Mr. Arthur Mitchell, in his admirable work, "The Past in the Present," has shown us how old arts degenerate as new arts arise. The reason is not far to seek. When our Indians were brought face to face with the civilization of the whites, the bright, intelligent, susceptible individuals and tribes dropped at once their old arts and took on the new. The old, the dull, the conservative clung to former things, which degenerated in their hands. On the whole there was progress, but many things in the onward mass were moving backward.

So it is with civilization at large—families, gentes, tribes—whole nations and races disappear; but new and better families—gentes, tribes, nations, and races take their places.

Mr. J. H. Blodgett said the remarks as to a sinking class of persons in this city and elsewhere, call to mind an investigation carefully made and recorded about 1810 in the city of Glasgow in connection with some of the benevolent operations of the Church of Scotland.

The classification then made was in these four groups: 1. A wealthy class, able to select and carry out their own plans of life in the main independently—one-sixth of the people. 2. An uprising class, struggling for better advantages for themselves and their children—one-third of the people. 3. A sinking class, tending downward except for helpful influences brought to bear on them by others—one-third of the people. 4. A sunken class, confirmed criminals and paupers—one-sixth of the people. Such investigations have a bearing upon discussions such as that of the Society recently upon our relative moral and physical progress.

NINETY-FOURTH REGULAR MEETING, April 7, 1885.

Major J. W. POWELL, President, in the Chair.

Dr. WASHINGTON MATTHEWS, U. S. A., read a paper entitled, "MYTHOLOGICAL DRY-PAINTING OF THE NAVAJOS."

ABSTRACT.

These are pictures of large size (10 to 12 feet in diameter) drawn in powdered substances on the sanded floors of the medicine lodges of the Navajo Indians of New Mexico and Arizona. They represent various gods and other mythical conceptions of this tribe. The pigments used are five in number: white, made of powdered white sandstone; yellow, of yellow sandstone; red, of red sandstone; black, of charcoal; and a so-called blue—but really a gray—of black and white mixed in proper proportions. To apply them the artist grasps a little in his hand and allows it to flow out between the thumb and the opposed fingers. When he makes a mistake he does not brush away the color, he obliterates it by pouring sand on it, and then draws the corrected design on the new surface.

The drawings are begun as much towards the center as the nature of the picture will permit, due regard being paid to the precedence of the points of the compass, *i e.*, the figure of the god in the east is begun first; that in the south, second; that in the west, third; that in the north, fourth. While the work is in progress the chief shaman does little more than direct and criticise; a dozen or more young men, who have been initiated into the mysteries, perform the manual labor. The pictures are drawn in accordance with established rules, except in certain well-defined cases where the painter is allowed to indulge his fancy. This is the case with the embroidered pouches, which the gods are represented as carrying. On the other hand some parts are measured by palms and spans, and not a line of the sacred design can be varied in them. Straight and parallel lines are drawn on a tightened cord. The naked forms of the mythical persons are first drawn, then the clothing is put on.

When the picture is finished it is the duty of the shaman to put corn-pollen on the lips and breast of each divine form and to set certain plumed wands around the picture. Then the sick person for whose benefit the whole ceremony is performed enters and has the colored dust from various parts of the pictured forms applied to

corresponding parts of his person to remove disease, and to have many other rites performed over him. When the patient has departed many of the spectators pick up and preserve the sacred corn-pollen. Some take dust from the figures on their moistened palms and rub it over their own bodies. Then the shaman obliterates the picture with a slender wand while he sings a song appropriate to this part of the ceremony. Lastly, the assistants gather the sand in their blankets, carry it to a distance from the lodge and throw it away. Thus in half an hour from the completion of the picture not a trace of it is left.

The lecturer has heard of seventeen great ceremonies of the Navajos in which pictures of this character are drawn. There are about four pictures to each ceremony—only one picture being painted in a day—and besides these great ceremonies there are minor rites with their appropriate pictures, smaller and less elaborate. The medicine men aver that these pictures of the great ceremonies are transmitted unaltered from year to year, and from generation to generation. This is doubtful, as no permanent design is preserved for reference and there is no final authority in the tribe. Furthermore, as the majority of the rites can be performed only in the season when the snakes hibernate, the pictures are carried from winter to winter in the fallible memories of men. It is probable, however, that innovations are unintentional and that changes are wrought slowly.

The lecture was illustrated with seven large charts, representing some of the pictures which the lecturer had seen. Of their meaning and symbolism there was given a full explanation, which included the description of many of the rites and the narration of many of the myths and traditions of the tribe.*

Following this paper Prof. Gilbert Thompson presented sketches of rude drawings, seen by him in a cave at San Antonio Springs, N. M. The walls of the cave were smoke-covered, but the drawings were distinct and plainly marked, etched in the stone surface and brought out with various colored pigments. Certain points of resemblance were indicated between these figures and some described by Dr. Matthews.

* A more extensive abstract appears in the "American Naturalist" for October, 1885.

DISCUSSION.

Mr. DORSEY said, referring to the mystic qualities attributed to the number four among the Navajos, that among the northern Athabascans the number five held the place accorded to four by the Indians of the Missouri river and Southwest.

Maj. POWELL said that great elaboration was to be observed in the myths of the North American Indian. The speaker at one time witnessed a ceremony in a Moqui village that lasted four days, including one day of feasting. A constant succession of nude figures with highly colored faces formed a marked feature of all the ceremonies. He saw different colored sands, meal, corn, and pebbles used in many ways in connection with the incantations of the Shaman, which were performed, as the speaker believed, to the end that rain and abundant crops might follow. The falling rain was represented by sprinkling the floor of the estufa. Among the Utes and Shoshones fully one-half of the nights, during six months of the year, is taken up with ceremonial gatherings and the relation of myths.

Col. MALLERY said that he found in Thomas V. Keam's Catalogue of Relics of the Ancient Builders of the Southwest Table Lands, a somewhat different arrangement of colors in symbolizing the cardinal points from that observed by Dr. Matthews: White, signified north; yellow, the east; red, the south; and blue, the west.

NINETY-FIFTH REGULAR MEETING, April 21, 1885.

Major J. W. POWELL, President, in the Chair.

The Secretary of the Council announced the election of Prof. A. H. Thompson, of the Geological Survey, and Mr. Charles N. Adams, of the Civil Service Commission, as active members of the Society; and informed the Society of the death of Dr. Harrison Wright, on February 20, 1885, at Wilkes Barre, Pa., and Col. P. W. Norris, on Jan. 14, 1885, at Rockland, Ky., corresponding members of the Society. Appropriate remarks upon the death of Col. Norris were made by President Powell, followed by Col. Mallery, who delivered a brief eulogy upon Dr. Wright.

Mr. H.W. HENSHAW read a paper entitled "MEDICINE STONES."*

DISCUSSION.

Col. MALLERY, referring to the evidence presented in the paper, that the objects generally classed as sinkers were used as ceremonial stones and amulets, remarked that amulets and fetiches had often been adopted from utensils and objects connected with daily life. He gave instances specially connected with the fish—commonly appearing towards the third century as an emblem of Christ, but derived from the worship of the Phœnician Dagon, and found still more anciently in Egypt, Nineveh, and India, with some relation to the productive powers of nature. The lingom stones were mentioned in this connection, also the bulla and the form called from its shape vesica (bladder) suspended to the necks of Roman boys, which was succeeded by the *Agnus Dei*, used in the same manner. Without attempting to trace an immediate association between these objects and those presented by Mr. Henshaw, his views are corroborated by the fact that stones similar in shape and size have been employed from high antiquity in many parts of the world for superstitious purposes, and that therefore it is unphilosophical to insist upon their exclusive design for mechanical or industrial uses among the tribes of North America, which are known to have universally been addicted to amuletism. Without any elaboration of symbolism the selection of the form might readily have been derived from the idea of "luck" connected with sinkers used on some special occasions.

Mr. DORSEY, referring to what Mr. Henshaw had said about the "Medicine Stones" and the down from the breast of a white goose, remarked that he had noticed among the Omahas, Kansas, and cognate tribes, some of the uses of this down from the white goose, and that in the gens or clan of the Earth-lodge Makers in the Omaha and Kansas tribes there were "White Goose (or Swan) people." In the Omaha gens referred to there are also Keepers of the Sacred Stones (or Mysterious Stones.)

He then gave a part of the traditon of the Sacred Pipes to the Omaha gentes: "The Earth-lodge people were visited by the seven old men bearing the pipes. When the gentes were finally organized half of these people were bad, and half were good. The

* Published in American Journal of Archæology. I. Pp. 105–114.

bad ones had some stones at the front of their lodge, and they colored them as well as their own hair, orange-red (zhee.) They wore the down of the white goose (or swan) in their hair, and branches of cedar around their heads, being frightful to behold. So the old men passed to the good ones, to whom they gave one of the pipes." According to Joseph La Fleche and Two Crows, there are four of the sacred stones, their colors being black, red, yellow, and blue. (One tradition is that the stones were made by the Coyote in ancient times, to be used for conjuring enemies.) In the Osage tradition, the four kinds of stone found at the first, were white, black, red, and blue (or green.)

In reply to a question put by the President, Mr. Dorsey said that among the Dakotas, Ponkas, and other related tribes, there was a worship paid to boulders found on the prairies, these being regarded as representatives of the Earth-god. When an Indian met one of them, he addressed it as "Grandfather," the same term that is applied by many tribes to the President of the United States (wrongly translated the "Great Father.") This term, Grandfather, is applied to supernatural beings. On addressing such a boulder, the Indian laid on it a small quantity of tobacco wrapped in a piece of cloth or skin, and then he smoked his pipe toward it, asking the Grandfather to help him in his journey or undertaking.

Colonel JAMES STEVENSON read a paper on the "MYTHOLOGICAL PAINTING OF THE ZUÑIS."

DISCUSSION.

Col. MALLERY presented the following account of Yuma ceremonies witnessed at Camp Verde, Arizona, as related by Dr. W. H. Corbusier, U. S. A.: "All the medicine-men meet occasionally, and with considerable ceremony make medicine. They went through the performance early in the summer of 1874, on the Reservation, for the purpose of averting the diseases with which the Indians were afflicted the summer previous. In the middle of one of the villages they made a round ramada—or house of boughs—some ten feet in diameter, and under it on the sand, illustrated the spirit-land, in a picture about seven feet across, made in colors by sprinkling powdered leaves and grass, red clay, charcoal, and ashes on the smoothed sand. In the centre was a round spot of red clay about ten inches in diameter, and around it several successive rings

of green and red alternately, each ring being an inch and a half wide; projecting from the outer ring, were four somewhat triangular shaped figures, each one of which corresponded to one of the cardinal points of the compass, giving the whole the appearance of a Maltese cross. Around this cross and between its arms were the figures of men with their feet toward the center—some made of charcoal with ashes for eyes and hair, others of red clay and ashes, etc. These figures were eight or nine inches long, and nearly all of them lacked some part of the body—some an arm, others a leg or the head. The medicine-men seated themselves around the picture, on the ground in a circle, and the Indians from the different bands crowded around them, the old men squatting close by, and the young men standing back of them. After they had invoked the aid of the spirits, in a number of chants, one of their number, apparently the oldest, a toothless, gray-haired man, solemnly arose, and, carefully stepping between the figures of the men, dropped on each one a pinch of the yellow powder, which he took from a small buckskin bag which had been handed to him. He put the powder on the heads of some, on the chests of others, and on other parts of the body, one of the other men sometimes telling him where to put it. After going all around, skipping three figures however, he put up the bag and then went around again, and took from each figure a large pinch of powder, taking up the yellow powder also, and in this way collected a heaping handful. After doing this he stepped back, and another medicine man collected a handful in the same way, others following him. Some of the laymen in their eagerness to get some pressed forward, but were ordered back. But after the medicine men had supplied themselves, the ramada was torn down, and a rush was made by men and boys, handfuls of the dirt were grabbed and rubbed on their bodies, or carried away. The women and children, who were waiting for an invitation, were then called. They rushed to the spot in a crowd, and grabbing handfuls of dirt tossed it up in the air so that it would fall on them, or they rubbed their bodies with it. Mothers throwing it over their children and rubbing it on their heads. This ended the performance.

Mr. Gatschet said: The Chiricahua Apache "sun circle," or "magic circle," is constructed for the purpose of curing those who have been "sun-struck," or as they express it, those who have become sick from the sun.

Conjurors will consent to construct a circle only when they are called upon by the sick person. The patient must indemnify the conjurors for the arrangements, and provide food for the Indians who congregate to witness the ceremony and participate in the dances. Frequently the sick person is compelled to borrow money to defray the expenses, and then he will kill his cattle to satisfy the appetite of the hungry crowd assisting in the great ceremony.

The conjurors do not always make the magic circles with their own hands. When they have it drawn by others they walk around superintending the work.

A few days before the time appointed for the ceremony the conjurors in charge send out heralds, each provided with several symbols called "nadu 'hkädä," or "God's messengers." One of these symbols is left with every head man or chief of an Apache tribe. Its purpose is to direct them to summon their men, women, and girls to appear and take part in the dances of the ceremony.

When the invited arrive, the nadu 'hkädä are brought back by them and set up in or near the center of the circle during the performances. The symbol is in the shape of a cross. The four arms thus point to the four cardinal points, and the feathers at the ends of each arm represent the birds which convey to the conjurors the dreams of the human figures set up within the circle.

The magic ring is made on the ground in a place carefully screened from mortal eye, and sometimes covered by a shed made of bent willow rods (called in Spanish "ramada".) The circle is properly speaking two concentric rings, and is composed of colored substances of various shades. The diameter of the ring is ten or more feet. Dry leaves of various trees are mostly used in effecting the different shades of color, and, if the weather permits, the conjurors go into the mountains to collect earth, clay, and colored sand for the same purpose. The clay being the same as that used for body paint.

The inner ring of the circle is called bās or nibās (round). The rim of the circles does not follow the line of a true circle but shows sallies and angles. The spaces in the angles are frequently colored. These colors when not of mineral substance are made by drying leaves in the fire and grinding them to powder. The angles or corners in the circle represent rays of the sun and the whole circle is an image of the sun. The effigies of four men, each painted with a different clay color are placed on the inside of the circle; they are called "God's people," or "divine people," and repre-

sent genii that can only be seen by the conjurers in their dreams. They stand on one leg only, the other leg being wrapped around the one on which they stand. This helps, it is said, to remain on their legs longer than by standing in any other way, since one leg adds strength to the other. On their heads they carry an ornament resembling two horns, which are in fact, as the name has it, two hats. The men represented by these effigies are supposed to dream and to convey the import of their dreams to the conjurors by means of birds called "God's messengers," each bird having the same colors as the effigies.

The effigy of the black man lies behind some black rays of the circle and is supposed to have charge of the whole ceremony. The effigy of the blue man stands at the end of blue rays. The effigy of the yellow man is at the end of yellow rays; and the white effigy at the end of white rays.

Before each of these effigies a sort of standard (nadá) is stuck up—about six feet high. They are carried about in the dances and their purpose is, as alleged, the same as our lightning-rods. They say the nadnai insure getting good health while dancing. The chief part of Indian religious ceremonies consist in dances which commence at sundown and continue till sunrise, with only three interruptions for meals. The dances take place at some distance from the magic circle and about a central fire. Near this fire may be seen the pile of firewood provided for the occasion, and on another side a group consisting of conjurors and men of the tribe. Close to the fire are the groups of dancers, male and female. In dancing they do not move about but skip up and down—a mode of dancing common to all Indians of North America. Smaller fires are blazing in a circle around and at some distance from the central fire. About these fires are gathered the people, old and young, while back of them are standing the horses that brought them to the ceremony.

Dances begin when the leading conjuror begins a song. At each new song a girl starts from one of the fires and directs her steps toward the males standing in the central group. She gently touches one man's shoulder and then returns to her family at the fire. This pantomime indicates a sentiment of love and is at the same time an invitation to the dance, which is responded to within a short time by the lucky young man, who is careful not to meet the looks of the girl's mother.

The ending of the ceremony is similar to that described in the Yuma ceremonies.

The cardinal points are symbolized among the Apaches thus:

East—Black.
South—White
West—Yellow.
North—Blue.

The sun in the east is called the "black sun." A wind gust or tornado is also called "black."

Ninety-Sixth Regular Meeting, May 5, 1885.

Vice-President Col. Garrick Mallery, U. S. A., in the Chair.

The Secretary of the Council announced the election of Hon. W. B. Snell, Justice of the Police Court, and Mr. L. J. Hatch, of the Bureau of Engraving and Printing, as active members of the Society, and informed the Society that the Council had determined to print Vol. III of the Transactions of the Society.

Col. F. A. Seely read a paper entitled "The Genesis of Inventions."

During the past few years unusual attention has been directed to the study of human inventions. The close relations between the amelioration of man's condition and the improvement of his mechanic arts have led to the consideration of the subject as one in which social science is concerned. It has been observed that institutions of every character—languages, laws, customs, philosophies, and beliefs—have been largely, if not wholly, the product of invention of somewhat the same character as that which has produced tools and machines. The term invention has acquired a broader scope, and includes every subject on which human thought and ingenuity and fancy may exercise themselves. Its study is therefore of no little consequence. It is no longer limited to the field of mere mechanics and physics, but embraces all that concerns whatever has been devised by men to satisfy the material and moral needs, either of the individual or of the mass in their various social relations. I propose to inquire what are the processes by which inventions are produced; what influences lead to them; what laws,

if any, they follow; and what results, immediate and ultimate, flow from them. I conceive that these inquiries are best pursued in connection with mechanical inventions. A parallel inquiry might be pursued in respect to inventions in the broader sense. In fact the study of savage society is, to a certain extent, such an inquiry.

Before proceeding to the consideration of the subject, it is important to call attention to the various meanings and shades of meaning of the word *invention*, which we have such constant occasion to employ. A late writer on Patent Law* refers to this in his opening chapter as a source of much confusion, since, as he remarks, it is not uncommon to find the word used in different senses in the same paragraph, even in the same sentence. He distinguishes four meanings of the word:

(1) The mental act of inventing.

(2) The thing invented.

(3) The fact that an invention has been made.

(4) The faculty or quality of invention.

It is scarcely necessary to illustrate these significations, since on a little reflection they become apparent. We may say of the sewing machine, *it was the invention of Howe*, referring to the mental process which produced it; we may say it *is a great or useful invention*, meaning the machine itself; we may say *the invention of it revolutionized the manufacture of clothing*, in which we mean the fact that it was made; and we may say of any particular form presented to us, *there is no invention in it over some earlier form*, in which we refer to the quality of invention as distinguished alike from the mental act, the concrete product, and the historical fact. In view of all these uses of the word and not to overload it further, I shall venture to suggest a new one to designate the study of invention. This study has not yet perhaps developed itself as a true science, though it appears to possess all the elements of a science. As a study of growing interest it is worthy of a name of its own, and, with all deference, I submit to the Society, as an appropriate name worthy of adoption the word *Eurematics*.† This should include the study not of arts, machines, laws or insti-

* Merwin. Patentability of Inventions. Boston. 1883.

† *Εὕρημα*, An invention. If the Greeks had been in the habit of philosophizing about inventions, they would have had an adjective, ʽευρηματικος, and the word would have found its place in English long ago, as has *eureka*.

tutions in themselves, but of them all in respect to their methods of growth and the means by which they have been developed and are still developing. This is a study which many are pursuing with eagerness and delight; and the need of a name for it clearly separating it from other kindred studies is every day more apparent.

It is my purpose to present in this paper a brief chapter in this science, following out and perhaps to some extent repeating some of the thoughts expressed in a paper presented to the Society two years ago,* in which I discussed the nature of the earliest human inventions, the original germs out of which they grew, and the steps and processes by which they were evolved or elaborated. Speculative as some of my suggestions may have been as to the nature of these primitive inventions, nevertheless the nature of the processes by which they were made is so inherent in all arts that it cannot be regarded as in any degree speculative. Possibly the inventions pointed out were not actually the first contrived by man, but whatever were the first, the way described is beyond doubt the way in which they were arrived at.

I propose in the course of this paper to discuss the development of the stone hatchet in its most finished form; but before doing so it is necessary to inquire into the nature of invention and some of the general principles it follows. Lying absolutely at the bottom of such principles are the following postulates, the A B C of Eurematics: Given any artificial implement or product, we must assume—1st, *that there was a time when it did not exist;* 2d, *that before it existed there must have been a creature capable of producing it;* and 3d, *that such creature before producing it must have been conscious of needing it, or must have had use for it.*

There can be no orderly discussion of the genesis of any art without recognizing the truth of these postulates at every step. Questions may arise upon resultant or collateral propositions, but, admitting all that can possibly be claimed for accident as an element in invention, these propositions are not to be questioned. They are fundamental, and no logical consequences that flow from them can be evaded.

The first proposition, that before any artificial product existed

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* An Inquiry into the Origin of Invention. Vol. II, Trans. Anthrop. Soc., Washington. 1883.

there was a time when it did not exist, is not startling, and may be passed over for the second: before it existed there was a creature capable of producing it. This is as much as saying that no product of art came into existence simultaneously with its producer, and seems to be no more startling a proposition than the first; and yet, if I rightly interpret the ideas of most writers, they have failed to grasp even so common-place a truth.

The third proposition, that the producer must have been conscious of needing the product, or must have had use for it before producing it, is not at first sight so obvious. In fact I believe the failure to grasp this truth is a great source of error and misconception among many writers. No one, however, who has given any thought to the nature of invention, has failed to observe that every step in the mechanic arts has grown out of a pre-existing want. Not necessarily out of a pressing need. Invention now-a-days does not wait for the call to be so urgent that waiting can be no longer. Long before this stage necessities are anticipated, and the means by which they are overcome often do not become indispensable till the very habits they engender make them so. Illustrations of this are all around us. The sewing machine, the reaper, the telephone—what could we do without them? And yet in our own generation we have done without them all. They have themselves created the conditions which have made them indispensable. But none of them came by accident. They have been, every one, the fruit of years of toil and thought and anxiety on the part of those who saw, what few clearly comprehended, the imperfection of the means employed to do the daily work of mankind, and studied to produce better means. This is the history of steam, of electricity, of railroads, of metal working, of pottery, of every art that has a recorded history. Prevision and calculation are so truly elements in the growth of all known arts that in asserting their universality we incur no more risk than did Newton in asserting the law of gravitation.

What then, it may be asked, is the place due to accident in invention? Notwithstanding a popular belief that many if not most of the great inventions have been the fruit of accident, it may be asserted that the contrary is true. Fortuitous circumstances, trifling unforeseen incidents, have in many cases doubtless suggested expedients which have led to the consummation of great inventions. It was an accident—the result of his poverty—which led Senefelder to write on a stone slab his family wash-bill, and so led to the inven-

tion of the lithographic process; but the accident did not occur, and could not, till long and persevering pursuit of a method of printing cheap music had brought together the polished stone, the ink, the acid,—all the materials necessary to accomplish the result. Possibly it was an accident which led Goodyear to the use of sulphur for the vulcanization of India rubber; but the accident, if such it were, did not occur till years of expense and toil and experiment with a great variety of materials had led the way to it. And the rubber and the sulphur and all the appliances necessary for the experiment were ready to his hand, all accumulated in the pursuit of his lifelong purpose. Such experiences are common, and familiar illustrations of them are found, as for instance, in the lives of Palissy, the Huguenot potter, and William Lee, the inventor of the stocking loom. In these the element of accident enters in some degree into the consummation of the invention; but in every case it is such accident as might have occurred a thousand times over without result to other men whose minds were not intent upon the invention. Lamps had swung for centuries in the Italian cathedrals, and men had idly counted their oscillations as they kept time to the tedious delivery of generations of dull sermons; but the isochronism of their swing, if observed at all, was not regarded till Galileo came.

The true and only field that philosophy can concede to accident in invention is that it supplements and sometimes abridges the labor, calculation, and time of the inventor. To a man filled with a steadfast purpose, all his senses alert to every means chance or calculation may present to accomplish it, the most trifling incident may furnish the clue, which has fled from him like an *ignis fatuus*. To another the same chances may come and go continually without result. And while it cannot be said that accident has no place in invention, it must be conceded that its place is completely subordinate to other elements. Great inventions have been the fruit of accident in the same sense and to the same degree that a ripened peach is the fruit of the rude blast that shakes it from the bough.

It is important in a discussion like this to keep clearly in mind the difference between invention proper and discovery. The function of the latter is to bring to light the material facts, and the natural laws, which the former applies to useful purposes; and in respect to discovery, the element of chance, of accident, is im-

portant. The progress of scientific discovery is marked at every milestone by the revelations of accidents, which the thoughtful mind of the inventor did not apply to practical ends till long afterwards, when the need had arisen. If it was an accident that led Galileo to the discovery of the isochronous oscillation of the pendulum, it was not till fifty years afterwards that this discovery was applied to regulate the movement of a clock. The phenomena of electricity that accident may have revealed to Galvani and Volta, are the basis of inventions that the most active minds of this decade are expending their best energies upon. It cannot be denied that in discovery accident has played an important part; but the more this fact is considered, and the more we consider the true function of discovery, the more strongly do we find the proposition confirmed that improvements in the arts are not the result of chance but of intelligent efforts to supply conscious needs. Hence I shall regard this proposition as conceded, and I pass to another.

(4) *Every human invention has sprung from some prior invention or from some prior known expedient.* Inventions do not, like their protectress, Pallas Athéne, spring forth full grown from the heads of their authors. This suggestion needs no argument when made regarding any of the modern inventions. Every one of them is seen by the most superficial observer to be built upon or elaborated out of inventions and expedients previously in use. It is only when we go back of these and study the expedients and appliances out of which they have grown, and whose history is unrecorded, that the proposition I contend for is not obvious. And yet there is not a single one of them which does not when studied exhibit in itself the evidences of a similar substructure. In the process of elimination we go back and back, and find no resting place till we reach the rude set of expedients, the original endowment of men and brutes alike. This is a truth which study more and more confirms, and from it the proposition stated may be deduced as one of the laws of invention.

It may be deduced as a corollary to this proposition, but at the same time a fact determinable by independent observation, that the generation of one invention from another is not immediate but always through one or more intermediate steps. The effect of every invention fundamental in its character is first to generate wants before unknown or unfelt. The effort to supply these wants leads to

new inventions.* These may be quite distinct in their character from the original invention to which they indirectly owe their origin. They are related to it only as means to supply some want to which it has given birth. I shall not pursue this branch of the subject. Illustrations will occur to all. There is hardly a branch of industry that has not felt the effect of inventions based upon wants created by the introduction of petroleum, or the general use of the telephone. Wood-working, mining, transportation by land and sea—all the avocations of men—have felt their influence, have found wants engendered by their use, and improvements have been made to meet these wants. The wants of primitive man were limited, and his inventions were accordingly few. As wants increased in number and intensity, inventions multiplied, and the numberless wants of modern civilized life are only paralleled by its numberless arts and expedients.

I set it down as a fifth proposition: *Inventions always generate wants, and these wants generate other inventions.*

A sixth proposition is that the *invention of tools and implements proceeds by specialization.* This is true to a certain extent of all arts, though perhaps not a universal truth regarding all invention. It results, as will be apparent on reflection, from the last proposition. A single tool may have a great variety of uses, but, if there is a sufficient requirement, men will not long be contented with one tool for those uses for which it is least convenient. It will be reserved for that to which it is best adapted, and other forms will be devised better suited for special uses; possibly the parent type may be found inferior for all uses to some of its modified forms, and it may, on the principle of the survival of the fittest, become obsolete. Look at the variety of tools on a joiner's bench, chisels, planes, saws, each especially adapted for its particular work, but all pointing back to a time when there was but one form of chisel, or plane, or saw. The "jack-plane" and "long-jointer" may each be made to perform the work of the other, but they do it very imperfectly. The primitive bench plane was like neither, but was the type of

* A curious instance of this is brought to my attention while writing this paper. In consequence of the expiration of the earlier patents on roller-skates, a great impetus has been given to their manufacture, the result being the exhaustion of the world's stock of boxwood of certain sizes used for rollers. And to supply the want so created hundreds of people are trying to invent a suitable and cheap substitute for boxwood for this purpose.

both. There is nothing more striking than the variety of cutlery on a well-furnished table. The time is not remote when one knife worn at the belt served the purpose of all these, so far as these purposes existed, and of many others; when the table knife was not differentiated from the dagger of the soldier or the tool of the artisan. A man then used one knife to cut out a leather sole, to shape his arrow, to carve his food, and to stab his enemy. Changes in modes of living have led first to the broader specializations; fashion, caprice, and increasing refinement to others; till one scarcely dares attempt to enumerate the various forms of carvers and table knives of various sorts differing in form and material, each adapted by some feature for its particular use, and each the result of some degree of invention, with which the tables of Europe and America are furnished. Undoubtedly this process has gone on ever since man became an inventor, and might be illustrated as perfectly, though not so profusely, in the implements and weapons of the savage as in those of civilized men. All study of invention must take account of it. As soon as men began to adapt sticks to their use by artificially pointing them they began to find in them various degrees of hardness, weight, length, and rigidity, qualities fitting them for diverse uses, and as skill and experience were acquired they fashioned them accordingly. Likewise when man had begun to employ flint flakes, and before he had learned to fashion them to his will, he selected from the splinters made by accident or by his own unskilled blows those which served best such diversified uses as he had found out.

My seventh proposition, and final one so far as this paper is concerned, is that *no art makes progress alone.* I venture to assert the universality of this truth from what is seen in the recorded history of all inventions. In the development of the mechanic arts, two or more arts distinct in their nature but having close interdependence make advance *pari passu.* If one lags the other is necessarily retarded. If one makes rapid progress the other springs forward with quickened impulses. An improved utensil or article of manufacture may be the result of or may lead to improved processes and tools and machines for producing it, or to improved means for its employment. The progress of the steam-engine was long retarded by the imperfection of iron-working machines, since perfect cylinders could not be produced. The progress of electrical invention has necessitated the invention of new machines and processes for insulating

wire. The introduction of illuminating gas has created a demand for metal tubing, and machines for its rapid and perfect manufacture. And so every step in every art is marked by one or more corresponding steps in other arts.

These general principles, imperfectly stated as they are, by no means exhaust the study of invention. They only lie at its threshhold. They are among the more obvious laws which inventions follow as they are every day presented to the mind of those who deal with them: so obvious, that I have found myself hesitating as to the value of their presentation in this form; a hesitation which is removed by observing that, so far as writers upon early inventions are concerned, they are unnoticed and apparently unknown. Further chapters in Eurematics might be devoted to the elucidation of other truths equally generic and universal, but more intricate and therefore less obvious. I might cite for instance the tendency of civilization to convert luxuries into necessaries, true not only of absolute civilization but of every stage of it or every step towards it. The effect of this tendency upon inventions is marked and positive. I might cite the fact that invention is stimulated by rewards and retarded by opposition, which history abundantly illustrates,—eminently the histories of France in the middle ages, of The Netherlands, of Great Britain, and of our own country. Another proposition might be that the truth regarding biologic evolution—that the type of any species which is to predominate is at its first appearance unconspicuous—applies equally to the evolution of arts. Many such propositions more or less recondite might be stated, the adequate discussion of which would require a volume; but I can afford to pass them by, as I have not set out upon an exhaustive study. The few propositions considered are enough for the present purpose.

I shall now discuss the progress of invention in a single direction, partly as a study in itself, partly by way of illustration of the doctrines I have enunciated. I have selected the stone hatchet for this purpose because in some of its ruder forms it represents the earliest human workmanship of which any knowledge has come to us, and also because in its rudest form it presents the evidences of being the fruit of long antecedent growth. Further than this I observe that primitive as it indeed is, and in its highest development rude and ineffective in comparison with the finished implement of this age of steel, the thoughtful student of invention sees in it the culmination for the time being of human art rather than

the beginning. For the purposes of this paper I regard nothing less than the hafted celt as the finished implement whose genesis I shall attempt to indicate.

I assume as the starting point the conclusion reached in my paper before referred to,* that the earliest mechanical process employed by man was the art of working wood by abrasion. This cannot be regarded as proven; absolutely proven it can never be; but it comes in as a link connecting what must have been in the history of primitive man with what is revealed to us regarding the man of the earliest stone age. This art, or something closely similar to it, appears as the immediate derivative of the original mechanical expedients of man in a state of nature, and of the wants engendered by his human characteristics. Tracing back the art of wood working we find no resting place till we come to the art in this condition. In short the more the subject is contemplated, and from whatever point of view, the stronger appear the probabilities, so strong that to my own mind they are convincing. Starting from this basis, what was the process, what the result sought, what the methods employed to produce it?

The object sought for was a pike, a strong, rigid, sharp-pointed stick or shaft adapted for use as an offensive and defensive weapon, a want early felt and hitherto imperfectly supplied by chance and nature. The means employed was a rough rock, a coarse sandstone or mill-stone-grit upon whose exposed surface the wood was rubbed or drawn back and forth until reduced as desired. A tedious process, but not more so than many of those employed to this day in the arts of savage life. We can imagine men coming from great distances to the inventor of this art with poles on their shoulders to be prepared in the new style. It would not at once be perceived that no special properties attached to this particular rock, that rocks having similar properties and perhaps better suited to the purpose were every where. The mind was dull in grasping the essential fact of the art, and perhaps for ages superstition and fetichism may have been engendered by this very improvement. It is easy to see, however, that it had created a new want, or perhaps intensified the old one. Pikes were liable to be broken, were subject to natural decay. They must be replaced, and new ones were always in demand. Their artificial production had increased the number of their

* An Inquiry into the Origin of Invention. Vol. II. Trans. Anthrop. Soc. Washington. 1883.

possessors, and the want of a ready means for the replacement was more widely felt. To the majority it was a new want. Hence among people widely scattered, more convenient and accessible means were sought for supplying the demand ; and in answer to this want came the discovery, perhaps the result of similar experiences and observations, that gritty rocks every where would yield the same results to similar manipulation by the hands of any one. And a further discovery followed close on the heels of this, that the jagged edges of flints and other hard rocks would by a manipulation but little varied perform the work better and faster than the gritty surface of the sand stones. A stick drawn forcibly over such a sharp edge has its surface scraped from it in thin shavings instead of being merely abraded as heretofore. This important step from abrasion to scraping, which is in fact cutting, was therefore reached before any cutting or abrading tool had been devised. Reached by slow steps, in answer to a felt want, but a want in no way pointing to it, it was actually the invention of another and quite distinct mechanical process. It was a better process, gave better results, and the weapon and the art of wood working made progress together.

We have advanced one step, man now has the notion of the cutting edge and its use. But it is part of an immovable bowlder or ledge, not always accessible, and the want of a convenient means always at hand is but partially supplied. The long pilgrimages which had to be taken to the primitive pointer of pikes were at an end, but the journeys though shorter still have to be made. How was the next step, resulting in the production of a portable cutting implement, to be accomplished?

It will be seen at once that in the use for a considerable period of the edge of a rock for cutting purposes it will become dulled. Other parts of the rock having exposed edges will be sought, and these will become dull in turn. This dulling process proceeds more or less rapidly according to the material applied to it; and as the harder woods were found to be in all respects more serviceable they were more generally used. We may conceive that at some time by the violent application of a hard piece of timber to an edge somewhat thinner than ordinary, the edge itself instead of being merely dulled is broken off, and to the pleasant surprise of the operator a new edge, sharp and clear, and better than the half-dulled one he had been using, makes its appearance. And he eventually learns that he can at any time produce a new edge by shivering off a piece

of the rock with blows. He is not long in learning that the part broken off has similar edges. If it be large enough to lie firmly he can employ it as he does the parent rock. If smaller, he may hold it firmly with his feet while he manipulates the wood upon it with his hands. Perhaps he can carry it away and use it at the place most convenient to him; when dulled he can shiver it by a blow or two and it is sharp again. And then at last by slow degrees, requiring ages perhaps, one can hardly tell how, but by the continuance of this process, he observes that these splinters struck from the fragment, these fragments of fragments, possess the same cutting edges as the original rock, and in a bit of stone not larger than his hand or his finger he possesses an instrumentality capable of doing all that he and his ancestors have been laboriously doing on the parent rock or clumsy fragment. He learns also that instead of dragging the wood over the edge, he can, with a totally different manipulation, hold the wood firmly and operate on it with the stone splinter, and the tool is invented.*

When I think of man in his primitive condition, as the logical necessities of this subject have compelled me to think of him, helpless, miserable, the prey of beasts, without tools, without means of defense except such as he shared with the beasts, and then think of him in the condition to which he is brought in this outline of his inventions, I find it impossible to adequately express my sense of the progress he has made. One effective weapon, its structure improved, and skill in its use acquired by generations of experience, and one cutting tool, even in the rudimentary form of an unfashioned flake, have separated him incalculably from the condition of his ancestors. His knife or hatchet, as we may henceforth call it, contained within it all the possibilities of the future, but for the present—his present—its capabilities were learned by constant lessons and with every new occasion. He had no want to which it did not minister. It not only served its first purpose to prepare his weapon, but it became itself a weapon. It served him to procure and prepare his food, both animal and vegetable, his shelter, his raiment, if he had reached the stage of wanting raiment. Its

* It is only by a loose construction of language that this can be called the invention of a tool. The tool, a mere flake of stone, had already long existed. The actual invention was an art or process quite distinct from any heretofore employed. The brief and more popular form of expression may be employed with this explanation.

acquisition was the greatest step he had taken in invention; and when we regard what has grown out of it, the infinite variety of cutting tools, implements, and machines, whose origin we remotely trace to it, and the unnumbered needs they supply, we cannot hesitate to ascribe to it the highest place among all the inventions of all time.

If the hafted celt was for the time the culmination of art, this is not less true, of its time, of the flint knife. As in man's rudest estate he used the expedients with which nature endowed him, selecting those best adapted to his immediate purpose, so now out of the diverse forms assumed by flakes and chips, he selects those best adapted for particular purposes. He is repeating what occurred in his earliest period, but with new and diversified wants, wider intelligence, and a greater range of material out of which to select. He finds blunt edges give satisfactory results in the old process of scraping wood, but he finds that thinner and sharper edges penetrate the wood deeper, and remove the superfluous material faster. He finds he can work more deftly, more conveniently, can put a finer point on his weapon, can apply the new tool to all parts of it, can reduce and trim the shaft as well as the point, can even sever the growing saplings to obtain his material. He finds that some forms can be made to penetrate and divide the tough skins of beasts, and carve their flesh. In fact, in whatever direction his necessities or inclinations lead him, he finds his knife in some form contributing to his comfort, his protection, and the supply of his wants. The possession of the tool has wrought out his mastery over nature.

This culmination in invention is but momentary. It is a milestone, a breathing place in the history of arts. But the march still goes on, and we find man still searching among fragments for forms adapted to his particular uses, but gradually learning by experience that by well-directed blows he can sometimes produce chips having special forms, and so fitted for special uses. But these are chips and flakes only. There is no attempt as yet at dressing or shaping stone. The rude forms they bear when shivered from the rock, are all that man has yet conceived in the structure of a stone implement. These rude forms seldom appear in our museums. They are the scoff of archæologists. They are not distinguishable from the work of the elements. In fact, the splinters thrown off by frost or fire may have been as readily selected for use as those formed by human agency. And as writers have agreed upon

the name *palæolithic* to indicate the age marked by the first traces of human workmanship in stone implements, we must recognize the *protolithic* age, in which stone fragments showing no trace of such workmanship were the common implements of mankind. The earliest age of wrought implements could never have come but for such a precursor. The rudest wrought forms did not appear till something of the same nature and used for the same purposes, but imperfectly adapted for their performance, had created the need of them and led up to the means for its supply, and the one thing which bore these relations to the earliest recognizable forms of dressed-stone implements was the unformed flake.

What were the steps from this form of flint knife, or scraper, or hatchet, to the hafted celt?

I formerly reached the conclusion that the original endowment of man could include no less than the stick and stone for striking and hurling, and the string or withe for tying or binding. In the course of this paper I have traced the synchronous development of the art of dressing wood, and of stone appliances for the purpose. With the advancement of these it is not to be supposed any former art or expedient was lost. On the contrary it is to be presumed that progress in them had been made corresponding to that we have been following. The club was better fashioned; approved forms of hurling-sticks may have been discovered and come into use. Greater skill may have been acquired in the use of the hammer-stone, and judgment in the selection of suitable forms either for crushing, or for splitting, and with more convenient hand-grasp. The flexible vines and strips of bark, with which primitive man lashed his frail shelter, his successor may have improved by rudely twisting the fibres or strands, or have supplemented by other materials, notably, after he had acquired the use of the flint knife, by strips of skin and animal tendons. The inventory of his possessions then would embrace the club and pike, each clearly specialized, the hammer-stone, not formed by art but selected, the stone knife, and strings of various materials. The pike, the hammer stone and knife may have been of many forms. Now it will be seen that these elements may be brought together in various ways so as to accomplish a variety of results, the elements in every case being a stick, a stone, and a string to bind them together, and the difference in result depending on the particular form of stick and stone. For instance the heavy end of a club is made heavier by lashing to it a hammer stone—result the mace. The pike is improved by securing to it a

pointed flake of flint. A flint flake too small for the hand is made effective by fixing it to a piece of wood, making a knife or dagger. A heavier sharp-edged fragment secured to a handle adapting it for striking, becomes the axe or hatchet. What immediate incidents or needs led to any of these combinations, I do not propose to guess. It is enough to have shown that at a period when man was as yet unlearned in respect to any dressing of stone beyond knocking off rude splinters from a rock, he may have had in his possession the means to produce, and was fully capable of producing, such implements and weapons as I have named. This being true, the same wants which might at any period of his history have led to their production may without violence be presumed to have done so then. They are in the line of his acquired arts, and the necessary links between these and the arts he is yet to acquire.

Whether these various combinations were made prior to actual working of flint it would be idle to speculate. It is more likely that neither preceded the other. While man was finding out how to use his possessions by bringing them together in new combinations, he was naturally improving them all. Having found the flint and other rocks of similar texture so far obedient to his power that they could be shattered, and new and useful forms produced, having acquired uses for these forms, having learned the purposes to which a sharp edge could be applied, and that a fresh one could be produced by knocking off the dulled one—it followed in due course, from experience, to form the new edge with less violent blows, with more judgment and dexterity, and, as the advantage of special forms became apparent, with a view to bringing it as close as possible to such forms. And all this time the old art of reducing by abrasion had not been lost; applying it now to the stone as finer and finer chipping suggested and provoked the desire for a smoother edge, the celt appeared, polished at first on its edge only, afterwards on its entire surface. There was no dividing line between the palæolithic and neolithic ages. If separated at all, it is by a broad zone through which the implements of both are found side by side. Neither was there any step from the finished celt to the hafted implement. The essential step, that of securing a stone in some form to a handle, had been taken long ago.

Lest it might be suggested that in order to sustain a theory regarding the developement of the arts, I have myself been led to invent steps in art that were never known to man, it is worth while to remark

that none of the steps I have set forth are imaginary. All of them are in existence and in use yet, in their appropriate places, often amidst the completest appliances of modern mechanic arts. If the primitive man sharpened a stick by rubbing it over a rough grit, he used the same means an artist employs to-day to produce a fine point on his pencil, and the same by which we sharpen all cutting tools. The scraping tool is one of the ordinary provisions of a joiner's outfit; but the use of a bit of broken glass is more common still. As the edge becomes dulled by use, the glass is simply broken and two fresh edges are formed. This is universal in civilized life, and a curious instance of it in savage life has just been brought to light by the Rev. Lorimer Fison, in his pamphlet on the Nanga or Sacred Stone Inclosure of Fiji, in which he relates often having seen "a mother shaving her child's head with a bit of glass, and biting a new edge on the instrument when it became dull." These original arts have never been lost. Probably it is a general truth regarding mechanic arts that no one of them once commonly acquired is ever again lost. It may be laid aside for a time or suspended, but it revives in some form; and I venture to think that much of the eloquence that has been expended upon the "The Lost Arts" has resulted from a very imperfect acquaintance with those that exist.

It is apparent that every step in the progress that has been recited resulted in an improvement in man's condition. The first improved weapon, club or pike or missile, was equivalent to so much greater strength of arm or length of reach. It augmented man's superiority over the brutes; it made his life less precarious; it put the means of securing food, shelter, and covering more fully within his power. His environment, to which he had in his primitive condition been completely subject, he now could to a certain extent control, could subject to himself. The first improved means of fabricating a weapon, the first tool or mechanical process, accomplished these results in an increased ratio. The step that made the cutting tool the possible possession of every man, which made the knife even in its clumsiest form a common tool, did for the whole race what the earliest steps did for a limited number, and made this amelioration general. The increased number of forms and varieties of tools and weapons, growing out of the diverse and manifold wants they were adapted to supply, were each steps in the betterment of his material condition, each an indication of progress; man's advance towards civilization, slow as it must have been, was

marked off step by step by the advances he made in his mechanic arts. The more he became independent of nature and capable of forcing her into his service the more time and inclination he found for the perfecting of his implements; and the more he perfected his implements the more capable he became of subduing nature. And this interaction has never ceased, it goes on to-day. But the achievements of to-day are not the conquest of savage beasts, nor the solution of the problems of food and shelter and warmth. We are overcoming time and distance; we are conquering the barriers of sea and mountain; we are finding out the more hidden forces of nature, and subjecting them. The fruit of our inventions is not seen in rough flakes of stone lashed by sinew to rude hafts, but in the mighty movement of the railway train thundering across the continent, or the click of the telegraph as London talks with Calcutta. And every step in progress has been a step in the improvement of man's condition from the first to the last. And so it shall be in the future.

Artists depict the genius of invention as a voluptuous female figure, in various stages of imperfect attire, attended by innocent boys in their primitive nudity, and with gear wheels and anvils and other rough equipments of the artisan in ill-assorted proximity. This is a feeble conception. The genius of invention is not a creature of delicate mould, but one of brawn and sinew. His voice is no gentle song of lullaby, but comes to us in the deafening clatter of Lowell looms and the roar of Pittsburgh forges. Mighty and beneficent and responsive to human wants—this is the kind of song he sings in his rugged rhythm:

"I am monarch of all the forges:
  I have solved the riddle of fire;
The amen of Nature to cry of man
  Answers at my desire.
I grasp with the subtle soul of flame
  The heart of the rocky earth;
And hot from my anvils the prophecies
  Of the miracle years leap forth.

I am swart with the soot of my furnace,
  I drip with the sweat of toil;
My fingers throttle the savage waste,
  I tear the curse from the soil;
I fling the bridges across the gulfs
  That hold us from the To-Be;
And build the roads for the bannered march
  Of crowned humanity."

DISCUSSION.

Mr. P. B. Pierce, discussing the paper, referred to some of the curiosities or phenomena of invention; for this science of *eurematics*, like every science, has its attendant phenomena. Indeed, that invention is a science is demonstrated by its attendant phenomena.

Invention is not creation; the first deals with matter direct; the latter supplies that with which invention deals. The student of eurematics, giving heed to what the history of his science has to teach, soon discovers the principles of the great law of evolution. Let him inspect the almost humanized giant that bears its load of living freight daily from Washington to New York in less than six hours, and what does he find, except that since the days of Watt the process of selection or differentiation has been intelligently going on! The clumsy, the crude, the ruder elements have been rejected; the harmonious, the simple, the efficient, and stronger have been utilized. Increment by increment complexity has given way to simplicity, until the perfected machine stands forth as we know it; that is to say, the machine we are pleased to call *perfect*, the selected excellence, the *summum bonum*, of all that experience and long use have taught to be best of those that have preceded it. Each inventor has contributed his mite, and lo! the grand result! And its maker, man, is he not perfecting himself along with that dull matter upon which he works and in which he achieves! Is he not, as described by the poet,

The heir of all the ages in the foremost files of time?

Is not matter reflex? Is Frankenstein in reality the monster his author protrayed him to be? Will not the science of eurematics, when once fairly beset by the persistent inquisition of scientific study and investigation, open wide the door of the temple that is even now ajar, and permit its disciples to enter and make intelligent conquest, under a full knowledge of its laws, where until now they have only been permitted to make occasional, random captures from the *vestibulum*, as it were?

The thousand forces of nature lie hidden within grasping distance; but for lack of systematic study they elude our clutch, escaping from our wiliest approaches as the thistle down upon a puff of air. This may not always remain so. The Lilliputians bound Gulliver with straws; let us ply Nature with pitiless interrogation till she yields

us the fullest knowledge of all her laws. For this is eurematics in its broadest significance; it is encompassing the laws of nature with material form and compelling matter to do the bidding of psychical energy.

But evolution does not account for all. There is in invention a synchromism that is almost mysterious. The present is the grand harvest time of all the seed that has been planted by the generations that have preceded us; but why the thoughts of inventive minds appear to move in batallions, all aiming at some common objective, seems at first view almost inexplicable. A given function is demonstrably demanded; a hundred minds set themselves at once, in all parts of the world, to produce the means for its satisfaction. With the almost universal diffusion of information that has come about with the art of printing, even in all languages and tongues, aided by the telegraph and the telephone, who fails to know in all the broad earth to-morrow morning what the chiefest want of to-day has been? Within one month's time from the great flour-dust explosion in the mills of Minneapolis, in May, 1878, there were over thirty inventions made for preventing the recurrence of such an accident, and all practically effective. Many of them were almost if not quite identical, although made by men having no knowledge even of each others' existence, and in all parts of the world! So quickly, when a pressing want is known, is the means supplied for staying the same. When the science of invention has been perfected, and every want has been given a means for its satisfaction, will not the highest type of invention then be the discovery of a new want, latent in the human soul, but never before developed?

Another feature of invention noticeable to an attentive observer is the isolation in which an important discovery is often times set. The evolution of the automatic grain binder of this day, from the sickle of Egypt and the Orient, is plain and familiar. To one who has witnessed the devouring knives of this latest type of human genius, hungrily levelling the yellow harvests of the great northwest and tossing the bundled sheaves backward in serried rows upon the stubble, and contrasts its action with that of the reaper in the time of Boaz, how far apart they seem separated! And so they are, wide centuries apart. But the quick mind of invention anticipated the want almost in the earliest day of the reaper. In the year 1854 two men invented, perfected, reduced to practice, and patented the

completed machine whose opportunity for use did not come until twenty-five years later. Like lonely islands arising out of the receding waters of an ocean, such inventions, though they may afterwards be the highest lands of great and fundamental enterprise, are lost for want of use. Although pioneers their inventors are without remuneration because they are too far in front of the needs of the world. The world itself is ever unready; the lines of necessity are conservative and strenuously refuse to make room for the new applicant for favor, even though full of promise.

Mr. Wm. H. Babcock said no one, on glancing over our patents, can fail to observe how many of the inventions covered by them are obviously outgrowths of those already in existence rather than contrivances adapted to meet any real want. A man sees a particular machine, or a description of one, and forthwith proceeds to devise a similar but slightly different construction. Thus there are, for example, more than three thousand patents on car couplers, most of them varying from others in a trivial degree, very few of them being actually in use. A large class of our inventions are of this incidental kind.

But another large class of inventions have grown mainly out of a distinct conception of a public demand, real, foreseen, or fancied, or of the practical needs of manufacture. Exclusive of certain sporadic and eccentric instances, inventors are either manufacturers, the men employed by them, or who expect to sell to them. All these are on the alert to note the drift of public taste and practical requirements. A manufacturer sees, or thinks he sees, that a new article, or a change in an old one, would meet with or lead to a considerable sale; or that a simplification of his machinery would enable him to reduce his force or his fuel; a factory hand finds that the machine with which he works has some persistent, annoying defect which a slight alteration would avoid; an outsider in a factory village forms his own theory as to what would give one competing manufacturer an advantage over another and knows that it would be well paid for; in all these influences the exertion of ingenuity is easily accounted for.

The effect of the public demand is curiously illustrated in the synchronism of invention. It frequently happens that men widely separated territorially and having no discoverable communication with one another make the same invention at the same time, or so nearly at the same time that priority cannot easily be determined.

The progress of a certain art has reached a point where a given step becomes inevitable, and like causes produce like results everywhere.

This shows, further, that the individual man is of less importance as a factor in invention, than his environment. Indeed invention in the wide vague popular sense can hardly be said to exist. Even our greatest inventions have proceeded by a succession of small increments. Each man puts a round in the ladder, and the next climbs on it to put in his higher up. The one who puts in the last round steps from it to receive the crown of success, although his contribution may have been the least of any; and his even more meritorious predecessors who failed, but made that success possible, are generally forgotten.

Invention for the pleasure of inventing is of prime importance in literature and art, and cannot be wholly ignored even in treating of mechanical matters. Many men delight in experimenting with machinery, combining element with element, adapting every part with every other and to the end in view. They find invention "its own exceeding great reward." Every one who deals with inventors can recall such enthusiasts, who are often men of notable if narrow ability, and, on the whole, the most interesting of their tribe.

Mr. A. W. Hart said: I am very glad that, among other things he has done, Col. Seely has put his foot down on the theory that accident is the mother of invention. This is a popular error which most of us may have sometime shared—certainly, I must admit it was included once in my catalogue of sins. What are called accidents are in reality normal results of a search or inquiry, or series of experiments, such, for example, in the geographical field, as the discovery of America by Columbus, or in the healing art, the prevention of cholera by inoculation with cholera germs if that is the correct term. In the way of a homely illustration, I will relate a personal incident. A friend proposed a walk to Arlington, and said we would look on the way for Indian arrow-heads. I assented but said that I never found an arrow-head in my life. "That is merely because you never looked for them," replied my friend. We went, and sure enough, found the arrow-head, and I found another the next walk I took in search for one. Now, while in a certain sense I may call that finding an accident, in the true and proper sense, it was none at all. It was the regular legitimate result of the search instituted. But for the preparation or plan and its systematic execution, the "accident" of discovery would never

have occurred. So inventions come when we are ripe for them and look for and strive after them—and then they are not accidents, but logical endings of systematic beginnings—just as the solution of a mathematical problem follows its working.

One may walk—as the savage does—over diamond or coal fields, rich bottom lands, or gold-bearing rocks, seeing nothing of their nature, contents or potentialities because intent on other things—of the hunt or war—and because not developed to any possible comprehension of anything more. But the civilized and mentally and scientifically developed man, going over the same ground might make valuable discoveries, for good to himself and his fellows, while losing sight of the beasts or the signs of presence of others that the eye of the savage takes in. The latter is therefore not to be charged with negligence, nor the civilized man with being the victim of an accident. So inventions come when we are ready for and seek them, —as apples fall into the basket we hold to catch them when ripe and ready to drop.

Mr. MURDOCH read a paper on the "SINEW-BACKED BOW OF THE ESKIMO."

All the branches of the widely-distributed Eskimo race now live in regions which are either treeless or else deprived of the ash and other elastic woods fit for making bows. The fact that the bow was in general use among the Eskimo previous to the introduction of firearms is one of the arguments that they have not always lived in the regions which they now inhabit, but have moved on from places where wood suitable for the purpose was to be obtained. As they gradually became settled in their new homes, probably before the different branches were so widely separated from the original stock as they are now, and as the simple bows which they had brought with them from their old country became worn out and had to be replaced, it was necessary to find some means of giving the needful elasticity to the brittle spruce and fir, frequently rendered still more brittle by a long drift on river and sea, followed by exposure to sun and rain on the sea-beach. In some places even driftwood is so scarce that bows were made of no better material than dry antler. The elastic sinews of several animals, especially of the reindeer, furnished the means desired of making an efficient weapon out of these poor materials. This is not employed in the way used by the Indians of the plains, who glue a broad strip of sinew along the

back of the bow, but is braided or twisted into a cord the size of stout whip-cord, which is laid on in a continuous piece so that there are numerous strands of the elastic cord running along the back of the bow so as to be stretched when the bow is drawn. The simplest or, so to speak, ancestral pattern of sinew-backed bow from which the types now in use are evidently derived is one in which there are a dozen or twenty of such plain strands along the back, running around the "nocks" and held down by knotting the end of the cord round the handle. Bows of this form, slightly modified by having the cords somewhat twisted from the middle, so as to increase their tension, are still to be found in Baffin Land, where many of the arts seem in a lower state of development than among the Greenlanders, on the one hand, or the Western Eskimos, on the other. Let us now consider how in course of time the different branches of the Eskimo race have improved upon this simple invention. Along the well-wooded shores of southern Alaska, from the island of Kadiak nearly to the mouth of the Yukon, where there is plenty of fresh, living spruce, they have chiefly increased the efficiency of the bow by lengthening and broadening it, and have paid but little attention to the sinew backing, contenting themselves with slightly increasing the number of strands, wrapping them round with a spiral seizing, which prevents them from spreading, and occasionally adding a few more strands which only extend part way to the tips, being secured by hitches round the bow. This makes the bow a little stiffer in the middle than at the ends, where less strength is required. On the other hand, the people who live along the treeless shores of the Arctic Ocean, from the Mackenzie river to Bering Strait, can obtain no wood better than the dead and weathered spruce which the sea casts upon the beach. Consequently, all improvements in the weapon were of necessity confined to the sinew backing, which has developed into a marvel of complication and perfection, while the bow itself is rather short and not especially stout. Starting as before with a loop at one end of the cord strands are laid on from nock to nock until there are enough of them to give sufficient stiffness to the ends of the bow. Then the cord goes only to within 6 or 8 inches of the tip and is secured round the bow by hitches, sometimes a very complicated lashing of as many as a dozen half hitches alternately in opposite directions, and returns to a corresponding place at the other end, where it is similarly hitched. In this way strand after strand is laid on, each pair shorter

than the preceding, and the backing constantly thickening towards the middle of the bow. When sufficient strands are laid on they are separated into two parcels, and with a pair of very ingenious little bone or ivory levers are twisted from the middle into two tight cables, so that the twist of the cords adds to the resistance to be overcome in drawing the bow. These are prevented from untwisting by a lashing at the middle which runs through the cable and round the bow in a sort of figure of 8. The end of the cord then makes a tight spiral seizing round the bow which not only keeps the backing from slipping, but serves to distribute the strain evenly and keep the bow from breaking. This pattern is probably the ultimate development of the sinew-backed bow. Not only is it difficult to imagine making a more perfect weapon from the material, but attention will no longer be paid to possible improvements in a weapon which is rapidly falling into disuse. As would naturally be supposed the region about Norton Sound, where the tribes of the Arctic coast meet those of Bering Sea, is a debatable ground, where bows of the two types described are found side by side, along with others partaking of the characteristics of both. If now we cross to St. Lawrence Island, we find Eskimos depending solely on driftwood, who employ another and most peculiar modification of the original type. They have lengthened the ends of the bow so that the original simple backing hardly reaches within a foot of either end, while these ends are bent up as in the Tartar bow, and separate backings are stretched across these bends.

The Eskimos of the mainland of Siberia, who have long maintained direct intercourse with the St. Lawrence Islanders and with the Eskimos of the Arctic coast by way of the Diomedes, show the evidence of this intercourse in the pattern of their bows, using either the peculiar St. Lawrence tpye, or purely American bows of the Arctic pattern, or weapons which curiously combine characteristic features of both.

DISCUSSION.

Mr. Bates said that the little blocks which are tied into the concave outer limb of several of Mr. Murdoch's bows are something more than a mere stiffener of the wooden portion. It is a truly mechanical expedient, to give efficiency to the tension member of the combination, which is the sinew. It not only acts as a strut to increase the leverage of the tension member, which is the

function of the strut in all combination trusses, but it shortens and straightens the line of the sinew, thus bringing its rigidity and elasticity into full play. In this, as in so many other instances of merely experimental evolution, the best results of abstract theory are arrived at.

Ninety-Seventh Regular Meeting, May 19, 1885.

Vice-President Dr. Robert Fletcher in the Chair.

The Chair announced the death of Count Giovanni Battiste Ercolani, of Bologna, Italy, a corresponding member, after which a memoir was read by Dr. E. R. Reynolds, who, in the course of his remarks, presented to the Society an embroidered Italian flag and a number of scarfs and mourning wreaths contributed by various scientific societies of Italy, of which Count Ercolani was a member. The Chair remarked that Count Ercolani would probably be remembered principally for his discovery that the circulation of the blood was known and promulgated prior to Harvey.

Dr. Matthews then read a paper upon "The Cubature of the Skull," which was followed by some inquiries by Dr. Frank Baker and Mr. Bates, leading to further remarks by Dr. Matthews.

ABSTRACT.

The lecturer discussed briefly the various methods which have been employed in the volumetric measurement of the cranial contents and pointed out their various defects. He then described a method which he had recently devised and employed in the Army Medical Museum at Washington.

After recording the weight of the skull it is varnished inside with thin shellac varnish, applied by means of a reversible spray apparatus. Artificial or accidental orifices are closed with India-rubber adhesive plaster. The foramena and fossæ are filled with putty. The skull is wrapped in a coating of putty an inch or more in thickness, which renders it water-tight. It is filled with water by means of a special apparatus in forty-five seconds and emptied in fifteen seconds. The rapidity of this manipulation in conjunction with the varnishing prevents soaking into the sinuses and the undue measurement of water which does not pertain to the

cranial cavity. The water is poured into a measuring glass of 2,000 c. c. capacity, and lycopodium is scattered on the water to define the true surface. The putty is taken from the skull; the latter is cleansed and placed in a dry, warm apartment until by slow evaporation it is reduced to its former weight and consequently to its former capacity. Then it is measured a second time to verify the results of the former measurement.

Hitherto anthropologists have chiefly employed solid particles, such as shot or seeds, in the cubature of skulls. Water had been tried by former experimenters without success, and abandoned—the objections to its use being considered insuperable. The lecturer, however, considered that by his method he had overcome the chief difficulties. Although the method is new and still susceptible of improvement, it is thought that the results—an average of one cubic centimetre difference between the first and second measurements—have not been excelled.

One of the bronze skulls of Professor J. Ranke, of Munich, was exhibited, and the claims of the inventor, as published in "Correspondenz-Blatt der Deutschen Gesellschaft für Anthropologie Ethnologie und Urgeschichte," September, 1884, were quoted. The lecturer had found one difficulty in using the artificial skull which Prof. Rauke had not suggested. The cavity varied greatly in capacity with changes of temperature. For a perfect conformity of measurements not only was it necessary that the water used should be certain specified heat, but the bronze skull, the various vessels used, and the atmosphere of the apartment in which the experiments were made should be of a corresponding temperature. At 4° centigrade the lecturer obtained for the bronze skull, estimating both by weight and measure, a capacity of 1,220 c. c., while at 14° centigrade he obtained 1,240 c. c. In no case did he get a result as high as that engraved on the skull, *viz:* 1,255.6 c. c. The skull was presented by Prof. Rauke to the Army Medical Museum.

A paper followed from Dr. BAKER upon "THE PRINCIPLES OF INTERPRETATION OF BRAIN, MASS, AND FORM." This paper was illustrated by numerous charts.

FROM SAVAGERY TO BARBARISM.

ANNUAL ADDRESS OF THE PRESIDENT,

J. W. POWELL,

Delivered February 3, 1885.

It is a long way from savagery to civilization. In the attempt to delineate the progress of mankind through this long way, it would be a convenience if it could be divided into clearly defined stages. The course of culture, which may be defined as the development of mankind from savagery to civilization, is the evolution of the humanities—the five great classes of activities denominated arts, institutions, languages, opinions, and intellections. Now if this course of culture is to be divided into stages, the several stages should be represented in every one of the classes of activities. If there are three stages of culture there should be three stages of arts, three stages of institutions, three stages of language, three stages of opinions, and three stages of intellections.

Three such culture stages have been recognized by anthropologists, denominated Savagery, Barbarism, and Civilization. But they have been vaguely characterized and demarcated. Savagery has been considered a low stage of culture, barbarism a middle stage of culture, and civilization a high stage of culture. In a brief address it is not practicable to set forth the essential characteristics of the whole course of culture; and it is intended on this occasion simply to characterize Savagery and Barbarism, and to define the epoch of transition. To this end it will be necessary to set forth the characteristics of savage art as distinct from barbaric art, and the nature of the change; to explain savage institutions and barbaric institutions, and how the lower class developed into the higher; to set forth briefly the characteristics of savage language and barbaric language, and the origin of the change; to show the nature of the opinions held by savages and the opinions held by barbarians, and to explain the reason of the change from one to the other; and finally to explain savage and barbaric intellections, and to show

how savage methods of reasoning were transformed into barbaric methods of reasoning.

The most noteworthy attempt hitherto made to distinguish and define culture-stages is that of Lewis H. Morgan, in his great work entitled "Ancient Society." In it these three grand periods appear — Savagery, Barbarism, and Civilization — each with sub-divisions. Morgan recognized the importance of arts as the foundation of culture, and his "ethnic periods," as he calls them, are based on art development. With him, Savagery embraces all that stage of human progress extending from the beginning of the history of man, as distinguished from the lower animals, to the invention of pottery. Barbarism then succeeds and extends to the invention of the alphabet. He adds that among some peoples hieroglyphic writing takes the place of phonetic writing, and civilization begins at this time. He then divides each of these periods into epochs which need not here be considered. In some of Morgan's works he connects the evolution of institutions with the development of arts, but to an imperfect degree, and without explaining their interdependence. He also, at different times, hints at the relation of linguistic development to arts; but he considers mythology to be too vague to afford valuable data for this purpose.

The scheme here presented differs from Morgan's in placing the epoch of demarcation between Savagery and Barbarism later on in the course of human culture; and it is proposed to characterize the stages, not by arts alone, but by all the fundamental activities of man.

The next most noteworthy attempt to define culture-periods is that by Lester F. Ward, one of the Vice-Presidents of this Society. In his scheme there are four stages of social progress, or social aggregation, viz:

"1st. The solitary, or autarchic stage;
2d. The constrained aggregate, or anarchic stage;
3d. The national, or politarchic stage; and,
4th. The cosmopolitan, or pantarchic stage."

Ward seeks to establish these as veritable stages on the basis of institutions alone. They are treated as stages of social aggregation, and not as culture-stages. The first, second, and fourth are purely hypothetic. I have elsewhere stated my reasons for not accepting the first and second stages; but, whether real or imaginary, they antedate all possible objective knowledge of the condition of man-

kind. The fourth stage is a prophecy, and though I believe that his prophetic vision is clear and that he sees a true picture of the future, it need not be considered here. His politarchic stage embraces all the course of human culture with which science may at present deal on a basis of observed fact, and it is this stage which is here divided into three parts—Savagery, Barbarism, and Civilization.

E. B. Tylor, also, has classified the stages of culture as Savage, Barbaric, and Civilized. The lowest or savage stage he defines "as that in which man subsists on wild plants and animals, neither tilling the soil nor domesticating creatures for his food." He considers that men arrive at the barbaric stage when "they take to agriculture," and pass from the barbaric to the civilized stage by acquiring the art of writing.

In relation to the epoch which separates Savagery from Barbarism, Tylor does not greatly disagree with Morgan. Morgan uses as a criterion of Barbarism as distinguished from Savagery the acquisition of the art of making pottery; Tylor, the acquisition of agriculture. But usually the two arts have been acquired at about the same time, and it seems probable that the conditions of life brought about by agriculture were necessary properly to develop ceramic art. If this is true, agriculture is the more fundamental. If stages of culture are to be established on conditions of art development alone, the invention of agriculture should doubtless be accepted as the plane of demarcation between the two lower stages; but if the culture-stages are to be based upon characteristics derived from all the classes of human activities, the separation between Savagery and Barbarism must be placed somewhat later on. Such a plane of demarcation has been adopted by me for a number of years, both in my publications and in the discussions and expositions informally presented to this Society from time to time; and it is my purpose to make a somewhat fuller exposition of my method.

All the grand classes of human activities are inter-related in such a manner that one presupposes another, and no one can exist without all the others. Arts are impossible without institutions, languages, opinions, and reasoning; and in like manner every one is developed by aid of the others. If, then, all of the grand classes of human activities are interdependent, any great change in one must effect corresponding changes in the others. The five classes of activi-

ties must progress together. Art-stages must have corresponding institutional, linguistic, philosophic, and psychic stages.

Stages of progress common to all the five grand classes of human activities may properly be denominated Culture-Stages, and such culture-stages should be defined by characterizing all these activities in each stage. This I shall attempt to do, but in a brief way.

ARTS OF SAVAGERY.

The very early history of mankind is covered by obscurity, through which conjecture peers at undefined forms; but when that portion of human history which rests upon a solid basis of known facts is reached, a succession of arts is discovered, each of which challenges attention and admiration. In the lowest stage of culture which comes within human knowledge, men understand the use of fire, and we may pretty fairly guess how they have learned of its utility. This early man also uses tools and implements of stone, bone, horn, wood, and clay, and by them adds skill to his hands. It is the genius of savage intellect that makes the hand more than a paw, that makes it an organ for the fashioning and the use of tools and implements. At this earlier stage man also knows how to protect himself from winds and storms and the cruel changes of the seasons by providing himself with clothing and shelter. He has also explored and experimented upon the whole realm of the vegetal world, and discovered in a more or less crude way the properties of plants, so that he knows those which are useful for food, the woods that are useful for fire, and the fibres that are useful for woven fabrics. In the same period of culture man has learned that the animals of the land and the waters are useful for food, and has discovered crude methods by which to kill and ensnare them, and has invented many simple instruments for hunting and fishing. Such is the state of the industrial arts in that stage of culture which we call Savagery.

INSTITUTIONS OF SAVAGERY.

Institutions relate to the constitution of bodies politic, to forms of government, and to principles of law; and in describing Savagery we must characterize the constitutions of savage tribes, the forms of savage government, and the principles of savage law.

In Savagery the tribe is always a body of kindred — actual kindred in the main; but, to a limited extent, artificial kinship obtains by

methods of adoption. In this stage of society no method is conceived in the human mind by which a number of men can be held together in one common body except the bond of kinship — the ties of consanguinity and affinity. The savage thinks and says, "My kindred are my friends, and he who is not my kin is my enemy," and upon this theory he acts.

The tribal state, therefore, is organized upon the basis of kinship. It is literally a bond of blood entwined in a bond of conjugal love, and the family organization thoroughly permeates the constitution of the tribal state. In this stage of culture the family, as understood in the civilized world, is unknown. The marriage of one man to the woman of his choice, and of one woman to the man of her choice, is unknown. The right of the father to his own children, is unknown. The husband does not take the wife to his own home; the husband is but the guest of his wife, who remains with her own kindred; and the children of the union belong to her, and over her the husband has no authority. The tribe is always divided into kinship clans. Each clan of this character is a group of people related to one another through the female line, and children belong to the clan of the mother, and submit themselves to the authority of the mother's brother or the mother's uncle. The husband of a woman is selected, not by herself but by her clan, to be the guest of the clan and the father of additional members of the clan. In this form of soeiety, then, a clan is a body of consanguineal kindred in the female line governed by some male member of the clan, usually the elder man. The clans constituting the tribe are bound together by ties of affinity. The methods by which they are thus bound vary from time to time and from tribe to tribe. In the simplest possible case a tribe is composed of two clans, each furnishing the other with husbands and fathers, and in such a case the men of the one clan are the guests of the other, are the husbands of the women and the fathers of the children of the other clan. In such a case the common government is a council of the elder men of both clans, or of chosen or hereditary representatives of both clans, and the council chooses the tribal chief. Such is the simplest possible form of tribal society.

This plan of the tribal state and form of government becomes very highly developed; there may be three, four, twenty, or fifty clans, with many curious ties of affinity, with many curious relations arising from marriage laws. The clan A may furnish

husbands to clan B, and clan B to clan C, and clan C to clan D, and clan D to clan A. It will be impossible to explain all the forms of kinship society in Savagery; but it is sufficient to say that everywhere the tribal state is organized on a kinship basis.

If two tribes form an alliance for offensive and defensive purposes, an artificial kinship is always established. Under such circumstances the tribes entering into the alliance make an agreement with one another what their relationship shall be. If two tribes are thus joined they may call each other brothers; then one will be the elder-brother tribe, the other the younger-brother tribe. Or they may assume the relationship of parent and child to each other, and the men of one tribe call the men of the other "fathers" and the women "mothers," &c. But all clan relations and all tribal relations are really or theoretically kinship relations. In all such bodies politic there is a perpetual conflict between tribal and clan prerogatives, and it is settled by different methods in different tribes and at different times; but, in general, crimes are of two classes in this respect: those over which the tribe has jurisdiction, and those over which the clan has jurisdiction. Sometimes the clan assumes almost supreme jurisdiction; at other times the tribe assumes almost supreme jurisdiction. All petty crimes, as they are considered in savage society, fall under the jurisdiction of the clan. It may be asked how a state of social organization so strange to us ever became established, and yet it may be easily seen that, anterior to the development of modern ideas and methods of government, it was the simplest way of settling difficulties, establishing peace, and consolidating peoples into bodies-politic that could occur to a people.

In the 34th chapter of Genesis there is recorded a proposition to organize a barbaric tribe:

"And Hamor the father of Shechem went out unto Jacob to commune with him.

. . . .

"And Hamor communed with them, saying, The soul of my son Shechem longeth for your daughter: I pray you give her him to wife.

"And make ye marriages with us, and give your daughters unto us, and take our daughters unto you.

"And ye shall dwell with us: and the land shall be before you; dwell and trade ye therein, and get you possessions therein."

In all stages of society, laws regulate conduct in those particulars about which men disagree. Wherever there is universal agreement there is no need for law, and when men disagree about the

actions of life, their actions must be regulated. Now, in early stages of society, the chief things about which men disagree are the relations of the sexes, personal authority, possession of property, and conduct relating to mythical beings. Their laws therefore relate, first, to marriage: and they avoid controversies in this respect by establishing the law that individuals themselves shall have no personal choice in the selection of mates, but that husbands shall be furnished to wives by legal appointment through the officers or rulers of the clan. Second, property rights are established by laws which make certain classes of the property belong to the tribes, other classes to the clans, and a very small part to individuals; and the property held by individuals cannot descend to other persons; and to prevent controversy in relation to personal property, it is established by law that every man's personal property shall be placed with him in his grave. Third, personal authority is established on seniority. The elder always has authority over the younger; and as the people in this stage of society have not yet developed arithmetic and records to such an extent that the ages of individuals are known, a curious linguistic device is established by which relative age is always known. Every man, woman, and child addresses every other man, woman, and child by a kinship term which always indicates relative age: thus, there is no term for brother, but a man in speaking to his brother always uses a term which signifies that he is an elder brother or a younger brother, as the case may be; and thus, through the entire system of kinship terms in tribal society no man can speak to another without addressing him by a term which, in its very nature, claims or yields authority. The younger must always be obedient to the elder. Fourth, laws involving conduct relating to mythic beings are very diverse and multifarious, and cannot be fully characterized. But one of the most essential of those laws concerns behavior in relation to the tutelar deity. Each clan has its tutelar deity and defends its honor, and punishes all impious acts or words against its tutelar god. And in savage society no man may speak disrespectfully of his neighbor's god, but may praise or defame his own, as that god is propitious or angry.

The general principle running through all these laws is this: That in order that men may live together in peace and render each other mutual assistance, controversy must be avoided; and in connection with this first principle, a second arises and runs through savage law, viz, when controversy has begun it must be terminated.

The methods of terminating such controversy are various, and may not here be entered upon. But, in Savagery, the struggle is for peace, and peace is secured by preventing and terminating controversy. Such are the institutions of Savagery.

THE LANGUAGE OF SAVAGERY.

It is not easy to characterize savage languages in such a manner that the subject may be clearly understood by scholars who are not specialists in philology. This is due to the fact that a false standard of linguistic excellence has been set up through the worship of Greek and Latin. These languages, at the time when they were taken as classical models, were very highly specialized, but not highly developed as compared with the languages of modern civilization. But having been taken as the models of excellence and the standards of comparison, erroneous ideas of the course of linguistic growth and of the value or excellence of linguistic methods have obtained currency. In order to understand clearly what savage, barbaric, and civilized languages are, and how they rank, it becomes necessary to eradicate these preconceived ideas, and this cannot be attempted in a short address. It can only be stated in a general way, and without hope that the statement will be fully understood, that savage languages have the parts of speech very imperfectly differentiated, that the grammatic processes and methods are heterogeneous and inconsistent, and that the body of thought which they are competent to express is greatly limited. But there is one linguistic characteristic of Savagery that may be made very clear; it is this: That simple picture-writing is found among savage peoples as a linguistic art, and that in such picture-writing conventional characters are rarely used. Hieroglyphs are never found among savage peoples, and of course alphabets are unknown.

THE PHILOSOPHY OF SAVAGERY.

It seems probable that, in the lowest stage of Savagery, all change, motion, or activity—in fact, all phenomena—are attributed to life supposed to exist in the objects exhibiting the phenomena. Thus, all things, animate and inanimate, are supposed to have life and to exercise will. But gradually, in the development of savagery itself, the animate and the inanimate are distinguished; and finally these ideas are usually woven into the grammatic structure of savage

languages. Still, in this stage of culture, the animate is supposed to act on the inanimate ; so that while life is not attributed to all things, all action is attributed to life — that is, unseen beings are supposed to actuate all nature and to produce all the phenomena of existence. Thus it is that the stars have spirits, the mountains have spirits, and all inanimate and vegetal nature, to a greater or less extent, is the abode of invisible beings. Superimposed on this is found an exalted conception of the wisdom, skill, and powers of the lower animals. In savagery the animals are considered to be the equals of man, and in some cases even his superiors. There is also a general belief that the form in which men and animals appear is but transitory and that these forms may be changed. They believe not so much in *transmigration* as in *transformation*. Then, through the principle of Ancientism, by which the remote past is exalted — in Savagery, Barbarism, and among the ignorant in Civilization alike — the ancients of the star, mountain, and river spirits, the ancients of the birds and beasts, are deified and worshiped. The most important characteristic of savage philosophy, then, is the exaltation of the lower animals, the worshiping of these animal gods, and the belief that they are the chief actors in the creation and history of the universe. Savage philosophy is best characterized by Zoötheism.

PSYCHIC OPERATIONS OF SAVAGERY.

Sensation is the recognition of external action upon the apparatus of the mind. When the olfactory nerves take cognizance of an odor, a sensation is received; but when the mind associates that odor with previous sensations of odor, and recognizes it as of some quality, or as belonging to some known object, it performs an act of inductive reasoning, and pronounces judgment that the odor is sweet, or that it emanates from some pleasant substance. When, therefore, we say that the odor of the rose is perceived, we fairly affirm that in that perception a train of reasoning has been pursued and a judgment formed thereon. By long exercise of the individual in the cultivation of the faculties of inductive reasoning, and by the inheritance of such faculties from ancestors, trains of reasoning of this character gradually come to be so spontaneous and so apparently instantaneous that the course of inductive reasoning is not recognized. The judgment is instantly formed, and the inductive reasoning is unconscious induction upon the data of sensation. Induction is the composition of data.

Again: a sound falls upon the ear; that is, many waves of sound beat upon the nervous receptacle which groups the sensations we call sound; the mind recognizes qualities in the sounds, and at the same time compares them with the memories of other sounds having the same quality, and the ear thus recognizes the voice of a friend. But there may be something more recognized, such as characteristics that express joy or sorrow, and the mind recognizes not only the voice of the friend but the state of his emotions. Now this process is wholly inductive, both in the perception of a known voice and in the perception of a known emotion. It is all a complex course of inductive reasoning, but that reasoning is so instantaneous that the facts which lie at the basis of induction, and the methods of induction, are not discerned, and the unconscious induction is called perception. When the eye is turned to look upon a horse it is affected by certain conditions of light, transformed by reflection from the object upon which the eye is directed. The different rays of light coming to the eye are of a multiplicity of kinds, exhibiting different degrees of light and shade and different degrees in the analysis of light into its constituent colors; thus, chiaroscuro and color strike upon the eye, the vast multiplicity of minute effects upon the eye are composed in the mind by an inductive process, and the inductive process goes beyond the composition of these facts to infer others. Perhaps the left side of the horse is turned to the eye, and the mind infers that there is a right side, that the hither side of the ear has a farther side, that beyond there is a right ear, and a right side throughout, so that the conclusion is reached that the object is characterized by bilateral symmetry. Still more than that, through that profound principle known as the correlation of parts, internal organs are inferred; it is concluded that the animal has a backbone, a heart, and other parts. All these facts, observed and inferred, are combined into a general conclusion by the mind that the object seen is a horse, and we say that a horse is perceived. Now this process of perception differs in no wise from any long and patient course of reasoning except in one characteristic, namely, that the process of reasoning is so instantaneous that the steps and methods do not arise in consciousness. The individual facts upon which the reasoning is based do not appear in severalty, but as forming integral parts of the whole; and the steps by which these observed facts are combined with previous knowledge, and reasoned upon from the basis of the principle of the correlation of parts, are unobserved.

The mind is unconscious of the facts upon which reason is based, and of the process of reasoning, but instantaneously reaches a conclusion. Thus perception is unconscious induction.

This may be further illustrated by facts familiar to all. The untrained arithmetician labors with a simple problem in addition; he steps slowly from one number to another with his eye and his mind's eye as he ascends the column; but an expert accountant glances his eye up and down the column and instantaneously states the sum; and that which was a slow inductive problem in arithmetic for the child and the ordinary adult is performed as an instantaneous process by the expert accountant; and that which was conscious induction in the one was perception in the other. In many ways and on all hands this fact may be illustrated, that perception and induction (or reflection, as it is usually called) are one and the same process in kind, but differ only in degree. *Perception is unconscious induction.*

It was necessary to explain this fundamental principle in psychology in order that we may properly characterize the psychic operations of Savagery. The psychic condition of a people can only be fully explained by setting forth fully the whole system of intellections, embracing perceptions, inductions, and inventions (or imagination, as the process of invention is more usually denominated in psychology), and also characterizing the emotions, the desires, and the purposes, so frequently denominated the "will." But it will be sufficient for our purposes here if we characterize the perceptions and inductions of Savagery; and it may be safely inferred that the imaginings, the emotions, the desires, and the purposes will correspond thereto.

Now the perceptions of Savagery are of a very rudimentary character and are greatly restricted. This can be shown in many ways, but two particulars will suffice for present purposes. The first is this, that the savage is unable to perceive a conventional meaning. He can perceive a horse, and he can even perceive the picture of a horse if its outlines are fairly drawn, but he cannot perceive a horse in a conventional character, like a hieroglyph or a written word.

Again: the savage can perceive numbers but to a very limited extent, but cannot perceive the relations of numbers; for example, he cannot add groups of numbers, as 3 to 5; but wishing to add 3 to 5, he first counts off carefully 5, and then adds the 3, one at a time—that is, he counts his addition. To subtract 3 from 8, he

subtracts one at a time until 3 are taken away, and subsequently counts the remainder to discover the 5. In like manner he cannot multiply, that is, add like groups to each other. Nor can he divide, that is, separate into like groups, but must in each case go through the process, not by considering abstract numbers, but by considering individual things, one at a time. Thus it is that in Savagery a very large field is included in conscious induction which belongs to perception in a higher stage of culture. There are many other mental characteristics of Savagery, but those given are sufficient for present purposes.

Savagery has been thus described with all the minuteness possible on such an occasion, and perhaps with sufficient thoroughness for present purposes. The savage has invented rude arts by which he obtains food, clothing, and shelter. He has invented a rude system of kinship society, with descent in the female line. He has spoken language, gesture-speech, and picture-writing, but is without hieroglyphic, syllabic, or alphabetic writing. He has a philosophy which informs conspicuous and important inanimate objects with spirit life, and which deifies the brute; and a mind whose perceptions are so slightly developed that conventional characters do not convey to him ideas, and his arithmetic is yet "counting." Such, in general, are the characteristics of all savage peoples that have been carefully studied by anthropologists. Now the question arises, how was this Savagery transformed into Barbarism; and what is that Barbarism?

In the lower stages of culture all progress rests upon the arts of life. To discover any great change in the condition of mankind we must look for the art-invention which was the efficient agency in producing the change.

If the early course of human progress be surveyed for the purpose of discovering the most important art-epochs, it will be safe to regard those of the greatest importance the effects of which are most clearly exhibited in the concomitant activities — that is, institutions, languages, opinions, and psychic operations. If an invention has but slight influence on these correlative activities, its importance may be questioned. But if an art-invention is discovered to have worked radical changes in all other activital departments, such art must be of the highest importance.

There are two arts intimately associated the invention of which causes a radical change in all of the departments of humanity,

viz, agriculture and the domestication of animals. Agriculture began in Savagery. Many savage tribes cultivate little patches of ground and thereby provide themselves with a part of their subsistence. This petty agriculture does not of itself result in any radical change; but when the art has developed to such an extent that the people obtain their chief subsistence therefrom, and especially when it is connected with the domestication of animals, so that these are reared for food and used as beasts of burden, the change for which we seek is wrought. It seems that extensive agriculture was first practiced in arid lands by means of artificial irrigation. In more humid lands the supply of food is more abundant, and the incentive to agriculture is less. On the other hand, agriculture is more difficult in humid lands than in arid lands. The savage is provided with rude tools, and with them he can more easily train water upon desert soils than he can repress the growth of valueless plants as they compete for life with those which furnish food. The desert soil has no sod to be destroyed, no chapparal to be eradicated, no trees to be cut down, with their great stumps to be extracted from the earth. The soil is ready for the seed. Throw upon that soil a handful of seed and then sprinkle it with a few calabashes of water once or twice through the season, and the crop is raised; or train upon a larger garden patch the water of a stream and let it flood the surface once or twice a year, and a harvest may be reaped.

Petty agriculture, such as I have described as belonging properly to Savagery, has been widely practiced in the four quarters of the globe among savage peoples, quite as much in humid as in arid regions; but the art seems not to have indigenously extended beyond that stage in any but arid regions. The earliest real agriculture known to man was in the Valley of the Nile, an almost rainless land; but the floods of the Nile were used to fertilize the soil. Again, in the land of Babylon, along the Tigris and the Euphrates, extensive agriculture grew up, but it was dependent upon artificial irrigation. Still farther to the southeast, in the Punjab, another system of indigenous agriculture was developed by utilizing the waters of the five great rivers. Still farther to the east an indigenous agriculture was developed on an extensive scale, all dependent upon artificial irrigation, as the Chinese use the waters of the Hoang-ho and the Yang-tse-Kiang. In South America the first system of agriculture was developed in Peru, all dependent upon artificial

irrigation; and finally, to the north of the Isthmus of Panama, in Central America and Mexico, agricultural arts were highly developed, and here also they were dependent upon artificial irrigation. From these six examples of high agricultural art, all the agriculture of the world has been developed; from these centers it has spread. The petty agriculture of humid lands never went beyond the utilization of little patches of ground in the forest glades until it was borrowed in a higher state from arid lands. Everywhere with the development of agriculture in the arid lands, the art of domesticating animals was associated, and everywhere such animals were raised for food, and to a large extent they were used as beasts of burden.

Now, it is to be noted that the animal industry eventually developed beyond the vegetal industry, and spread more widely, and many tribal peoples became herdsmen and nomads before they came to be agriculturists. The art of domesticating animals was more easily borrowed, especially in humid regions, than was the art of agriculture.

These industries enabled mankind to obtain a far more generous subsistence and more thorough protection from unfriendly nature. They thus caused a great increase in population. They also constituted the first great agency for the accumulation of wealth, by creating it in giving value to land, by creating it in flocks and herds, and by storing it through the discovery of methods by which the wants of the future could be met. By planting fields the wants of to-morrow and all the days of the year to come are served; and when the young of animals are reared, provision for future years is made, and thereby men learn to accumulate.

This change in the arts of life, and the increase of population resulting therefrom, entirely changed the constitution of society. In savage society, when mother-right prevails, a tribe is a group of classes or clans living together in a village that is easily moved from time to time. If a colony departs from a tribe, a segment of two or more clans goes away and starts a new village, and the clans again live as a village community upon the same plan as the parent tribe.

Now, let us suppose that a tribe separates by clans, so that each goes off by itself; a curious condition arises therefrom: first, it results in the divorce of all marriages, because husband and wife are always of different clans; and for the same reason the father is

separated from his children. In such communities there is often a partial separation by clans of this nature: in savage society the men of a clan often go off together on a hunting or fishing excursion. Sometimes these excursions or travels are prolonged for weeks or months. In such cases the men often take their wives with them, and under these circumstances the women are separated from their clan and kindred and are not under the control of clan authority, but fall under the temporary control of their husbands and fathers. Now, if we could suppose a state of affairs where this separation of women and children from kindred and clan authority becomes permanent, it is manifest that the power of clan authority would wane, and the authority of the husband and father would grow. Such a condition of affairs results from extensive agriculture by irrigation and the care of extensive flocks. It must be remembered that in this stage of society property is communal; that is, property in the main belongs to the clan. A flock of sheep, a herd of cattle, a band of horses; is the property of the men of a clan. When such property becomes so large that it will occupy for its sustentation a large valley, the men to whom it belongs will necessarily be occupied all the time with its care and protection, and they must have their wives and children with them in order that domestic life may be possible. Under such circumstances it results that women and children are gradually taken from the control of those persons who had previously been supposed to be their natural protectors, their clan kindred, and fall under the control of their husbands and fathers, who are members of other clans. The same result has always been produced by the segregation of the male members of the clan from the tribe through agriculture by irrigation. The circumstances are these: In this early agriculture the agricultural implements are very crude, and great hydraulic works cannot be undertaken. It is thus necessary to attempt the control of only the small streams, and the men of each clan will therefore select some small stream and occupy the little valley through which it runs and upon which its waters are trained; the men of one clan, with their wives and children, occupy a distinct valley, the male members of another clan another valley, and the tribe is thus segregated into groups, the male members of each group belonging to the same clan and having with them their wives and children. The women and children being thus severed from clan authority, fall under the authority of their husbands, and mother-

right, or descent in the female line, is changed into father-right, or descent in the male line; and thus is established the patriarchy, a form of society with which we are all familiar, as it is very clearly set forth in the post-Noachian history of the Bible.

Under this form of society kinship bonds are still preserved, but they are of a different nature. First, descent is transferred to the male line—that is, children belong to the clan of the father, and are controlled by him instead of by the mother's brother, or the mother's uncle; second, the husband is no longer the guest of the wife and her clan. At first the wife is the guest of the husband and his clan, but gradually this relationship of guest and host is changed to the relationship of master and owner, and the husband becomes the owner of his wife, and finally the owner of his children. They are considered to be his property; they are responsible to no one but himself—that is, the tribe does not hold the wife and children responsible for their acts, but holds the husband responsible for them. (It is impossible in an evening's address to characterize fully the causes and the consequences of the change from enatic to agnatic descent, but the statement here given is perhaps sufficient for present purposes.)

Another great change is effected, the increase of wealth which has been described multiplies the relations between men arising from the possession of property. And these are relations about which men disagree, and therefore they must be regulated by law. The state, therefore, comes to be organized in part on a property basis; hitherto it has been organized wholly upon a kinship basis. The plan of the structure of the state is thus changed. The laws, too, are enlarged to regulate the relations that arise out of ownership.

And yet another change is effected. Some clans prosper and increase in wealth; other clans fall into poverty. With this increase of wealth and desire for wealth, labor becomes of value, because it can be converted into wealth, and the poor are employed by the rich, and the relations of the employer and the employed are established. Out of this grows the relationship of master and slave, and ranks or grades are established in society. With this grows ambition for wealth and power, and tribe wars on tribe to drive away its herds and to take possession of its accumulated property, and captured peoples become slaves, and the chiefs of conquering tribes extend their authority over conquered tribes, and gradually great

chiefs become great leaders in war and gather their retainers about them, giving to them protection from without, and claiming in compensation for the same fealty, tribute, and service under arms.

Such is a brief outline of the characteristics of tribal society in barbarism, brought about through the cultivation of the soil and the domestication of animals.

THE CHANGE IN LANGUAGE.

The great changes wrought in arts and institutions which have been described doubtless had their influence on languages, as the new ideas required new means of expression. While in the present state of knowledge it is perhaps not possible to set forth clearly the resultant sematic and structural effects upon any language, in linguistic arts important effects are discovered.

In the lower status of culture, here denominated savagery, picture-writing was highly developed; but in the transition to barbarism, picture-writing was transformed into ideographic writing. In the earlier stage a slight tendency to conventionalism is discovered; but in ideographic writing the original pictorial signs are conventionalized to such a degree that it becomes an important linguistic art, by which ideas may be recorded and transmitted from person to person and from generation to generation. It must be understood that the evolution of picture-writing had all along been in the direction of ideographic writing, but a great impulse is given to this tendency by the enlargement of human activities in the arts of life and the institutions of society. This is discovered in many directions, the chief of which may be here enumerated.

1st. The increase of property demands increase in the methods of identifying property and of substantiating ownership.

2d. The separation of clans and the distribution of cognate peoples over large areas of territory demand means of intercommunication other than that of direct oral conversation; and

3d. Nomadism, which is the direct result of the domestication of animals, makes men travelers, and so enlarges their horizon of observation that some method for the record of events becomes necessary. Under such stimulus, picture-writing speedily develops into ideographic writing.

THE CHANGE IN PHILOSOPHY.

In savagery, mythology develops into a high form of zoötheism.

The beasts are not gods, but many of the gods are beasts — the ancients of beasts, the prototypes or progenitors of the living animals. The rudiments of physitheism also exist in the worship of the heavenly bodies, the winds, and other natural phenomena personified.

When animals become beasts of burden they are degraded; they are discovered to be inferior beings, and the mysteries of animal life are largely dispelled; and by the development of agriculture man becomes more dependent upon the sun, the seasons, and the weather. The heavenly bodies and meteorologic powers and phenomena grow in importance and become more and more the subject of interest and speculation, until the personifications of natural objects in the heavens and natural phenomena in the seasons and the weather are deified, and the tribal worship presided over by medicine-men and prophets becomes a religion based upon physitheism. The occult lore of the people is composed of stories of the sun, moon, and stars; of thunder, lightning, and the rainbow; of the storms, clouds, and winds, and of dawn and gloaming.

There is another important development in the religion of barbaric peoples. With the establishment of the patriarchy the patriarch comes gradually to be the great power, and worship of a clan tutelar deity is changed into ancestral worship — the worship of the ancient chiefs or patriarchs; ancestor gods and ancestral worship replace tutelar gods and tutelar worship. Barbarism, then, is properly characterized by domestic ancestor worship and tribal nature worship.

THE PSYCHIC CHANGE.

The enlarged plane of human activities already outlined causes an important development in psychic activities. First, perception is enlarged. This is seen in the fact that people at this stage are able to read hieroglyphs; they can perceive meanings in conventional characters. Again, stimulated by the accumulation of wealth, arithmetic is developed beyond the counting stage, and man can add a number of units to a number of units, and can subtract numbers from numbers, and divide numbers by numbers. In savagery, men learn to count; in barbarism, men learn arithmetic, and can at once perceive the simpler relations of numbers. The entire field of human thought is greatly enlarged, and with this enlargement there may be observed a nicer discrimi-

nation of phenomena, and a grouping of phenomena on a new system of analogies.

From the foregoing brief characterization it will be seen that barbaric culture implies a somewhat high state of agriculture and the domestication of animals, one or both. It implies that patriarchal institutions have been organized, that descent is in the male line, that ranks in society have been established, and that new laws regulating property have been enacted. It implies that the people use hieroglyphs. It implies that domestic worship is ancestral worship, that tribal worship is based on physitheism, and that the phenomena of the universe are attributed to nature gods. And finally, it implies that men can perceive meanings in conventional signs, and that arithmetic has been invented.

The statement I have hitherto made rests on the postulate that the progress of culture has been essentially along the same line in all times and places. The facts accumulated by the researches of modern anthropologists fairly establish this. It is true there has been much variation in the order and steps of culture, but this variation has been confined within certain limits. The chief variation lies in the fact that all races have not made progress to the same extent. Some tribes are yet savages; other tribes are yet barbarians; and some peoples have attained civilization.

The common origin of mankind, otherwise denominated the unity of the human race, is a conclusion to which the modern science of anthropology gives abundant evidence. Although the diversity among men is so great that no two are alike, yet this diversity is restricted to narrow limits. The units of the mass of humanity are discovered to be homogeneous in essential endowments to such an extent as almost to startle the student who studies man in all lands and at all times.

Primitive men had a common origin, but early in their history they differentiated into biotic varieties, characterized by the conformation of the skull, the proportions of the skeleton, the color of the skin, the structure of the hair, the attitude of the eyes, and other biotic peculiarities. Had this tendency to differentiate continued through the entire course of human culture, species would have been established, but early in the period of human history the tendency to differentiation was checked and a return to homogeneity initiated. Thenceforth the progress of mankind has been by methods radically differing from the methods of biotic evolution as exhibited among plants and animals.

This return to biotic homogeneity is due to the development of human activities, which make men depend one upon another in such a manner that the welfare of one involves the welfare of others, so that no man may claim the right to live for himself, but every man lives and labors for the good of his kind. The fundamental principle of animality is supreme selfishness; the fundamental principle of humanity is mutual assistance.

As man is an animal, in systematic biology he may be grouped with other animals as determined by morphologic characteristics. He has a head, body, and limbs; he has organs which perform the functions of biotic life; and when we consider man in this aspect the study is a part of biology. Man is more than animal by reason of his activities; man is man by reason of his humanities; and when we study him in this aspect the subject is anthropology.

Henceforward human evolution differs radically from biotic evolution as exhibited among plants and animals. Animal evolution has been accomplished by the survival of the fittest in the struggle for existence. By this method animals were adapted to environment, and in the course of this adaptation they differentiated into a multitude of species, genera, families, and orders. Animal evolution, then, has these three characteristics: first, the agency of evolution was the survival of the fittest in the struggle for existence, brought about by over-population; second, the fittest that survived were adapted to environment; and third, progress resulted in immeasurable variety, carried to the utmost degree. In all of these characteristics human evolution differs radically from animal evolution.

First, man has not progressed by the survival of the fittest in the struggle for existence. Man does not, to any important extent, compete with plants and the lower animals, but he utilizes them, developing such as he will in directions that best subserve his interests, and gradually destroying others from the face of the earth. Nor does man progress by reason of competition within the species. When the highwayman and the traveler meet, the robber is not always killed; and when races battle with each other, the strongest and the best go to die. In the course of human history, in a few localities and at a few times population has been overcrowded, but in the grand aggregate the world has never been fully peopled, and man has not crowded upon man for existence.

While man has not progressed by the struggle for existence, he

has progressed by his endeavor to secure happiness; and in this endeavor he has invented arts, institutions, languages, opinions, and methods of reasoning — that is, he has progressed by the development of five great classes of human activities. In the establishment of these activities, he transfers the struggle for existence from himself to his activities, from the subject, man, to the objects which he creates. Arts compete with one another, and progress in art is by the survival of the fittest in the struggle for existence. In like manner, institutions compete with institutions, languages with languages, opinions with opinions, and reasoning with reasoning; and in each case we have the survival of the fittest in the struggle for existence. Man by his invention has transferred the brutal struggle for existence from himself to the works of his hand.

Again, man has not been adapted to environment. There is no aquatic variety of man, no aërial variety, no tropical variety, no boreal variety, no herbivorous or carnivorous variety. On the other hand, man has adapted the environment to himself — that is, he has created for himself an artificial environment by means of his arts. He can sail upon the sea and live on the products of the sea, and he utilizes the denizens of the air and the plants and animals of the land. He protects himself from great heat and great cold and in a multitude of ways creates an artificial environment. And this he has done to such an extent that were he suddenly to lose his control over the environment gained through his arts, he would speedily perish from the earth.

Again, among the lower plants and animals the course of adaptation to environment led progressively to the differentiation of species, until a multiplicity of biotic forms covered the earth. The method of human evolution by endeavor to secure happiness through human activities, which resulted in the creation of an artificial environment, checked the tendency of the animal man to differentiate into distinct species, and the interdependence and solidarity that were established through these activities tend more and more to restore the units of mankind to pristine homogeneity. This is accomplished biotically by a constant interfusion of streams of blood, as men are commingled and intermarried throughout the world. When races of higher culture spread civilization over inferior races, the admixture goes on at an increased rate. The blood of the American Indian is to a large extent mixed with the blood of the European, and especially is this true where Latin peoples

have established themselves. The African tribes transplanted in America are rapidly bleached by the synthetic chemistry of social life. When three generations more have passed, it may not be possible to find a drop of pure Indian or negro blood on this continent. Civilization overwhelms Savagery, not so much by spilling blood as by mixing blood, but whether spilled or mixed, a greater homogeneity is secured.

This return to homogeneity is accomplished by the spread of arts from their centers of invention to the circumference of their utilities. As an art is expressed in material form, it is an object-lesson readily learned. It may be that the tongue of the inventor can be understood by no people but those of his own tribe, but his handiwork needs no interpreter; and so arts are spread from land to land, and those who engage in common arts are trained by homogeneous methods.

This return to homogeneity is accomplished by the spread of institutions from tribe to tribe and from nation to nation, for waves of conquest have rolled again and again over all lands, and when civilization is reached institutions and institutional devices are transplanted, for civilized men are ever engaged in comparison and ever striving to select the best.

This tendency to homogeneity is accomplished by linguistic communication, for with the progress of culture men come to speak more and more in synonyms, and dominant languages are spread far beyond the boundaries of their native lands; and thus there is a tendency to homogeneity of tongue.

This return to homogeneity is accomplished by the spread of opinions, for the opinions that influence the highest of the race come ultimately to influence all; and scientific philosophy is rapidly spreading to the uttermost parts of the earth.

And finally this homogeneity is accomplished by the spread of the same methods of reasoning, the same psychic operations. Homologic methods of reasoning, by which the truth is reached, are steadily replacing analogic methods, by which myths only are invented; and as gradually the same facts are brought to the light of all mankind, and the same processes of reasoning are pursued, men are gradually becoming occupied in the same mental activities.

Thus it is that if we consider man biologically, or man in relation to his activities, expressed in arts, institutions, languages, opinions, and reasoning, we discover that the tendency to the differentiation

of species has been checked, and that a tendency to homogeneity has been established.

To recapitulate: Human evolution has none of the characteristics of animal evolution. It is not "by the survival of the fittest" in the struggle for existence, but it is by human endeavor to secure happiness; and in this endeavor man has transferred the struggle for existence from himself to the works of his hand and mind. It is not by adaptation to environment, but by the creation of an artificial environment. It does not secure differentiation into varieties and species, but establishes a tendency toward homogeneity.

By the division of labor men have become interdependent, so that every man works for some other man. To the extent that culture has progressed beyond the plane occupied by the brute, man has ceased to worked directly for himself and come to work directly for others and indirectly for himself. He struggles directly to benefit others, that he may indirectly but ultimately benefit himself. This principle of political economy is so thoroughly established that it needs no explication here; but it must be fully appreciated before we can thoroughly understand the vast extent to which interdependence has been established. For the glasses which I wear, mines were worked in California, and railroads constructed across the continent to transport the product of those mines to the manufactories in the East. For the bits of steel on the bow, mines were worked in Michigan, smelting works were erected in Chicago, manufactories built in New Jersey, and railroads constructed to transport the material from one point to the other. Merchant-houses and banking-houses were rendered necessary. Many men were employed in producing and bringing that little instrument to me. As I sit in my library to read a book, I open the pages with a paper-cutter, the ivory of which was obtained through the employment of a tribe of African elephant-hunters. The paper on which my book is printed was made of the rags saved by the beggars of Italy. A watchman stands on guard in Hoosac Tunnel that I may some time ride through it in safety. If all the men who have worked for me, directly and indirectly, for the past ten years, and who are now scattered through the four quarters of the earth, were marshaled on the plain outside of the city, organized and equipped for war, I could march to the proudest capital of the world and the armies of Europe could not withstand me. I am the master of all the world. But during all my life I have worked for other men, and thus I am

every man's servant; so are we all — servants to many masters and masters of many servants. It is thus that men are gradually becoming organized into one vast body-politic, every one striving to serve his fellow man and all working for the common welfare. Thus the enmity of man to man is appeased, and men live and labor for one another; individualism is transmuted into socialism, egoism into altruism, and man is lifted above the brute to an immeasurable height. Man inherited the body, instincts, and passions of the brute; the nature thus inherited has survived in his constitution and is exhibited along all the course of his history. Injustice, fraud, and cruelty stain the pathway of culture from the earliest to the latest days. But man has not risen in culture by reason of his brutal nature. His method of evolution has not been the same as that of the lower animals; the evolution of man has been through the evolution of the humanities, the evolution of those things which distinguish him from the brute. The doctrines of evolution which biologists have clearly shown to apply to animals *do not apply to man.* Man has evolved because he has been emancipated from the cruel laws of brutality.

The evolution of man is the evolution of the humanities, by which he has become the master of the powers of the universe, by which he has made life beautiful with æsthetic art, by which he has established justice, by which he has invented means of communication, so that mind speaks to mind even across the seas; by which his philosophy is the truth of the universe. Man is man because of the humanities.

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INDEX

TO THE

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BY

F. W. TRAPHAGEN,

FORMS PART OF

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SMITHSONIAN MISCELLANEOUS COLLECTIONS.

SMITHSONIAN MISCELLANEOUS COLLECTIONS.

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TO THE

LITERATURE OF COLUMBIUM:

1801–1887.

BY

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PREFACE.

The following "Index to the Literature of Columbium" has been published upon the recommendation of the Committee on Indexing Chemical Literature, of the American Association for the Advancement of Science. In its original draft the word "niobium" was used in place of "columbium;" but upon suggestion from members of the Committee the author consented to a change. It is well known that the name columbium, originally given by Hatchett, has clear priority; while "niobium," and its supposed twin "pelopium," grew out of errors made by Rose. Although in European treatises the name "niobium" has generally been adopted, no good reason for the substitution has ever been offered, and all the accepted rules of nomenclature demand the retention of columbium. This note has been written at the request of Professor Traphagen.

F. W. Clarke.

INDEX TO THE LITERATURE OF COLUMBIUM, 1801-1887.

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OF

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FOR THE YEAR

1887

BY

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BIBLIOGRAPHY OF ASTRONOMY: 1887.

BY WILLIAM C. WINLOCK.

The following subject-index of astronomy for 1887 was originally compiled as an appendix to a general review of the progress of astronomy during that year, and though not exhaustive, it may, perhaps, be found a useful reference list. Important contributions to astronomy published during 1887 in scientific journals and transactions of societies, as well as all more elaborate publications that have come to the compiler's notice, have been included—a few titles being taken from reviews, or book-catalogues. Observations of asteroids and comets, except those of the comets of 1887, have generally been omitted.

The prices quoted are usually from Friedländer's *Naturæ Novitates*, in German "marks" (1 Mark = 100 Pfennige = 1 franc 25 centimes = 25 cents, nearly.)

The abbreviated titles will probably be readily understood by those familiar with scientific periodicals without special explanation, beyond the following list of less obvious contractions.

Abstr. = Abstract.	Lfg. = Lieferung.	pt. = part.
Am. = American.	M. = marks.	r. = reale.
Bd. = Band.	n. d. = no date.	Rev. = review.
d. = die, der, del, etc.	n. p. = no page.	s. = series.
ed. = edition.	n. F. = neue Folge.	sc. = science, scientific.
Hft. = Heft.	n. s. = new series.	sh. = shilling,
hrsg. = herausgegeben.	not. = notices.	sup. = supplement.
il. = illustrated.	obsns. = observations.	v., vol. = volume.
j., jour. = journal.	Obsry. = Observatory.	
k. k. = kaiserlich königlich.	p. = page.	
	pl. = plates.	

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Göttingen, Feb. 24; *Ibid.*, 116: 221.

Greenwich, Feb. 28; Month. not., 47: 275.

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Comet 1887 III (Observations of position of). 1887—*Continued.*

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Orwell Park, Feb. 28, Mar. 13, 14, 16, 18, 19, 23, 27, Apr. 10; Month. not., 48: 61.

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Comet 1887 IV = Comet 1887 *e.*

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Bothkamp, June 15, 16, 24; Astron. Nachr., 117: 133. June 28, July 25; *Ibid.*, 215.

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Cape of Good Hope, May 19, 21, 23, 24, 27, June 8, 9, 11, 13, 14, 16, 17; Astron. Nachr., 117: 339.

Dresden, May 19; *Ibid.*, 43. May 22; *Ibid.*, 59. June 13; *Ibid.*, 133. July 16; *Ibid.*, 215.

Geneva, May 19; *Ibid.*, 43.

Gohlis, June 13, 14, 16–19, 22, 25; Astron. Nachr., 117: 214.

Göttingen, June 15–17, 22; *Ibid.*, 133.

Greenwich, June 12, 18, 19; *Ibid.*, 215.

Hamburg, June 16, 17, 19; *Ibid.*, 119, 133.

Harrow, June 12, 15, 17, 19, 22. Month. not., 47: 550.

Kiel, May 14; Astron. Nachr., 117: 31. May 16, 21; *Ibid.*, 43.

Kremsmünster, May 15, 26, June 13, 15, 18, 19, 23, 24, 25, 27, July 12; *Ibid.*, 118: 107.

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Nice, May 14; Astron. Nachr., 117: 43. May 14, 17, 18, 20–23; Bull. astron., 4: 225. May 27, July 7, 11, 18, 23; *Ibid.*, 380.

Nicolaief, May 14, 15, 17, 18, 21; Astron. Nachr., 117: 55.

Orwell Park, June 9, 10, 12, 13, 15, 17, 18, 20, 22, July 11, 12, 13, 14, 18–21, 24, 27, 28; Month. not., 48: 61.

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Scarborough, May 20, 21, 29; Month. not., 47: 498.

Comet 1887 IV (Observations of position of). 1887—*Continued.*

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LECTURE X.

A CLINICAL STUDY OF THE SKULL.

BY

HARRISON ALLEN, M. D.

DELIVERED MAY 29, 1889.

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S. P. LANGLEY,
Secretary Smithsonian Institution.

SMITHSONIAN INSTITUTION,
WASHINGTON, *March*, 1890.

INTRODUCTION.

It would be difficult to mention a single phase of the manifold expressions that belong to disease in which the study of the face and brain cannot enter; but to the laryngologist and to the neurologist many portions of the head must be of especial interest. The laryngologist can examine the mouth, nose, and the pharynx; the neurologist the surfaces of the crown, which afford him guides to the peculiarities of the interior of the skull. Since much which pertains to both of these branches of medicine is of comparatively recent growth, a study of the osteology of the head cannot fail at the present time to be useful.

An accurate impression of the superficial characters of the skull can be received from examination of the living subject. Reliance must be made upon these characters in fixing the relations of the soft parts; hence the ranges of variation in these characters should be known, and as full a knowledge as possible be obtained by study of the cranium. In the paper herewith submitted an attempt is made to treat of these relations and ranges of variation. The author's interest, at first, was confined to the diseased conditions of the facial region and of the vault of the pharynx, but the interest gradually widened and soon embraced the normal anatomy of the entire head.

The method recommended by him is as follows: First, to study carefully a character as detected in the living subject, then to examine all crania available and endeavor to ascertain in what guise the same structure may re-appear, and subsequently to formulate such descriptions as can be deduced from the data; second, to bring together the material gleaned while examining crania, which appears to be of interest, to illustrate the nutritive processes at work

in forming or maintaining the different parts of the skull each to the others.

Many of the characters obtained in this manner are of necessity minute; so, indeed, are the distinctions upon which the anthropologist relies. The obliquity of the palpebral fissure, the color of the iris, the distribution of the hair, or the characters furnished by the individual hair may be mentioned in this connection.

The significance which can be attached to the study of variation either in the study of race grade, or in the large question of evolution of organic forms, is of course conceded. The last named cannot be determined until extended series of data have been collected. If, according to Engel (Untersuchungen über Schädelformen, p. 121), uncultivated primitive races exhibit few variations in the composition of the skull as compared with the more advanced, we may be prepared to accept Retzius' dictum that individual differences become greater in proportion to the higher intellectual development of a nation. Preservation of such facts as the disposition of the minute plates and processes of the interior of the nasal chambers and of the base of the skull, the description of suture changes, of the depressions made by small veins, and of the minor deviations in size of paired structures may have an outcome as interesting as those derived by discovery of structures which exist on a larger scale.

The lack of fixation of characters should not of necessity diminish their value. Beginnings of characters are always facile and indeterminate. This is nature's process.

The effects of diseased action, although their manifestations be apparently insignificant, are also worthy of study from the standpoint of the biologist as well as that of the pathologist. When produced from other than traumatic causes, these effects have distinct value. They may indicate modifications of the processes of life, which are of the same kind as those furnished by the anatomy of normal parts.

The extent to which variation in normal anatomy is an exciting cause of disease is difficult to determine. All things remaining the same, it may be said that the most variable parts are seen in the regions which are in extremes of specialization, as in the nasal chambers of man, and that these chambers are degenerate as compared to many mammals where the range of variation is small. The pre-disposition of nasal disease in man cannot be rationally dissociated from the proneness of the parts which enter into the composition of the nose to vary. If this statement can be depended upon, the publication of all details of structure in the nasal chamber becomes essential.

This essay is a contribution to the morphological study of diseased action. The writer trusts that increasing interest may be awakened in the proposition that medicine for the most part is a science based on biology. The study of biology should not be the preparatory work of the trio only, but should be the subject of unceasing assiduity in every phase of medical research. The study of anatomical variation in the human frame is a phase of biology, and it is held in this connection to be a subject as important as any other which may claim the attention of the student of etiology of disease.

The materials upon which the essay is based were found in the collections of the Academy of Natural Sciences of Philadelphia and of the College of Physicians of Philadelphia. The letter C, in absence of other signs, will indicate that a specimen so named is to be found in the College of Physicians (Hyrtl Collection); the remainder are in the Academy. (The last named are often indicated by the letters A. N. S.)

The determination of percentage of frequency of any anatomical peculiarities has not been attempted. The writer has been content to give the numbers and nationalities of the specimens referred to.

The entire number of specimens of crania in the Academy is 1,750, and in the College of Physicians 156.

The following exhibit the arrangement of the subject-matter of the essay:

The malar bone.

The lower jaw.

The norma-basilaris.

The basi-cranial angle.

The posterula.

The nasal chambers.

The vertex—its sutures, eminences, depressions, general shape, etc.

Remarks on the sutures other than those of the vertex.

The foramina.

The grooves caused by blood vessels.

The cranial ridges, processes, etc.

LECTURE X.

Delivered May 29, 1889.

A CLINICAL STUDY OF THE SKULL.

BY HARRISON ALLEN, M. D.

THE MALAR BONE AND THE ZYGOMATIC ARCH.

The malar bone is one of the most conspicuous of the superficial characters of the face. At the outer and lower margin of the orbit the external surface, as well as the posterior and zygomatic borders, can be separately distinguished. The bone as it enters into the composition of the lower border of the orbit is discussed elsewhere.

The consideration of the external surface will be undertaken at this place. The chief points to consider are, first, its inequality, and, second, its obliquity.

1st. The inequality of the surface is simple in character. It is comprised in the lower part, this being at times raised so as to form a rounded projected eminence. It is less pronounced in the negro than in the Caucasian, and is entirely absent in the child. When the cranium is examined the inequality is seen to answer to distinct differences in texture of the superficies—differences varying in individuals, but never entirely absent.

Throughout the series of examinations made with this object in view—*viz.*, of determining the variations in the upper and lower part of the bone—it was found that from simple differences in superficial texture it was an easy transition to the detection of differences in the deeper texture of the two parts; that thence to attempts at the for-

mation of suture-lines which extended along the boundaries of the parts to a groove along the entire length of the bone on the posterior surface; and that finally the observer was led to the study of specimens which showed the separation of the bone by a perfect, open suture. The details of the description adopted will appear in the reverse order of the appearances as given above.

The existence of a suture in the malar bone has been occasionally noted in the skulls of various races. J. B. Davis,[1] in 1872, contributed a short note on the subject, of which the following is an epitome: The author refers to the presence of the suture in nine skulls of Asiatics and negroes. The suture is often met with in the skulls of the Dyaks of Borneo. It is rarely met with in modern skulls, but more frequently in the skulls of ancient Europe. Prof. Wenzel Gruber[2] subsequently enumerated twenty-one examples and gave the literature of the subject.

The collection of the Academy of Natural Sciences contains ten undescribed examples of a distinct bone occupying the lower part of the malar, or lying at the malo-squamosal suture. Seven skulls were found which exhibited these peculiarities. No. 1255, Ostrogoth, showed a separate malar bone on one side. No. 1442, Peruvian, exhibited a separate malar ossicle at the malo-zygomatic suture, on both right and left side of the skull. No. 1690, Peruvian, a distinct transverse suture crossed the malar bone of the right side. No. 83 (Atacames), Peruvian, a transverse suture was seen on the left side; a less distinct one on the right side. No. 1305, Peruvian, a small ossicle was seen at the malo-zygomatic suture on both sides of the skull. No. 753, Seminole, and 540, Pawnee, a similar ossicle was noted on the right side. No. 460, Malay, exhibited an imperfect division of one of the malar bones into two parts. This specimen is not enumerated with the foregoing. In the United

[1] Journal of Anthropological Institute, vol. 1, p. clvi.
[2] Das Zweigetheilte Jochbeine. Vienna, 1873.

States Army Museum (No. 309, Chickasaw) the malar bone was double on both sides.[1]

Four examples were seen in which a suture began at the malo-zygomatic suture, and, advancing forward, was lost a short distance from the posterior end of the bone. These were No. 1424, Peru, on both right and left sides; No. 1506, *ibid*, on the right side, and No. 1434, *ibid*, on both right and left sides.

In a single example, No. 1369, *ibid*, a skull of an old female, a foramen perforated the right malar bone a little in advance of the malo-zygomatic suture and appeared to be in the line of the transverse suture, though no trace of the line could be discerned either in front or back of the opening.

Thus twelve examples can be cited, all of American origin, which exhibit transverse malar sutures more or less complete as seen from both the inner and outer surface of the bone, and two in which the bone was double.

But when the inner side of the malar bone is examined a much larger number of skulls exhibit the transverse suture. No attempt was made to ascertain the entire number of examples last named. While a search was instituted, with another object in view, the inner surface of the malar bone was at the same time examined, and out of the entire number examined, it was detected in fifty-one crania. In eight of these it existed on both right and left sides.

The distribution of selected examples among the races was as follows: North American Indians, one each of Chinook,[2] Lenape,[3] Menominee,[4] Naas,[5] Shawnee,[6] California Indians,[7] one unnamed;[8] Peruvians, ten;[9] Anglo-American, one;[10] Mexicans, three;[11] negro, one;[12] Egyptians, five;[13] Circassians,[14] Roman,[15] Arabian,[16] Nubian,[17] one each.

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[1] I have since seen an additional example in a skull (No. 53) in the anatomical collection of the University of Pennsylvania.

[2] 462. [3] 40. [4] 44. [5] 213. [6] 1210. [7] 1683. [8] 204.
[9] Nos. 567, 1704, 1298, 1303, 1025, 941, 447, 11, 891, 1426. [10] No. 17.
[11] 1005, 1004, 1515. [12] 548. [13] 997, 778, 799, 814, 768.
[14] No. 762. [15] No. 248. [16] No. 776. [17] No. 829.

In all of these specimens a delicate line could be traced forward from the posterior to the antero-inferior portion of the bone. It was of precisely the same character in the examples in which the outer suture was distinct.

The inner surface of the malar bone will not infrequently exhibit a concavity below the line of the suture. This concavity is distinct even in specimens in which the suture is nowhere evident. Such a peculiarity is well seen in a Peruvian skull (No. 1407). A similar disposition was noticed in the skull of the Hyrtl collection (No. 92).

The small ossicles named above as occurring at the malo-zygomatic suture tend to break up the uniform smooth surface of the inner aspect of the malar bone—a disposition which may exist even in the absence of a separate ossicle. The squamosal element may be long and irregular and extend forward, along the line occasionally taken by the suture, nearly to the maxilla.[1] This was seen in a Peruvian (No. 1506), which showed an incomplete suture externally, and in the skulls of two Creek Indians (Nos. 652 and 75).

The following measurements were made to indicate the proportionate size of the upper and lower parts of the malar bone. The numbers have been arranged in the order of the size of the upper part of the bone, this being the smallest in the first example named and the largest in the last:

Upper . .	2 c. 8 m.	Hindoo.
Lower . .	0 “ 9 “	
	3 “ 0 “	Tahitian.
	0 “ 8 “	
	3 “ 2 “	Esquimaux, 1561.
	0 “ 8 “	
	3 “ 2 “	Marquesas, 1531.
	1 “ 5 “	
	3 “ 3 “	Esquimaux, 1559.
	0 “ 6 “	

[1] Wenzel Gruber (Archiv. f. Anat. u. Physiol., 1873) has given elaborate attention to this subject as studied in modern European crania.

Upper . .	3 " 3 "	Hindoo, 1047.
Lower . .	0 " 7 "	Lapp., 1551.
	3 " 3 "	Chinese, 426.
	1 " 1 "	
	3 " 3 "	Malay, 47.
	1 " 3 "	
	3 " 5 "	Burmese.
	0 " 8 "	

Upper . .	3 c. 8 m.	left side.	Cretin of Hyrtl collection.
Lower . .	0 " 10 "		
Upper . .	3 " 9 "	right side.	
Lower . .	0 " 8 "		

The measurements were taken from the middle of the fronto-malar suture to a line indicating the boundary between the two parts of the bone as defined by the change in texture of the surface. No sutures existed in the specimens selected.

In the Hyrtl collection of crania, in the College of Physicians of Philadelphia, a Chinese skull (No. 13) showed a complete external suture on both right and left sides of body. A Cretin (No. 7) showed a complete suture in the bone of the left side and an incomplete one on the right side.

A Siamese skull (No. 39) retained an incomplete partial external suture on both bones, with entire posterior grooves.

In No. 67 the malar and zygomatic processes nearly met on the posterior surfaces of both bones. A similar groove was seen in a Japanese skull (No. 50).

In a skull (No. 77) a distinct posterior fissure was seen in both bones.

In a skull of a Hollander (No. 10) a foramen was noted in the maxillo-malar suture.

The malar bone is thus found to exhibit a disposition for the lower part to become distinct from the upper. The disposition is more frequently seen on the inner than the outer surface, and in all

instances is more pronounced posteriorly than anteriorly. The line of the suture when complete answers with an approach to accuracy to the attachment of the masseter muscle; and the existence of the suture might in some instances be found associated with the traction of this muscle.

In skulls which had been in a measure disintegrated by the action of the air, and sunlight and heat—in a word, which had been "weathered"—a distinct texturing was seen at the two parts of the bone. A beautiful instance of "weathering," demonstrating the texture of the bone, was seen in a skull of a California Indian (No. 1683), as well as in a Peruvian (No. 939, A. N. S.) In like manner fractures of the bone as shown in a skull of a Tahitian (No. 1016, A. N. S.), indicate the same difference in texture of the two parts.

This difference, in brief, is as follows: The superficial lines of the upper part are concentric, or nearly so, with the orbital margin, and the interior is composed of rounded cancelli, while the superficial lines of the lower part are parallel with the inferior free margin of the bone and the cancelli are coarsely laminated.

2d. The obliquity of the malar bone depends more upon the lower border than the upper part, and is associated with a change in the zygomatic process of the squamosa. The arch being viewed from above, the entire inner contour of the process last named can be seen in skulls in which the malar (especially at the lower part) is much deflected, as in Peruvians, while the posterior part only of the inner contour can be seen in skulls of low degree of malar deflection, as in negroes.

The degree of obliquity is independent of size. It is marked in in a Tschutki skull, A. N. S., where the malar bone is small.

In skulls of high degree of malar deflection, as in Malays and Chinese, the under surface of the zygomatic process of the squamosa is inclined inward from without and outward from below, while those in which the degree of deflection is small, as in negroes, it is nearly straight.

Such data indicate that with the deflection outward of the lower part of the malar there are associated distinctive changes in the squamosal part of the zygoma.

The malar bone of the fœtus at term shows all the essential peculiarities of the adult bone, (the swelling mentioned on p. 7 alone being absent,) even the minute spine at the posterior margin of the orbital process, (occasionally retained in the adult,) being present.

THE POSITION OF THE MALAR BONE AT THE SPHENO-MAXILLARY FISSURE.

The frequency with which the malar bone may enter into the spheno-maxillary fissure (by the processus marginalis) is subject to much variation. Froment (quoted by Henle) found it to enter into the fissure in nearly one-fourth of all skulls observed by him. In the following skulls of immature subjects—*i. e.*, below the age of sixteen years—forty-one in number, the association was present in thirty-one instances; hence I conclude that the exclusion of the malar bone from the fissure is more frequent in adult life than in youth.

The skulls in which the process was excluded were distributed as follows: One each in a Utah, Sioux, Seminole Indian, Chinese, Caucassian, Egyptian, and a Sandwich Islander skull. The remaining three skulls were unnamed.

THE LOWER JAW.

By careful inspection almost the entire outline of the lower jaw can be made out in the living subject. Even without preparation, the degree of projection of the chin can be seen in profile, as also the extent to which the angles are developed; but careful examination with the hand, especially when accompanied with oral inspection, greatly aids the observer in determining the form of the bone.

The most marked variation in the form of the jaw is seen in the depression which lies in advance of the insertion of the masseter

muscle. I have ventured to call this the *antegonium.* This depression can be easily detected by the finger. When the antegonium is well defined the mentum is always high. In a word, the vertical measurement at the anterior end of the horizontal ramus being large, the measurement at the posterior end is small. In one example examined the anterior measurement was 3^{c} 9^{mm} and the posterior 2^{c} 3^{mm}. In another example the anterior measurement was 3^{c} and the posterior 2^{c} 2^{mm}.

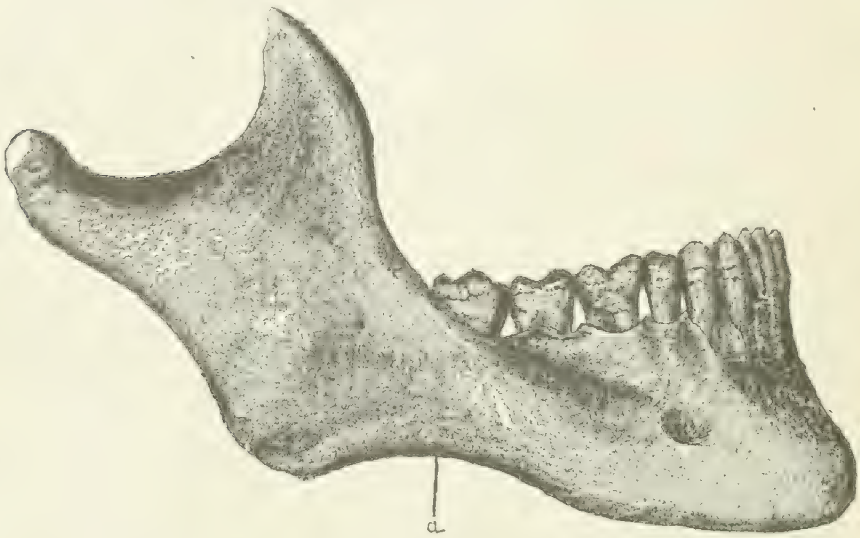

FIG. 1.—The lower jaw, showing the antegonium, and high mentum; *a*, the antegonium. (No. 200, A. N. S.)

This variation is often met with in patients and is generally seen in Cretins. It appears to be a result of rapid growth of the bone. To what extent the facial artery and vein may exert pressure on the bone to form the depression is not known. The molar teeth are often tilted forward in specimens of the bone which exhibit the antegonium, and in some instances the teeth wear transversely instead of obliquely.

The finger being placed in the interval between the tongue and the lower jaw-bone, one can detect with ease the mylo-hyoid ridge. The

base of the coronoid process can be outlined in emaciated subjects.

The condyles of the lower jaw are the most variable of any part of the bone. Of necessity the general shape of the articular surfaces cannot be made out in the living subject, but the tubercle to which is attached the external lateral ligament can easily be felt. When the lower jaw is depressed the finger can define the outer half (nearly) of the condylar surface.

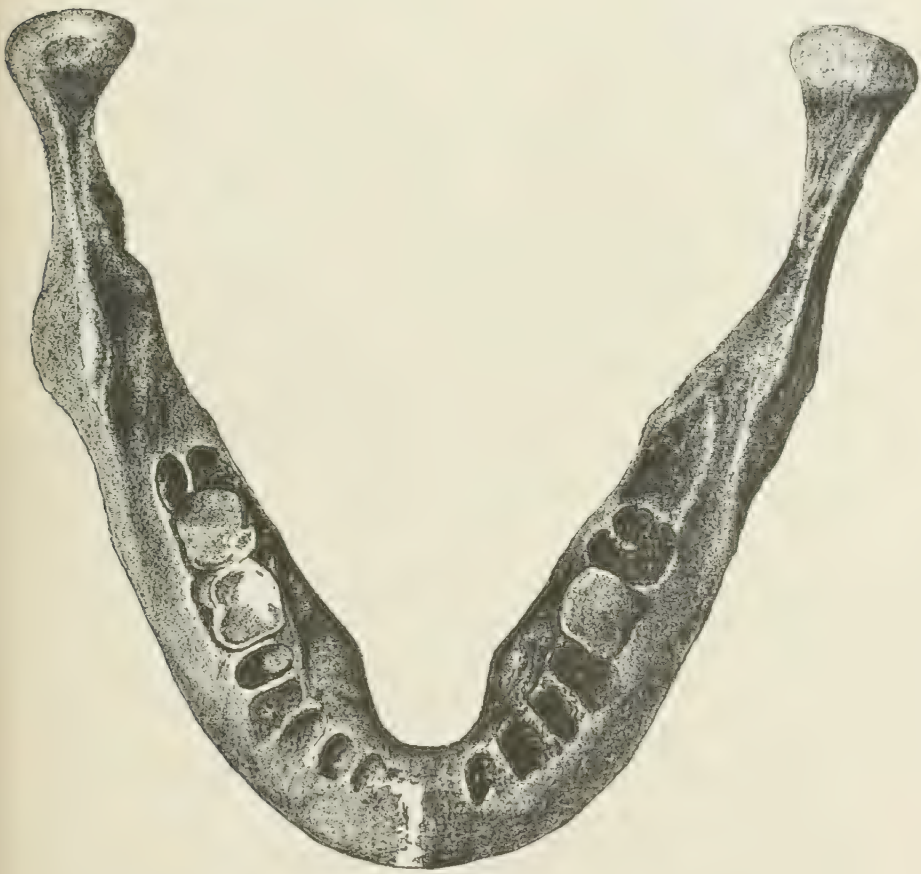

Fig. 2.—Lower jaw of an Esquimaux, showing hyperostosis on the lingual aspect of the horizontal ramus. (No. 173, A. N. S.)

In the specimens of the lower jaw of the cranium of an Esquimaux in the A. N. S. an elongated swelling was noted lying on the

lingual aspect of the ramus from the first molar to the canine tooth. In the skull of a young adult the swelling was mammalated, each nodule answering to the socket of a tooth. In the remaining bones, three in number, the swelling was uniformly convex, and extended to a line which was nearly equal to that of the bottoms of the sockets. The bone constituting the swelling was firm in consistence, but did not appear to be the result of inflammation. Out of thirty-four Esquimaux crania in the Army Medical Museum the hyperostosis was absent in one only.

In the living subject the distance from the angle of the bone to the firm muscle-mass about the cervical vertibræ often differs on the two sides. It is commonly greater on the left side. When separated from the attachments and relations, the bone does not exhibit the degree of asymmetry, which corresponds to the peculiarity named. It is true the left ramus may be deflected slightly outward to correspond with the increase of left-sided deviation of the superior dental arch, but no amount of dental variation could correlate with the apparently gross change at the angle as is seen during life. The explanation lies not in the maxillæ, but in the cervical vertebræ, especially the atlas, the left transverse process of which is the smaller.

The disposition for the left angle of the lower jaw to project to a degree much greater than the angle of the right side has been found by me to correspond also to the relation between the right and left sides of the hyoid bone. In a word, the entire left greater cornu deviates to a greater degree from the median line of the bone than does the right. The same remark is applicable to the two sides of the thyroid cartilage. These parts can be felt in the neck of the living subject. I have notes of several cases in which an irritation of the lower part of the pharynx appeared to be associated with the pressure of the posterior free end of the right great cornu against the mucous membrane. I have never detected similar points of irritation on the left side. The tentative conclusion I have drawn

from the facts is that the right greater cornu of the hyoid bone has a tendency to be pressed in against the wall of the pharynx, while the left appears to have no such disposition.

THE NORMA BASILARIS.

The norma basilaris embraces the skull when viewed from beneath. It is the least natural of any of the normæ, for the parts back of the foramen magnum are included in the region of the neck and are separated from the occiput by inconstant lines, while the facial parts are included in the mouth. The parts intermediate to the occiput and the face (the lower jaw will be considered as absent) constitute the true "base of the skull" as limited by physicians in studying the skull in the living individual. It includes studies of the important region of the pharyngeal vault. Variations in the norma basilaris, as might be expected, are seen in the occipital, facial, and intermediate regions, which do not of necessity correlate with one another, but express oftentimes entirely distinct, if not opposing, tendencies.

In order properly to consider the relations of the somewhat incongruous elements of the norma basilaris it is important to recall the significance of the parts.

Taking the union of the squamosal, tympanic, petrosal, and styloid elements to form the temporal bone as an illustration of the fact that early union between bones is an evidence of their affinity, then the following statements become tenable :

The skull of the child at the sixth year exhibits the bones of the face united completely to one another and to the sphenoid and frontal bones. Thus, since they unite with the facial elements sooner than with the occipital, squamosal, or frontal, they may be said to have closer relations with them. The bones of the face (excepting, of course, the lower jaw) unite with the sphenoid and the frontal bones to form a single segment or piece, while the remaining bones—the parietals, temporals, and the occipital—are

separate. The association of bones above named will receive in this connection the name of the *anterior cranial segment.*

The squamosal portion of the temporal bone unites with the malar bone, while the element first named is in articular union with the lower jaw, thus a natural series on the side and the base of the skull is constituted. The most intimate relations of this series are with the bones of the anterior segment rather than with the parietal, and it may receive the name of the *squamoso-malar series.*

The petrosal elements early unite with the squamosal, but never exhibit inclinations to unite with the occipital or sphenoid bones.

The occipital and parietal elements are also distinct, and nothing can be claimed to show their disposition to unite in any definite manner to one another or to any of the groups above named of cranial bones.

In reviewing the above facts it is seen that the sphenoid and frontal bones have facial affinities; that the squamosal and malar bones form a natural series, which tend to embrace the lower jaw, but that nothing in the attempt to demonstrate affinities by their predilections in articulation can be shown for the parietal or occipital bones.

Conceding that variations in bones are to be studied in connection with the changes in the groups to which they belong, it follows that the variations of the face should include those of the sphenoid and frontal bones; that the squamosal, malar, and inferior maxilla should be studied together, and that the remaining bones cannot be studied as a whole.

The anterior segment can be easily separated from the parts lying back of it by the line of the occipito-sphenoid junction. When the junction is obliterated a hypothetical *transverse line*[1] joining

[1] The transverse line answers necessarily to the place of the former suture which unites the sphenoid and occipital bones. Some writers assert that the transverse depression seen in the adult skull is not sutural, but muscular. This is not the case. The two lines are distinct. They are clearly seen as such in No. 87 Carniola (College of Physicians) and obscurely so in many specimens.

the tips of the sphenoidal tongues can be substituted for it. The production of this line across the norma will traverse on either side the alisphenoid, at the region of the oval foramen, the articular eminence, and the root of the zygoma.

The transverse line can be intersected at its centre by a hypothetical *longitudinal line*, which, passing through the mid-point of the *basion*, can be produced so as to divide the norma into a right and a left part.

The points of the greatest interest in the region are the asymmetry of the sides of the superior dental arch, the relative positions of the oval foramina of the sphenoid bones, the position of the anterior border of the articular eminence in connection with the transverse line, the depth of the zygomatic fossa, the thickness of the malar bone, the size of the bulbo petrosus—*i. e.*, the rounded swelling of the free part of the petrosa—and the angles formed by the axes of the tympanic bones and the petrosa with the longitudinal line.

The left side of the dental arch has been found more frequently expanded (it embracing a larger curve at the position of the first and second molars) than is the right. With this expansion is associated a diminished depth of the zygomatic fossa, a weaker articular eminence, and a thinning of the zygomatic arch, as compared with the same parts on the right side. The temporal ridge is also the weaker on the expanded side. The significance of the above facts appears to be as follows: The side of least expansion of the dental arch is the stronger side; hence a dental armature which is straight, or nearly so, is stronger than one which is curved.

The base of the alisphenoid on the stronger side inclines to be carried back farther than is the case on the weaker; but this is variable.

The angles formed by the axis of the tympanic and the petrosal elements vary on the two sides, but appear to be independent of the changes in the anterior segment and the squamoso-malar series. The asymmetry in the sides of the foramen magnum, and in the

distance from the basion to the mastoid process, and to the transverse process, are also variable without reference to other basic structures.

In some specimens the differences between the measurements of the anterior cranial segment and those of the occipital bone suggest that the rates of growth in the two parts of the skull have been determined by independent causes.

Thus, when the base of the skull (norma basilaris) is carefully inspected, it is evident that the parts on the sides of a median line are not always of equal value in size; also that the parts of the anterior segment may vary in a manner different from those of the parts posterior to it. In a word, while the contrast of right and left measurements are often discernible, the preponderance is not always the same in the two parts. In some examples the left side of the norma is wider throughout, though this is infrequent. In others the left side of the structures posterior to the anterior cranial segment is the wider, while the right side of the anterior segment is best developed. This is a common disposition. In the group last named the increase of the base of the alisphenoid (especially in a backward direction) is associated with a narrowing of the petrosal space—*i. e*, the space between the alisphenoid and the occipital bones. When this is seen the left side of the dental arch is often more deflected than the right, the right malar bone is the larger and encroaches to a greater degree on the inferior orbital margin, and the surfaces of origin of the right masseter and temporal muscles are the better marked.

The production of the transverse line intersects the foramen ovale at a point near its posterior margin or at one entirely back of the opening. The left side of the incisive foramen is often the larger, and the suture between the palatal plates of the maxillæ is not in line with the basion, but lies to the left. It appears to be probable that the muscles of mastication of the right side are more powerful than are those of the left. Hence the muscular impressions are here most marked and the malar bone is the more robust. The base of the right alisphenoid appears to be forced back, and by

harmonious distribution of the blood-vessels is increased in its diameters, while the angular process becomes wider.

That the left dental arch is more deflected may be the result of a diminished tonicity in the masticatory and buccal muscles on this side. This hypothesis agrees with the fact that the left frontal eminence is commonly the smaller.

The following measurements have been taken in illustration of the data as above stated. No one specimen illustrates all the points, nor is this to be expected in so variable a form as the human skull. When in a given example a measurement is omitted it may be understood that the result is negative, and not that the measurement conflicts with the views as already given.

No. 916, negro, A. N. S., aged 16 years:

Longitudinal line overlies median suture of hard palate.

Transverse line lies back of oval foramen, left; in front of the foramen, right. The line is 4^{mm} back of anterior border articular eminence, right, 8^{mm} left.

Distance from longitudinal line to outer border first molar tooth, 30^{mm} right, 28^{mm} left.

The left petrosal element has an angle of 60°, the vaginal process 50°, and the tympanic bone 20°. The right petrosal element has an angle of 60°, the vaginal process 45°, and the tympanic bone 25°.

Other peculiarities of this cranium included the lachrymal crest joining the maxilla; the perpendicular plate of the ethmoid bone rudimental, with deep triangular notch reaching back to the second molar tooth; the longitudinal palatal suture straight—the atlas anchylosed to occiput.

No. 917, negro, A. N. S., aged 21 years:

Longitudinal line overlies hard palate 2^{mm} to left of longitudinal suture. Transverse line 2^{mm} behind anterior border articular eminence, right; at level of border, left.

Distance from longitudinal line to outer margin first molar, left, 30^{mm}; right, 28^{mm}; to outer margin 2d molar, left, 32^{mm}; right, 30^{mm}; to outer margin 3d molar, 30 left, 30 right.

Angle of left petrosal element, 50°; left vaginal process, 50°; left tympanic bone, 90°. Angle of right petrosal element, 50°; right vaginal process, 40°; right tympanic bone, 90°.

Left zygomatic fossa, 26^{mm} deep; right, 28^{mm} deep.

Left zygomatic arch at suture, 4^{mm} wide; right, 6^{mm} wide.

Other peculiarities: Large, thick perpendicular plate of ethmoid bone; septum straight; longitudinal palatal suture not straight; jugular and carotid foramen smaller on left than right; also the canal for tensor tympani muscle; bregmal and post-bregmal portions of sagittal suture deflected, the latter about 60°; lachrymal crest inferiorly produced, but not touching maxilla; lingual process of sphenoid bone absent on right.

Distance from middle point of tympanic bone to base of malar process of maxilla, 8^{c}; from alveolar process, left, 6^{c}, and alveolar process right.

The following notes were taken to indicate an occasional character which varied in size on the two sides of the norma:

No. 127, Turk (Col. of Physicians):

Left angular process, 8^{mm}; right, 10^{mm}.

Stephanion more interrupted on the right than the left side.

No. 132 (Col. of Physicians):

Left side palate deflected.

Right, depth of zygomatic fossa, 16^{mm}; left, 17^{mm}.

No. 92, Uskoke (Col. of Physicians):

Left, width from basion to outer border transverse process of occipital bone, 44^{mm}; right, 41^{mm}.

Left, depth of zygomatic fossa, 20^{mm}; right, 23^{mm}.

Left, width of malo-zygomatic suture, 6^{mm}; right, 8^{mm}.

Left, width of bulbo-petrosa narrower than on right.

All parts of the right side of the vertex smaller than left.

No. 94 (Col. of Physicians):

Left basio-transverse (*i. e.*, measurement from basion to outer end of the transverse process of occipital bone), 40^{mm}; right, 43^{mm}.

Left carotid foramen, 5^{mm}; right, 6^{mm}.

Left supra orbital margin more inclined than on right.

Left portion of incisive foramen the larger.

Left side dental arch but slightly larger than on right.

No 114, Elba (Col. of Physicians):

Left dental arch most expanded.

Left basio-transverse, 41^{mm}; right, 43^{mm}.

Left zygomatic fossa, 21^{mm} deep; right, 24^{mm} deep.

No. 73 (Col. of Physicians):

Left dental arch most expanded.

Left basio-transverse measurement, 41^{mm}; right, 40^{mm}.

Left spinous process of sphenoid, 4^{mm}; right, 5^{mm}.

Left angular process from base of lingualis to sphenoido-squamosal suture, 22^{mm}; right, 22^{mm}.

No. 77 (Col. of Physicians):

Left dental arch most expanded.

Left basio-transverse, 36^{mm}; right, 40^{mm}.

Left zygomatic fossa, 23^{mm} deep; right, 25^{mm} deep.

No. 117:

Left dental arch most expanded.

Left zygomatic fossa, 17^{mm} deep; right, 21^{mm} deep.

Temporal ridge greatly interrupted on right; not interrupted on left.

No. 34, Krim. (Col. of Physicians):

Right side dental arch expanded. Distance from base of lingual process to sphenoido-squamosal suture, left, 26^{mm}; right, 25^{mm}.

Transverse line barely reaches back of the rounded form of foramen ovale, or right, while lying well behind the foramen, or left. Left

jugular and carotid foramina smaller on left than on right. Zygomatic fossa, right, 24^{mm} deep; left, 25^{mm} deep.

No. 6 (Col. of Physicians):

Depth of zygomatic fossa, 15^{mm} right, 17^{mm} left.

Width of anterior lacerated foramen, 10^{mm} left, 6^{mm} right.

In cranium of giant in College of Physicians the zygomatic fossa is 30^{mm} deep on right, 22^{mm} on left. The malar bone lacks 2^{mm} of reaching infra-orbital foramen, right, and 5^{mm} on left.

No. 50, Japanese (Col. of Physicians):

Base of alisphenoid, $21\frac{1}{2}^{mm}$ right, 20^{mm} left; no asymmetry of dental arches. Transverse line 2^{mm} back of foramen ovale, left; crosses foramen at anterior third right.

No. 545, Malay (A. N. S.):

The foramen ovale is crossed by the transverse line at the posterior margin on the right side, and at the middle, on the left, the line crosses the articular eminence 4^{mm} back of anterior margin.

The antero-posterior line answers to the middle of the socket of the left central incisor.

Right spinous process, 5^{mm}.

Left " " 5^{mm}.

Right base of alisphenoid, back of foramen ovale, 5^{mm}.

Left " " " " " 2^{mm}.

Right from spinous foramen outward, 9^{mm}.

Left " " " " 7^{mm}.

Right width of zygoma at suture, 5^{mm}.

THE POSTERULA.

Since the introduction of the rhinal mirror as an aid to the examination of the pharynx, the region known as the naso-pharynx can be inspected with almost the same ease as any other portion of the body. Many of the features of interest which relate to this portion of the pharynx are of a character which can be analyzed only by reference to the cranium. In the naso-pharynx are de-

tected the outlines of the delicate vertical plate of the vomer, the sphenoidal surfaces of the internal pterygoid processes, the posterior ends of the middle and inferior turbinated bones, and the vault of the pharynx as defined for the most part by the alæ of the vomer and the occipital process of the occipital bone.

In a communication to the American Laryngological Association which I made in 1888 I called attention to a portion of this region which extends from the plane of the posterior nares to the posterior limit of the vomerine alæ, and defined on the side by the internal pterygoid plates, and proposed for it the term *posterula.* Subsequent study has confirmed me in the value of this portion of the naso-pharynx being restricted as a distinct clinical region, and I here venture to show the close relation which exists between it and the morbid condition of the interior of the nasal chamber. I will, in addition, discuss the subject of the basio-cranial angle—that is to say, the clinical interest arising from the angle formed between the posterula on the one part and the inclination of the basilar process of the occipital bone on the other. The separate heads in the description of the posterula include the following:

The under surfaces of the body of the sphenoid bone.

The vomer as seen in articulation with the sphenoid and palatal bones.

The posterior nares or choanæ. (For note on Choanæ see Nasal Chamber.)

The region of the spheno-turbinals.

In no other portion of the skull do so many elements combine as in the naso-pharynx.

The basi-sphenoid and pre-sphenoid here unite. The spheno-turbinals lie in front of the under surface of the sphenoid elements and pass backward a variable distance above the palatals and vaginal processes, and forward along the sides of the meso-ethmoid. The vomer articulates with the body of the sphenoid bone. The borders of the alæ unite in a variable manner with the sphenoidal

of the region lies the basi-sphenoid junction, which is the last of all the sutures in this region to close.

The under surface of the basi-sphenoid, up to the sixth year, is convex in the centre and grooved or fluted on the sides. The convexity serves to receive the concave surface of the vomer, and the flutings accommodate vessels and nerves. The vaginal processes, the body of the sphenoid bone, and the sphenoidal processes of the palatals, later in development, convert these grooves into canals.

In some examples of crania[1] the under surface of the sphenoid bone continues to be convex in adult life.

In others (972, Negro; 1043, Pawnee; 726, Seminole; 1009,

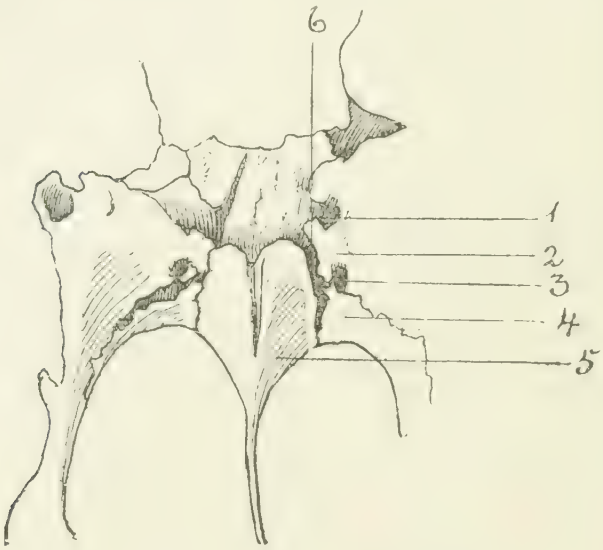

Fig. 3.—The posterula of a German (No. 1188, A. N. S.), showing failure of the vaginal process of the sphenoid bone and the palatal bone to reach the vomer. On the right side a fissure exists between the parts named.

1. Lateral superior foramen.
2. Vaginal process.
3. Lateral inferior foramen.
4. Palatal bone.
5. Vomer.
6. Interval between vomer and vaginal and palatal elements.

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[1] Nos. 438, 912, 27, 69, Seminole; 732, Seminole; 951, 953, Narrag.; 43, Menom.; 118, Lenape; 741, Mandan.

Ottawa; 1188, 1063, German; 746, Minitari; 947, Araunian) the vaginals, and sphenoidal processes of the palatal bones do not reach the vomer, or may be entirely absent.

From this condition of retained juvenile feature the most characteristic departure is to have the under surface of the body of the sphenoid bone slightly rugose (757, Otoe; 1233, Miami; 19, Bengalee; 693, Narragansett).

A few examples may be named in which the surface is moderately convex. On the other hand, it may be flat. The variety last named includes a large number of examples which are of especial interest, since no civilized race is represented (53; 651, Araucanian; 1227, Blackfoot; 1451, Australian; 1029, Fiji; 1342, bastard Malay; 435, Malay; 990, Maya; 1315, N. A. Indian; 730, Seminole; 935, Narragansett; 204, Chinook; 605 Sioux; 142, 905, 913, 973, 654, Negro).

FIG. 4.—The posterula of an adult North American Indian (No. 1322, A. N. S.), showing a median vomero-basilar foramen, in addition to the two lateral foramina.

1. Lateral superior vomero-basilar foramen.
2. Vaginal process.
3. Lateral inferior vomero-basilar foramen.
4. Palatal bone.
5. Vomer.
6. Median vomero-basilar foramen.

In one instance the surface is hyperostosed (No. 78, Menominee).

In three examples the under surface is concave in the centre, and two large canals retained at the sides (1322, Potawatomie; 1229, Upsala; 1228, Upsarooka).

The *vomer* is more or less concave at the upper surface, and is adapted to the convex surface of the sphenoid bone; but the method of union of the two bones is not as simple as the above statement would imply. The posterior part of the vomer, including the wings, may be without union to the sphenoid bone. The two bones are thus separated by an interval, which is variable with the shape of the body of sphenoid itself. The arrangement suggests that during life either blood-vessels or indifferent tissue occupied the intervals, or that hyperostosis at the anterior part of the sphenoid—probably at the line of the pre-sphenoid—had forced the vomer down and thrown it off from attachment to the posterior part. In immature crania the vomer is very generally removed from the sphenoid posteriorly. The disposition seen in the adult skull may be a retention of a juvenile character. If it is not so it is remarkable that the region so commonly exhibits this retardation in nutrition, for it is comparatively rare to see any other arrangement of the parts.

From among all the crania examined but 75 exhibit a departure from the above plan—*i. e.*, in this number of specimens only did the vomer articulate directly with the sphenoid throughout. Is it not strange that the description of the union generally accepted should be that of the entire union? Is it not suggestive that the retardation of the processes of development of the region should be greater in the crania of civilized races than among primitive people? The greater number of examples of entire union were found in the ethnological collection of the Academy of Natural Sciences.

While it is true that the lower animals uniformly exhibit the simple form of union, and on that account the plan may be considered as an instance of reversion, it must be stated that in 65 immature skulls (none, however, of the negro or his congeners) exam-

ined in the collection of the Academy not a single one was seen of such union, while all the examples showed by the extent of the groovings on the side of the body of the sphenoid bone, and by the degree of convexity of the central part of the surface, that the type was that of the variety described in my notes as "convex, hyperostosed," or "open posteriorly."

If we accept the theory that the arrangement seen in primitive races is the same as in many lower animals, and therefore that the earlier races of men more readily resemble the skull of mammals generally, we are forced to conclude that development is more rapid in the primitive races than in civilized, and that a phase of development which is transient in savage races becomes permanent and fixed in the civilized.

It is not unlikely that the retention of the juvenile characteristics in a large proportion of skulls of civilized man may be associated in some individuals with enlargement of the adenoid tissue at the roof of the pharynx, and that these characters of the sphenoid bone and the vomer may be due to the veins which pass from the mass, effecting anastomosis with the nasal venous sinuses and the spheno-palatine veins, and thus tending to keep up the large vascular tracks which lie between the body of the sphenoid bone, its internal pterygoid plate and the palatal bone. The size of the gaps left by failure of the pterygoid and palatal plates to unite with the vomer, as compared to the width of the posterula, is not insignificant. (See Figs. 3, 4, 6, 7.)

The primitive vomer is chiefly found, as above mentioned, in the crania of savages, while the hyperostosed vomer with incomplete sphenoidal union in those of civilized people. In addition it may be said that the last-named group includes a larger number of associated anatomical variations than is the case with the first named—a conclusion in harmony with the statement already quoted, which is attributed to Retzius, that individual differences become greater in proportion to the higher intellectual development.

In illustration of the lack of uniformity of description of the region of the posterula the following citations are made:

Quain's Anatomy (ed. 1876, p. 51): "The alæ are lifted posteriorly, and articulate edge to edge with the lamella projecting inwards at the base of the internal pterygoid plate."

L. Holden (Human Osteology, 1869, p. 101): "The diverging edges of the fissure, called the "wings," fit into the little furrows beneath the vaginal processes of the sphenoid bone."

Ph. C. Sappey (Traite d'Anatomie Descriptive, p. 214) describes the alæ as the borders of the groove by which the vomer articulates with the sphenoid bone, and further states that they are received in the groove on the internal surface of the base of the pterygoid process.

F. O. Ward (Human Osteology, p. 89) describes two projecting laminæ of the sphenoid bone overlapping and retaining the vomerine alæ.

FIG. 5.—The posterula of a North American Indian (No. 951, Narragansett), showing entire union between the vomer with the sphenoid bone.

1. Lateral superior foramen.
2. Vaginal process.
3. Lateral inferior foramina.
4. Palatal bone.
5. Vomer.

At the risk of repeating a few phrases the following detailed statements are here made: The crania named below are examples of

entire union of the vomer with the sphenoid bone: Of North American Indians—541, 542, 1054, 1233, 1056, 1052, Miami; 44, 563, 454, 747, Menominee; 118, 876, 40, Lenapé; 739, 741, 742, 644, Mandan; 462, 457, Chinook; 950, 951, Narragansett; 1227, Blackfoot; 897, Mohawk; 1730, Seminole; 91, Columbia R. Ind.; 1214, Ohio Ind.; 461, Chickasaw; 1006, Ottawa; 747, Minitari; 1210, Shawnee; 1,315 unnamed. Of South American Indians—601, 652, Araucanian. Of Negroes—1315, Golgon; 549, 974, 913, 967, 968, 648; "Oceanic Negro," 435. Of other races—573, Kowalitsk; 94, Chinese; 1300, 572, Sandwich Islanders; 1342, Malay; 1244, Hottentot; 1029, Fiji Islanders, and 969, 1263, 563, 142, 247, 400, 421, 1338, unnamed.

The following embrace examples in which the sphenoid bone and the vomer are united, with the exception of a small portion of the vomerine wings and the space between this lifted part, and the flat, small sphenoid body. In essential features the group is the same as the foregoing: Of negroes—905, 900, 904, 961, 968, 1102, 923, 993, 909, 973, 971, 972, 927, 916, 909; two of the negro group marked 1093, Golah, and 580, Macua. Of North American Indians—708, Seminole; 954, Narragansett; 1009, Ottawa. Of other races—1311, Bengalee; 550, Chinese.

From among 164 skulls examined complete apposition of the vomer to the sphenoid bone was found in 94 instances. In connection with the lack of union between the vomer and the sphenoid bone may be named the frequent instances in which the parts of the region become hyperostosed. The vomer is often of great thickness at the alæ and upper part of the posterior border.[1]

The vaginals, as they extend medianly from the vertical plate, are often massive and present a marked contrast to the thin brittle plate commonly found in this situation.[2]

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[1] 944, 746, Minitari; 744, Blackfoot; 740, Mandan; 977, Araucanian; 204, Chinook; 605, Sioux; 407, Miami; 115, Lenapé; 692, Carib; 693, Narragansett; 895, Mohawk.

[2] 97, 960, Negro; 112, Naples, nine years (College of Physicians); 113, Genoa, *ibid.*

They may overlap vomer as follows: 692, Carib; 963, 903, Negro; 240, Australian; 87, Peruvian; 737, Otoe; 1281, Peruvian, 12 years old; 986, Irish, 16 years old.

FIG. 6.—The posterula of an Irish girl, aged 16 years (No. 986, A. N. S.), showing an extensive fissure between the vaginal process of the sphenoid bone and the palate bone on the right side. Both these parts fail to join the vomer. On the left side the same processes not only join the vomer, but in one place tend to overlap.

1. Vaginal process.
2. Palatal bone, a conspicuous interval is seen lying between this element and the vomer.
3. Vomer.
4. Process from vomer joining the vaginal process to form an irregular union.

Open spaces indicate the failure of union between the vaginal and palatal elements and the vomer.

The shaded space back of the vomer indicates the inequality of level between the vomer and the body of the sphenoid bone.

The vaginal and sphenoidal plates may be firmly united throughout, or permit a foramen of varying size to appear between them. The plates may be depressed below the plane of the vomerine alæ.[1]

As a rule, the plates agree with the alæ in general character—*i. e.*, when the plates are hyperostosed the vomer is also; but the process may be reversed, and the plates be thin when the alæ are

[1] 572, Sandwich Islander.

thick.[1] The plates as a rule conceal the backward extension of the spheno-turbinals, and lie over and protect the veins and nerve of the canal. Thus they are apt to be pushed downward with the vomerine alæ in instances of unusual imperfection of the sphenoido-vomerine union.

When both the sphenoidal processes of the palatines and the vaginal processes are defective, the spheno-turbinals are seen distinctly exposed, thus showing the value of the plates named in covering and in strengthening the body of the sphenoid.[2]

The vaginal processes rarely extend backward beyond the vomer, and evince a disposition to approach toward the median line.[3]

It is well to remember that it is possible to have the image of the choana as seen in the rhinal mirror narrowed by thickening of the internal pterygoid plate.

The lower end of the vomer may exhibit the tendency, so commonly seen in the lower animals, of joining the palatal bones in advance of the posterior nasal spine. The vomer may be fully two millimetres within the nasal chambers.[4] I have seen it recede still farther within the chambers in the living subject. The exact position of the vomer becomes a matter of importance in determining the degree to which the inferior turbinated bone projects from the nasal chamber into the naso-pharynx.

The wings of the vomer may be united as the bone lies against the sphenoid, or a well-defined notch may be defined between them.[5]

A faint ridge is often seen extending vertically on the vomer. It answers to the line of union of the sphenoidal process of the pal-

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[1] 1205, Seminole.

[2] Examples are seen in No. 113, Genoa, 12 years old; 19, Ruthene, 7 years old; American, No. 58, 6 years old (Col. of Phys.), and a Hindoo, aged 8 years, No. 32, A. N. S. No. 89 (Col. of Phys.), an adult skull of an Adrian, shows the same peculiarity.

[3] No. 60, Austrian, 16 years old (Col. of Phys.).

[4] 1315, Golgonda; 605, Sioux; 1227, Blackfoot.

[5] 44, 1220, Menominee; 106, 407, 522, 542, Miami; 952, 953, 733, Narragansett; 207, Puget Sound Indian; 604, Seminole; 436, Chetimache; —, California Indian; 670, Chinese; and in 1205.

atal bone and the vaginal process. The surface in front of this ridge divides the region of the sphenoid base into two portions; that in front of the crest is nasal and that back of the crest is pharyngeal. The relative size of the nasal and pharyngeal spaces is variable. In 47 examples the nasal part equalled the posterior in extent. In 15 examples[1] the nasal part equalled two-thirds of the entire region. In one specimen (No. 956) the nasal portion was little less than one-half. In 971, 973, 974 (Negroes without locality), the nasal portion was one-third. It equalled one-fourth to one-fifth of the whole. In a fourth Negro (No. 983) the nasal portion was one-third on the left side and one-fourth on the right.

THE REGION OF THE SPHENO-TURBINALS.

In instances of absence of union of the sphenoidal process of the palatal bone with the vomerine ala in adult crania, the posterior end of the conch is seen lying upon the base of the sphenoid bone. In the skull of a male Australian in the American Museum of Natural History I have seen this process extend back to the suture between the defective sphenoidal process of the palatal bone and the vomer. I have in several instances seen the same disposition in the skull of the adult gorilla.

An adult Esquimaux cranium, Army Medical Museum, exhibits the conchs entirely free from the sphenoid bone posteriorly, and the suture, which united them to the palatals, open.

Is it not a tenable hypothesis that many of the unusual dispositions of the canales basis vomeres may be correlated with delayed union of the sphenoidal conchs? Is it not probably true that the defects of union between the vaginal and sphenoidal process and the vomer are associated with exceptionally large or numerous veins between the body of the sphenoid bone and the processes named? If this be conceded, but one step more is required to be taken to explain the frequency of congestions and hyperplasiæ in the nasal

[1] 975, 926, 1093, 913, 972, 928, 648, 433, 978, 916, negroes without locality; 648, negro of Liberia; 423, negro of Mozambique; 707, Seminole; 1063, 1064, German.

chamber when there is a history of adenoid disease at the pharyngeal vault. (See p. 27.)

FIG. 7.—The posterula of a North American Indian (Upsarooka, No. 1228), showing large foramina for the transmission of veins. (See also Fig. 4.)

1. Lateral superior foramen.
2. Vaginal process.
3. Lateral inferior foramen.
4. Palatal bone.
5. Vomer.

In a well-defined group of cases characterized by excess of tenacious mucus in the pharynx, a disposition to vascular obstruction in the nasal chambers, a sensation of weariness, if not of pain, in the sides of the neck (which is especially liable to ensue upon a moderate use of the voice, as in reading aloud and in singing), it is found that the roof of the pharynx is occupied by small growths which do not appear to differ, either in locality or consistency, from the adenoid growths found in the same locality in young persons. I have seen many instances in which the symptoms narrated had existed for many years entirely disappear twenty-four hours after the pharyngeal vault had been rasped by the finger nail, the vegetations removed, and the surface subsequently entirely restored by the removal, with the forceps, of bits of remaining tissue. It is reasonable to suppose that the increase of nasal congestion in such cases is dependent upon unusual freedom of communication which

exists between the veins of the nasal chamber and those of the pharynx by means of defects of the canales basis vomeres. Certainly it is a fact that no thickening of the walls of the choanæ occurs in such cases, and that the communication between the nose and the pharynx, if it takes place at all, does so at planes below the mucous and sub-mucous tissues.

THE BASI-CRANIAL ANGLE.

In the living subject the angle at which the basilar process of the occipital bone joins the body of the sphenoid bone can be frequently detected by the finger being inserted in the naso-pharynx. An interruption of the contour-line between the two structures can be often detected. In individuals in whom the angle is high the entire region of the naso-pharynx is narrowed posteriorly, owing to the fact that the inclination of the basilar process renders it easy for the velum to ascend toward it. This is especially marked in subjects which exhibit prominences on the bodies of the second and third cervical vertebræ.

The basi-vomerine angle, when high, places the parts to a great disadvantage should the naso-pharynx be the seat of diseased action. Tenacious secretion forming, either in the nasal chambers or in the naso-pharynx, the material is apt to lodge at the apex of the narrowed space, to resist any effort on the part of the patient to dislodge it, and to make it difficult so to do on the part of the physician.

A high angulation of the process in the living subject would predispose, *à priori*, the naso-pharynx to those distressing conditions which result from the contact of the velum to the posterior wall of the pharynx.

It may be surmised that irregular union of the occipital and sphenoid bones or their separation by a wide interval will be found associated with adenoid growths on the pharyngeal vault. Clinical experience confirms this; but, as far as I know, a careful dissection of a subject in which adenoid masses exist is yet lacking to complete our knowledge of their localization.

The high inclination of the basilar process would be at first sight a condition which would correlate with other cranial structures, but I have not found this to be the case in the living subject or in the crania which I have examined. It is true that in some crania the high angulation is associated with an inclined vomer, as seen at its posterior free border, and a high palatal crest. At one time I was prepared to assert that the high angle of union of the basilar process with the sphenoid bone and the vomer was associated with certain changes in the nasal chambers and in the proportions of the face, but examinations of more extensive series of specimens than those at my command will be required before any definite conclusions on this subject can be secured. To be enabled to harmonize the shape of the naso-pharynx with a series of fixed landmarks of the nose and face would be a most valuable desideratum and one to which I respectfully invite the attention of observers.[1] (See remarks at the end of the lecture on clinical measurements.)

The existence of a high angle with a large conceptaculum cerebelli is sometimes noted, as well as a low angle with a small degree of convexity or descent of the conceptaculum;[2] yet exceptions to the association can be found in sufficient numbers to forbid a correlation being established between the two.

In the skull of Sandwich Islanders and some Esquimaux the basio-cranial angle is low. Since the human fœtus at term exhibits uniformly a low angle or none, the adult crania which retain it may be said to be retarded in this particular.

Lissauer[3] has delineated and described the angle created by the union of the basilar process and the vomer. In a Tartar this angle

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[1] Dr. Jno. M. Mackenzie (Arch. of Laryngology, IV, 164) describes examples of obliquity of the plane of the posterior nares. No mention is made of its correlation with peculiarities of occipito-sphenoidal union.

[2] The funnel-shaped chamber which lies within the embrace of the conceptaculum and is defined anteriorly by a basilar process has been made the subject of special study by Lissauer. (Archiv. für Anthropologie, 1885, 16, Figs. 4 and 5.)

[3] Loc. cit. XV, Fig. 9, p. 18.

is 74 degrees, in a Cassube 98 degrees, and in a Negro 136 degrees. The difficulty in making a rhinological examination with a mirror placed in the naso-pharynx where the angle is 74 degrees, or approximately so, would evidently be much greater than when the angle is 136 degrees. Very commonly (as already remarked) a high degree ef angulation is associated with a large tubercle upon the body of the second cervical vertebra, which tends to diminish the diameter of the pharynx at the place at which the mirror is used.

THE NASAL CHAMBERS.

The study of the nasal chambers in the living subject presents facilities of determining by anterior and posterior inspection the following points:

By *anterior inspection*, the floor of the chamber—the degree it is depressed below the plane of the lower margin of the nostril.

The premaxilla—the degree it enters into the composition of the septum, and the size of the prominence it may make at the floor directly back of the plane of the lower margin of the nostril.

The septum—how it may be deflected either to the right or to the left, and whether the entire septum, or a part only, be deflected.

The inferior turbinal—the degree it may approach the floor and the septum; the relation its superior border holds to the middle turbinal.

The middle turbinal—the contrast between the vertical edge at the anterior and the part back of this border—whether the anterior part is inflated or laminar; whether the lower border is inflected or straight. The posterior part of the median surface, whether it is concealed by the inequality of the septum, or whether it is outlined as far as can be seen. The lateral (external) part, whether it is concealed in the recess which lies back of the plane of the ascending process of the superior maxilla or is distinctly outlined. The uncinate process, whether it is placed parallel to the lateral wall of the chamber or transverse to it. The bulbous anterior border of the lateral mass of the ethmoid bone, whether it is or is not visible.

By *posterior inspection* (by the rhinal mirror)—whether the choanæ are of unequal size; whether the left middle turbinal is more vertically disposed than is the right; whether the vomer is distinctly contoured, or the contour is indistinct by reason of lateral swellings; whether the inferior turbinals are protruding into the naso-pharynx; whether the superior turbinals are or are not visible; whether the choanæ and the septum at the choanæ retain the embryonic form.

From this long list it may easily be inferred that the interior of the nasal chamber yields many points for elucidation.

In the study of the nasal chambers of the cranium the parts, while assisting at every stage the demonstration as made in the living subject, soon awaken in the mind of the observer separate lines of inquiry. It is not, therefore, desirable to confine observation to the clinical field, and I have arranged the results of my research under heads which appear to be more convenient.

The bones of which the chambers are composed will be treated under different heads. Many of the examples selected showed more than one peculiarity. In a Peruvian skull,[1] for example, the middle turbinals, anteriorly, were small and primitive, the left bone being the smaller. The septum was deflected to the left along the entire length of the ethmo-vomerine suture. The left uncinate process was anchylosed to the ethmoid cells.

Each of the points named in the foregoing description will appear under a distinct heading. The parts which have been made the subject of special inquiry are the following:

1. The Middle Turbinated Bone.
2. The Parts which enter into the Composition of the Septum.
3. The Choanæ.
4. The Floor of the Nasal Chamber.
5. The Deviations of the Septum.
6. The Region at which the Frontal Bone forms Part of the Nasal Chamber.
7. The Anterior Part of the Lateral Mass of the Ethmoid Bone.

[1] No. 1705.

1. *The Middle Turbinated Bone.*

The middle turbinated bone is divided into two parts—an anterior one-third and a posterior two-thirds, nearly. The anterior part is best seen when the nasal chamber is examined from in front, and the posterior part is best seen from behind.

The anterior third. This portion is a plate of bone which ranges parallel to the perpendicular plate of the ethmoid bone or is deflected outward at an angle. The free lower border may be abruptly bent in or out. The entire anterior part may become inflated. It is often of greater density than the posterior part (is often covered with spines or is greatly roughened), and may be separated therefrom by a decided change in the inferior contour-line. At times a groove cuts off the anterior from the posterior part. This was well seen in the skull of an Ottawa Indian.[1]

In the infant the anterior part is always thin and compressed. It is parallel to the perpendicular plate, as above described. The same disposition exists in many of the skulls of later childhood and in those of adult life. Out of 188 skulls in which the anterior part was examined the plates were as given in 70.

The inferior or free border may be flat and wide, so that the appearance of the part might be compared to a wedge whose apex is directed upward. Of this variety 60 examples were observed.

Instead of being flat and wide the inferior free border may be bent abruptly upon itself or may present an acute projection which is directed either inward or outward. Eleven examples of such conformation can be cited. They were distributed among the races as follows: Seven of Peruvian[2], one each of German[3], ancient Roman[4], Iroquois[5], and Mexican[6] origin.

It is an interesting fact that no example of median inflection of the lower border was noticed in the examination made of 78 negro skulls.

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[1] No. 1009. [2] Nos. 957, 30, 228, 1432, 100, 1475, 631. [3] No. 1190.
[4] No. 248. [5] No. 119. [6] No. 1015.

The border may be inclined on both right and left sides, as was noticed in a Columbia River Indian[1] and in a Peruvian.[2]

In a second Columbia River Indian[3] the deflection is seen to a marked degree on the left side, and in a Peruvian[4] on the right.

Among other forms of inflated middle turbinals may be named one which resembles the terminal stroke of the German letter 𝔐. In others the shape is the same, but the direction reversed.[5] In yet another the bone is parallelogrammatic, and may be a centimetre in width.[6] A symmetrical disposition of such a form of inflation is seen in the skull of a negro.[7]

Infrequently the anterior portion is slightly though uniformly inflated, and is distorted in the shape of a crescent, or is even S-shaped.[8] As a rule the inflation begins at the free anterior border and involves the entire portion; but it is in some examples confined to the part immediately back of the anterior border, which remains narrowed and compressed, as in the infant.[9]

As already remarked, changes in the superficies are found confined to the anterior portion, for the posterior part is uniformly smooth or marked by vessel-grooves only. Numerous bristle-like processes are found occupying the surface in some instances.[10]

The inflected part may be seen at any section of the surface of the anterior portion. In a Madagascar[11] and a Peruvian[12] skull a large spine with a broad base projects from the outer side. I have often met with a similar spine in the living subject.

The flat interior border may even be inflated in common with the

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[1] No. 573. [2] No. 1366. [3] No. 377.

[4] No. 1447. [5] No. 976.

[6] The "bulbo-ethmoidalis," by which term I embrace the inflation of the anterior limit to the ethmoidal mass, is distinct from a bulbous inflation of the pedicle of the middle turbinal, which is occasionally met with, but is not included in the description as given above.

[7] No. 976.

[8] For examples see Peruvians, 1490, 1462, 1458; Narragansett, 693; Naumkeag, 567; San Miguel, 1636; Nantucket, 101; Ancient Mexican, 1003.

[9] Peruvian, No. 84. [10] Peruvian, No. 1460.

[11] No. 1306. [12] No. 1465.

rest of the portion, and as a result the part be club-shaped, or, being flat on the inner, become markedly convex on the outer surface. In the skull of a Mexican[1] a spur on a deflected septum on the right side is firmly indented in a club-shaped right middle turbinal. In most examples, however, of inflated anterior portions with deflected septa the parts are conformed one to the other.

With respect to the right and left disposition of the varieties named nothing can be said. A compressed primitive form of the anterior portion may be associated with a fellow of the same kind or of any of the forms already given.

In thirty examples of the anterior portion studied in skulls of children under eleven years of age the following disposition is noticed:

In six the anterior portion is compressed and primitive; in eleven the plane is the same as above, but the lower border was abruptly inflected; in ten the lower border is flat and the portion more or less wedge-shaped; in two a moderate amount of diffused inflation, and in one a marked degree of the same is present.

The extent to which the anterior portion may advance into the external nose varies greatly. In a Cimbrian and a Peruvian the bone is placed well within the region of the ascending process of the maxilla. I have observed the same peculiarity in the living subject.

Notwithstanding that the interior of the nose is protected by stout bony barriers, the parts may be distorted or destroyed by a variety of circumstances. In the practice of preparing the body for burial adopted by the ancient Peruvians masses of woolen or vegetable fibre were used to plug the nasal chambers. Lateral distortions of the turbinals are occasionally seen to be due to this cause.[2]

When bodies are left exposed to the air immediately after death dipterous insects deposit ova in and about the nostrils. The ravages made by the larvæ of the insects often destroy the turbinals. In a specimen of a skull of a Mandan Indian the anterior half of

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[1] No. 1430, aged six years. [2] No. 690.

the lateral mass of the ethmoid bone is eaten away, and the entire remaining portion of the bone is closely packed with the pupa cases.

The Posterior Two-thirds.—While the anterior part of the middle urbinal is apt to be vertical and compressed, so the posterior part is often horizontal at its upper part.[1] The scroll, indeed, may be said to be an inferior volute from a horizontal line, as in the scroll of the Ionic capital. Hence, the term turbinal is characteristic of a portion only of the bone.

The outer border of the posterior end of the horizontal part is usually notched. It is probable that the notch is for the accommodation of a vessel which is in connection with the structures occupying the spheno-palatine foramen. Occasionally, as is seen in a Bengalese skull,[2] a delicate bridge of bone converts the notch into a foramen.

The ledge-like upper border of the middle turbinal may be inclined or nearly vertical. These different shapes can be indicated (even when the turbinal is absent) by the direction taken by the upper crest on the palatal bone.[3]

In the posterior nares the ends of the middle turbinated bones, in the great majority of instances, are symmetrical and more or less curved. In 150 out of 234 skulls of adults examined the parts are of the kind described.

In 44 specimens the scroll presented a semicircular outline thus:)|(. The curved lines represent the middle turbinals and the vertical line the vomer. Of this number, 3 exhibit the upper part of the middle turbinal horizontal, instead of curved.

The varieties in which the middle turbinals were long and placed high up in the choanæ are included in the list of the symmetrical

[1] The horizontal part is most likely homologous with the ledge of the nasal chambers of quadrupeds where it separates the olfactory from the respiratory tracts.

[2] No. 25.

[3] The middle turbinal may lie well within the nasal chamber, some distance from the plane of the posterior end of the inferior turbinal. Example, No. 679, Esquimaux.

forms above given. In 90 examples of all races the middle turbinals are thus placed. Of this number 78 were Negroes and 2 Hottentots.

In 47 the left middle turbinals are smaller and more vertically placed than the right, and exhibited a small horizontal upper border. In 11 of these the left bone is compressed laterally and is straight. The left contour line of the septum is angulated in three of these examples. A similar peculiarity is met with in 4 immature skulls. The remaining 36 crania show various degrees of increased obliquity and curvature of the left bone over the right, and in a number of ways a diminished surface.

2. *The Parts which Enter Into the Composition of the Septum.*

The septum has been studied in the present paper for the most part in connection with its disposition to deviate from a vertical plane.

Several minor points were observed during the examination, which will be first recorded.

The Perpendicular Plate.—The perpendicular plate advances forward to a variable degree. Even in the adult—it was seen in a Bengalese skull[1]—the anterior end of the plate may be placed as in the young subject. Yet in some specimens, as witnessed in a North American Indian,[2] the plate was in advance of the plane of the anterior nasal aperture. The nasal plate of the frontal bone may be concealed by the advancement of the perpendicular plate of the ethmoid bone beneath. This was noted in a Circassian skull.[3] In a second skull of the same race the plate is also well advanced and of enormous thickness.

The plate may reach the nasal bones by a small surface, or may touch the bones along their entire lengths. The latter disposition is well seen in a Negro[4] and a Peruvian skull.[5]

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[1] No. 25. [2] Upsarooka, No. 1228.
[3] No. 762. [4] No. 914. [5] No. 413.

In the skull of an Araucanian[1] a large opening is detected in the perpendicular plate at a point directly back of the nasal plate of the frontal bone. Openings elsewhere in the perpendicular plate are so common that no special mention of them need be made.

The Vomer.—When the vomer at the posterior nares is not at the level of the openings, but lies at its lower part a little way within the chambers, the bone may be said to be *recedent.* It is a reversion effect, since it is commonly seen in the skulls of carnivora and in important groups of ungulata. (See p. 31.)

In a Peruvian[2] skull of five years and a Bengalese[3] skull of six years this recedence may be said to be present. The same peculiarity is seen in the adult skull of a Narragansett Indian,[4] an Assiniboin,[5] a Golgonda,[6] a Sioux,[7] and a Blackfoot.[8]

Recedence is so marked in a Maltese[9] skull that the bone unites with the maxillary crest at the maxillo-palatal suture. There is no upward extension of the spine of the palatal bone. The exact position of the vomer at the choanæ in determining the posterior projection of the inferior turbinated bone is of clinical importance.

The vomer may have two grooves—one for the triangular cartillage (it may be so obliquely placed as to appear to belong to the parieties) anteriorly, and one for a vein placed far back on the side. Examples of the obliquely placed groove for the triangular cartilage are seen in an Araucanian[10] skull and in several skulls of North American Indians.

3. *The Choanæ.*

The choanæ vary remarkably in form and dimensions. They may be as large as 23^{mm} long by 13^{mm} wide, or as small as 13^{mm} long by 6^{mm} wide. Usually wide and of a rectangular form inferiorly, as the borders join the transverse palatal process, they may be oval. The larger varieties include the shape first named, and the smaller one the shape last named.

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[1] No. 63. [2] No. 1492; [3] 48; [4] 951; [5] 1554; [6] 1315; [7] 605; [8] 1227. [9] No. 117 (Col. of Phys.). [10] No. 651.

The smaller varieties exhibit relatively long palatal crests when, indeed, they occupy one-third or one-half of the septum at the choanæ plane. Since this arrangement is seen in the fœtus at term, and the openings are oval or sub-rounded, it is fair to assume that the small oval variety is a form of arrested development. In a case of atresiæ nasi seen in an adult I detected this variety of choanal shape. The small oval form is so often met with in clinical studies that the conclusion may be tentatively drawn that it aids in retaining mucus in the nasal chambers, and in this way an anatomical factor may materially aid in establishing a morbid state. For examples in adult crania see a skull of a Miami Indian[1] and a Menominee.[2] In immature skulls, an Armenian,[3] an Austrian,[4] a Czech,[5] a Genoese,[6] a Sandwich Islander,[7] a Ruthene,[8] and a Neapolitan.[9]

. It is well to remember, as already stated, p. 29, that it is possible to have the image of the choanæ, as seen by the rhinal mirror, narrowed by thickening of the internal pterygoid process of the sphenoid bone.

4. *The Floor of the Nasal Chamber.*

In many subjects the plane of the lower border of the nostril is higher than that of the floor of the chamber. The inferior turbinal lies a variable distance within this depression. The finger when inserted into the nostril will not, in such cases, enter the inferior meatus, but will pass into a space which is defined by the septum on the one hand and the upper part of the inferior turbinals on the other. An example of the skull showing the depressed floor is seen in a Menominee[10] Indian.

5. *Deviations of the Septum.*

When it is recalled that the bony septum is composed not only of the vomer and the perpendicular plate of the ethmoid bone, but

[1] No. 1052. [2] No. 1222. [3] No. 58, Col. of Phys., 6 years.
[4] No. 60, Col. of Phys., 16 years. [5] " 80, " " 17 "
[6] " 113, " " 12 " [7] " 143, " " 16 "
[8] " 19, " " 7 " [9] " 112, " " 9 "
[10] No. 44.

of the frontal bone at the region of the vestibular roof, and small portions of the maxilla and of the palatal bones, it follows that if it is possible for defects to arise from faults of union, more than a single place for such defects must be sought for; or, if by mere distortion any one of the parts may be found out of the straight line, the localities at which such deviation may occur are many.

In point of fact the consideration of some of the lines of suture and plates of bone need not be regarded. Deviations at the region of the frontal spine and at the region of the palatal bone almost never occur, but in the remaining component parts they are of frequent occurrence and are apt to occur are as follows:

The perpendicular plate of the ethmoid bone.

The perpendicular plate of the ethmoid bone and the vomer acting as one factor.

The vomer.

The ethmoido-vomerine suture.

The maxillary crest.

As a rule, it may be said that deviations result from two structures differ in nature uniting one with another under unfavorable conditions. The perpendicular plate of the ethmoid bone may be bent on a broad curve, while all the remaining parts are normal. This is well seen in a Chilian skull,[1] in a Hindoo,[2] and in an Arab.[3] In the skull last named the plate is bulged to the left.

The perpendicular plate may be in the position described and the vomer be bent with it. No hyperostosis need exist at the suture. This is well seen in a Peruvian skull.[4]

The perpendicular plate and the vomer may be straight, but not lie in the same vertical plane. In this way a "fault" is defined between the two. This peculiarity also is shown in a Peruvian skull.[5]

The vomer may exhibit an angulation on the side, posteriorly—*i. e.*, at a point near the choanæ—and is, therefore, best seen from

[1] No. 1699. [2] No. 432. [3] No. 499.
[4] No. 1465. [5] No. 403.

behind. The septum may be in other respects straight. The apex of the angulated part often presents a groove which closely resembles the sulcus found in localities marked by the course of vessels. In the skull of an Ottawa Indian[1] the ethmoido-vomerine spur bears a groove which is continuous with a distinct canal posteriorly. The following specimens of skulls may be referred to, in each of which the groove is present on the left side: A Columbia River Indian,[2] two Peruvians,[3] and an Anglo-American.[4]

In one additional skull—that of a Peruvian[5]—the angulation and groove are on the right side.

That the chamber to which the septum inclines should be the smaller is shown by many examples.[6]

Deviations to the right side are seen in two Peruvians,[7] an Afghan,[8] a Circassian,[9] an Armenian,[10] a Finn,[11] and a Utah Indian.[12]

The disposition for the ethmoido-vomerine suture, as well as the maxillary crest at the triangular notch, to be hyperostosed and to present spur-like projections to the left side are such striking features in the majority of crania that no more than a recognition of their presence is here demanded.[13]

In a skull of a Ruthene (No. 19, Col. of Phys.) from a child seven years old the perpendicular plate and the vomer slip by one another, are not united, but are simply in apposition. The apposed surfaces are 3^{mm} long. If the degree of variation had been expressed in resistance at the line of normal union, it is difficult to see how deflection could have been avoided. Adult skulls not infrequently show the nasal surface of the frontal bone with the nasal process retaining the long plate of bone in place of the short, compressed spine, as is usually described. Examples of this conformation are seen in three Egyptians,[14] two Peruvians,[15] and one each of Circassians,[16]

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[1] No. 573. [2] No. 1363 and 1407. [3] No. 62. [4] No. 67.

[5] Egyptian, No. 819; Circassian, 765, 498; and a Malay, 459.

[6] Nos. 412, 1407; [7] 1333; [8] 762; [9] 790; [10] 1543; [11] 140.

[12] See a paper by the writer, Amer. Journ. Med. Sci., April, 1880, 70.

[13] Nos. 799, 819, 804, aged 16 years. [14] No. 432.

[15] " 642, 1137. [16] " 25, aged 12 years.

Hindoo,[1] Bengalese,[2] a North American Indian (Lenapé), an[3] Anglo-American lunatic, and one unnamed.

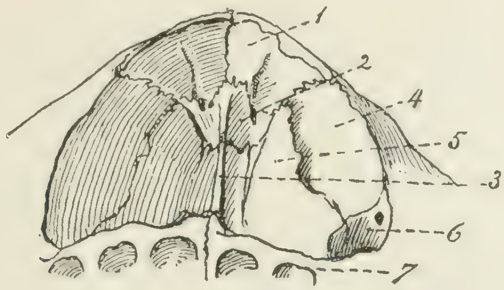

FIG. 8.—View within the anterior nasal aperture of an adult negro (No. 927, A. N. S.)

1. Nasal bone.
2. Frontal bone, forming at this place a keel instead of a spine.
3. Perpendicular plate of the ethmoid bone.
4. Ascending process of the maxilla.
5. Lateral mass of the ethmoid bone.
6. Inferior turbinated bone.
7. Alveolar process.

Thus ten well-defined examples of the nasal plate of the frontal bone were met with. With reference to this conclusion it is stated I have met it in 56 out of 76 negro skulls, and it would appear that we have in the nasal plate a valuable guide to the identity of this race. These facts lead me to consider

6. *The Region at which the Frontal Bone Forms Part of the Nasal Chamber.*

The frontal bone as it enters into the composition of the nasal chamber is usually described in forming a nasal spine.[4]

I have found that in the child the nasal portion of the frontal bone is of a different form from that described, and that in the adult

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[1] No. 763. [2] No. 40.

[3] Hoffman's "Lehrbuch der Anatomie des Menschen;" describes the "pars nasalis" as yielding a sharp process of variable length—the spina nasalis superior—which extends between the nasal bones and the perpendicular plate of the ethmoid bone. This description may be accepted as representative of those found in the text-books.

numbers of examples may be cited which do not answer to the accounts given by writers.

In the child, from the fourth to the eighth year the nasal portion is never furnished with a spine, but, in its place, with a plate which extends the entire length of the interval between the nasal and ethmoid bones.[1] The plate joins the perpendicular plate of the ethmoid bone inferiorly. A shallow groove on either side of the plate defines the roof of the nasal chamber at this place.

The nasal plate of the frontal bone is very rarely united to the perpendicular plate of the ethmoid bone. That there exists in the nasal chamber, in the races other than the Negro, an occasional, and in the Negro a frequent, absence of bony union between the two component parts of the septum, is an interesting fact.

Good examples of such apposition without union are seen in Nos. 951, 957 (Narragansett Indians), No. 651 (Araucanian), and No. 13 (Chinese). In the Army Medical Museum at Washington out of twenty Negro crania the parts above named are open in fifteen.

Care should be taken not to confound a fissure of absorption in the perpendicular plate with the form of retention as above described. A defect of this kind is noted in a Peruvian skull.[2]

Among the examples in which the conversion of the nasal plate into the nasal spine takes place it is interesting to observe the great size which may be attained. In a Negro[3] the spine was found to be nearly as large as the nasal bone. In two Araucanian[4] skulls the processes are also very large.

The nasal spine is found in an Afghan[5] skull to form part of the periphery of the external nose where it was lodged between the nasal bones.

Good examples are also seen in an Egyptian[6] and in a Nubian[7] skull.

[1] Good examples are presented in Nos. 426, 670, Chinese (A. N. S.).

[2] No. 1705. [3] No. 914.

[4] Nos. 790, 792. [5] No. 735.

[6] No. 1317. [7] No. 829.

That deviations from the vertical plane, which so commonly occur in the nasal septum, might be connected in some way with the changes that take place in the region of the nasal plate is not improbable. It is known that the parts at the root of the nose are exceedingly firm, and that the nasal bones vary greatly in diameter from the outer to the inner surface. It is also known that the perpendicular plate of the ethmoid bone is of inconstant proportion, but on the whole tends to advance. Hence, the nasal plate of the frontal bone may be compressed between these opposed directions of growth; but if the naso-frontal parts are preternaturally fixed the perpendicular plate of the ethmoid bone may be deflected, or the entire septum be forced to expand in a region whose boundaries have been already fixed.

The external nose during the period of transition from childhood to adult life changes greatly in shape. It is probable that at this time the substitution from the nasal plate to the nasal spine takes place, and that the deviation in some way correlates with the shape of the nasal bones in the adult. In the negroes, in whom the nasal bones are small and flattened, both at the root and the bridge (the juvenile shape), the process in question retains the plate-like form, while in other races the prominence of the root and bridge is associated with increased frequency of change of the nasal plate to the nasal spine; but in the alteration last named the increase of septal deviation is also to be noticed, and an obliteration of the harmonic apposition of the spine with the perpendicular plate of the ethmoid is likely to occur.

Enough has been observed to warrant the tentative conclusion that a cause for deviation of the septum (especially in that portion of the septum into which the perpendicular plate of the ethmoid enters) exists at the junction of the nasal spine of the frontal bone and the ethmoid, together with the rate and character of the change in the forms of the nasal bones.

While this is a conclusion which the premises in many instances validate, it is true that no one explanation suffices for the explanation of all deviations. (See p. 45.)

7. *The Anterior Part of the Lateral Mass of the Ethmoid Bone.*

This region, as a rule, has a narrow border. The superior border of the middle turbinal and the base of the uncinate process here unite. Occasionally, as is seen in a Peruvian skull,[1] the three structures are separated by a large globose surface, which forms the boundary of the most advanced of the ethmoid cells.

The Uncinate Process.—The uncinate process is flat and usually lies on the plane of the outer wall of the nose. In a low type of skull (this is well exemplified in a Hottentot,[2] in which it is firmly united to the inferior turbinal) the process may be found lying transverse to the long diameter of the nasal chamber, and of such dimensions as almost entirely to conceal the large middle turbinal. This disposition is seen in the left side of a skull of a Negro,[3] and in a second from Santa Barbara, Cal. In two Peruvian[4] skulls the uncinate process on the left side is united to the ethmoid cells.

The degree to which the uncinate process extends in an antero-posterior direction is subject to considerable variation. It may be in contact anteriorly with the inferior turbinal, so that an opening on the lateral wall of the chamber alone exists between the pedicle of the uncinate and the ascending process of the superior maxilla. It may be entirely free from the inferior turbinal at this section of the chamber, so in place of a foramen a long interval is found between its antero-inferior limit and the maxilla and the inferior turbinal. The extent to which the opening into the maxillary sinus is narrowed is also subject to variation. The opening appears to be the smallest in the prognathic and the largest in the orthognathic form of crania.

THE VERTEX.

The sconce or crown constitutes in thelanguage of craniology the vertex. The main parts comprising it are so easily determined by

[1] No. 1432.
[2] No. 1107.
[3] No. 964.
[4] No. 1705, 1432.

palpation that, so far as they are concerned, the clinical and anatomical study can be pursued on identical lines. Respecting the details, especially such as are seen in the sutures, it is only necessary to say that the topography of the general surface has been based, by common consent, on the arrangement of the parts at or near the sutures, and I have concluded to give the details of such localization the first place.

The names proposed for the suture-divisions, eminences, and depressions are easily adapted to the nomenclature of Broca. While it is acknowledged that multiplicity of terms is undesirable, I see no way out of the difficulty in presenting new names, since accuracy of description is impossible without them.

It is hoped that by their aid not only the vertex, but the scalp as well, can be mapped out for clinical purposes.

The sagittal, coronal, and lambdoidal sutures show peculiarities of the several parts entering into their composition which are worthy of special description.

To speak first of the *sagittal suture,* it is found that the portion which answers to the parietal end of the anterior fontanel and to the suture a short distance back from this opening is simpler in composition than the adjacent part of the suture.[1] It measures 1 to 2 centimetres in length. It is convenient to call this the *bregmal* portion.

The second portion of the sagittal suture is the longest and contains, as a rule, the largest serrations. These are either denticulate or lobate. The line answers to the region of the parietal tubera, and measures from 4 to 6 centimetres in length. In the normal cranium it represents the highest portion of the glabello-inial curve, and may receive the name of the *intertuberal portion of the sagittal suture.*

The part of the intertuberal portion which lies back of the bregmal for a distance of 1^{c} to $1^{c}\ 5^{mm}$ is often of a distinct type of ser-

[1] Out of the 66 negroes' crania with open sutures examined 21 retained sinuate and 45 serrate bregmal portions.

ration and may be deflected from the line of the intertuberal portion. It corresponds nearly to the position of a depression which is commonly symmetrical on either side of the suture as seen on the endocranial surface. When well marked it may receive the name of the *post-bregmal portion.* In Negroes it is commonly merged in the intertuberal.

The third portion of the sagittal suture is the *obelion* of Broca.[1] The parietal foramina lie on the sides and serve as guides to this the *obelial portion.*

Broca describes the obelion as having a length of 2^c, measuring, as it does, 1^c either way from the foramina. The suture is very commonly harmonic, while it may be sinuate, serrate,[2] or lobate, but rarely the last named. The vertex, as a rule, is rounded or ridged at the sides of the obelion, which thus appears to be depressed.

The fourth and last portion of the sagittal suture also appears to be depressed. It extends from the obelial to the lambdoidal suture. The serrations are coarse, and are often composed of denticles which exceed in length any seen in the foregoing divisions of the sagittal suture. In the growing subject it is often the thickest part of the suture. It measures from 1 to 2 centimetres in length and may be called the *post-obelial portion.*

The *coronal suture* is constantly divided into three parts—the internal or ental, which answers to the anterior fontanel; the middle or mesal, and the external or ectal. The internal is simple or wavy; the middle is denticulate and extends from the internal third to the stephanion, while the external or ectal is again simple, and lies between the stephanion and the pterion. It is covered by the temporal muscle. The external or ectal may remain open while the remaining portion of the suture is obliterated (No. 38, Col. of Phys.). In some subjects, notably the Negro, the middle portion

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[1] Instructions Craniologiques et Craniometriques, Paris, 1875, p. 24.

[2] Out of the 55 crania of negroes in the collection of the Academy of Natural Sciences 35 exhibited sinuate obelial portions and 20 serrate.

becomes simple when it runs forward parallel to the temporal ridge for a short distance before crossing it at the stephanion.

In an Esquimaux skull (No. 200, A. N. S.) the line of the temporal fascia crosses an almost simple coronal suture 28^{mm} from the bregma. The stephanion is practically unseen.

Kuppfer und Bessel Hagen, in 281 skulls from East Prussia, found the coronal suture running along the temporal ridge a short distance before crossing it in 5 per cent. males and 6 per cent. females. In the skulls of the insane these observers noted the disposition in 40 per cent. W. Sommer (Virchow's Archiv., vol. 90) in a similar examination found this disposition in 17 per cent. of males and 7 per cent. of females.

The *lambdoidal suture*,[1] like the coronal, is divided also into three parts, which may be named, in a similar manner, the endal, mesal, and ectal. Of these the ectal is the simplest in composition, and the mesal the most denticulated. Wormian bones, when present, are commonly situate in one or the other of these divisions, and not at their lines of juncture. The divisions appear to be subject to greater variation than in the cases of the sagittal and coronal sutures.[2]

W. Sommers (loc. cit.) found the lambdoidal suture concave forward in 90 per cent. of skulls of the insane, and 10 per cent. convex. No mention is made of the eminence which I have named meso-lambdoidal. It is fair to assume that it was present in those

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[1] Broca practically makes similar subdivisions of the coronal and lambdoidal sutures in his method of studying the relations which exist between the cranium and the cerebrum. (See Revue de Anthropologie, v. 1, p. 36.)

[2] In No. 461, Clickitat (Columbia river) and 730 (Seminole) the lambdoidal suture is completely occupied by a number of Wormian bones. The divisions of the sutures, as above named, are lost, and the entire region presents an elliptical figure. In No. 208, Nisqually, A. N. S., the suture is nearly straight and with few serrations. Out of 60 negro crania examined the lambdoidal suture was straight, or nearly so, in 21, and arranged as described above in 39. In Esquimaux crania the outer part of the lambdoidal is much smaller than is usually found in skulls of other races, and the meso-lambdoidal is less convex forward.

in which the suture was convex, inasmuch as this convexity is most marked in, if not confined to, the mesal part of the suture. (See *infra.*)

THE EMINENCES AND DEPRESSIONS OF THE VERTEX.

The eminences of the vertex which have been separately named are the frontal, the parietal or the tuberal, and the occipital. In addition, I venture to name five others, as follows:

The meso-coronal.
The metopic.
The para-tuberal.
The meso-lambdoidal.

The *meso-coronal eminence*, lies on the frontal bone just in advance of the meso-coronal portion of the suture, about two centimetres above the stephanion. It may involve the suture itself, when the corresponding part of the parietal bone is also elevated. It is marked in many Peruvian crania, but is often absent in the skulls of Negroes and Esquimaux.

The *metopic eminence* is a median elevation of the frontal bone over the interfrontal suture. It is inconstant, but may amount to a conspicuous carination which can be seen often in the living individual.

The *para-tuberal eminence* is a rounded elevation which lies between the parietal tuber at its posterior limit and the obelion. It is commonly present. It is least developed in the Esquimaux.

The *meso-lambdoidal eminence* lies on the parietal bone in advance of the lambdoidal suture at its middle portion, or it may cross the suture and involve the occiput. It is marked in synostotic crania of the criminal type. It is very well seen in a skull of a Krim.[1] In some crania it appears to be continuous with the tubera.

In No. 1561, Esquimaux (A. N. S.), the vertex is marked by a large adventitious but distinct swelling (measuring 2 centimetres

[1] Coll. Phys.

long by 1 wide), which lies between the tuber and the lambdoidal suture. In No. 1562, of the same race, an elevation extends from the tuber to the sagittal suture. It limits the inclination of the parietal bone towards the occiput.

The temporo-frontal eminence.—Under this head may be mentioned a swelling which is felt occasionally in the living subject directly to the outside of the temporal ridge as it is defined on the frontal bone. It forms a low obtuse prominence, measuring about 3 centimetres in diameter. It is best discerned in young individuals, since in adults it is obscured by the massive temporal muscle. I have found the temporo-frontal eminence, so frequently in Peruvian crania that it may be included among the characters distinguishing them. In a Marquesas skull, in the A. N. S., a similar prominence is marked.

The *depressions* which can be detected on the vertex are arranged as follows: In advance of the bregma; this constitutes the *pre-bregmal.* At the centre of the fontanel, or embracing in a general way the region of the fontanel; this is the *bregmal.* At the line of the coronal suture and the part directly back of it; this is the *coronal.* At the broad interspace between the frontal bone and the tubera; this is the *post-coronal,* and appears to be an extension of the foregoing. An apparent depression is defined at the obelion.

The coronal depression has been described by Prof. J. Cleland (Philosoph. Trans., vol. clx, 1870). It can be easily defined in the living subject. Abundant means are at hand for confirmation of this statement. Children exhibit the peculiarity as well as adults. It is generally seen in short high heads, which also retain a short sagittal suture and an abrupt curve to the mid-vertex. Rolleston (British Barrows, 1877) names skulls which show this peculiarity "cut off;" it appears to be the same variety as is described by Lissauer (Archiv. f. Anthropologie, 1885, p. 9) under the name of "sagittal Krümmung."

When the two coronal depressions are associated with large tu-

bera and para-tubera, and the interval between them (viz., the obelion and the post-ebelion) is on a lower plane than the occipital angle, the variety of skull named by Prof. Cleland, "trilobate," is defined. Trilobate skulls have been found by Prof. Rolleston[1] in the barrows of England. In the College of Physicians, No. 87, Carniolian, and No. 10, Hollander, exhibit the peculiarity. I have detected one in a Peruvian, another in N. A. Indian (No. 747, A. N. S.), and a third in a Tschutchi Indian (No. 3, A. N. S.). An imperfectly developed form is seen in a Nantucket Indian child aged 12 years. W. H. Flower gives an example in Catalogue Osteol. Collection, Col. of Phys. and Surgeons, Lond., 1879, 172. The natiform skull of congenital syphilis appears to be of the same nature as the tribolate.

The *post-coronal depression* is often associated with the general roundness and fullness of contour of the frontal bone just in front of the coronal suture. This is well seen in No. 1492, Peruvian (A. N. S.), aged five years, and in 890, *Ibid.*

Instead of the coronal depression being marked the bregma may be greatly depressed, the sagitta shortened, and the occiput knobbed. Such crania are frequently seen, and in the living subject make it exceedingly difficult to determine accurate measurements from the line into which the bregma enters. The subjects are apt to exhibit hyperostosis of the sutures of the hard palate, and to have small choanæ. Examples are seen in two Italian skulls in the College of Physicians (Nos. 110 and 113).

Occasionally a depression is seen above the temporal ridge and corresponds to the curve of this elevation. It is well seen in an Esquimaux cranium (No. 677, A. N. S.).

The Ridges of the Vertex.—The ridges of the vertex are those at the sagittal suture, above the temporal ridge, and at the sides of the obelion and the post-obelion.

The ridges of the sagittal suture constitute the carinations de-

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[1] "The precipitous dip downward of the posterior half of the parietals which is so characteristic of brachycephaly generally.—*Ibid*, p. 682.

scribed by anthropologists. They may be restricted to the subdivisions of the sagittal as above proposed. Thus the post-obelial and the intertuberal parts are often separately and distinctly carinated. The bregmal and post-bregmal parts may be carinated, while the rest of the sagitta is normal. The post-obelial, obelial, and the posterior half of the intertuberal parts have been found to be carinated, together with the bregmal and post-bregmal, the anterior part of the intertuberal alone remaining normal. The carinated portion of the sagitta may extend the entire length of the suture, excepting only the post-obelial. This arrangement is ad. mirably seen in the figures of a woman's skull in Welcker's monograph (*infra*, xiii, Figs. 1, 2, 3, 4).

The ridge which conforms to the temporal ridge is relatively infrequent. It is found in heavy male skulls as far as my observations go. It should be easily felt in the head of the living subject. The enormous lateral ridges of Uintatherium are probably developments of the temporal ridges, thus showing the extraordinary influence muscle-traction can exert over bony surfaces. If the exact degree of influence of all the muscles having bony attachments could be measured, osteology would be placed upon a philosophical basis.

Instead of the sagittal suture at the obelion and the post-obelion being depressed it may remain unchanged. The margins of the parietal bone remain also unchanged, while a ridge-like elevation of bone passes obliquely from the sagitta, at the end of the intertuberal portion, backward and outward to the meso-lambdoid eminence. Such conformation is well marked in the skull of a Chinese in the College of Physicians. In a living individual retaining such a peculiarity it is highly probable that a large triangular depression could be felt at the posterior part of the vertex.

THE STUDY OF THE INTERIOR OF THE VERTEX.

The interior or endo-cranial view of the vertex confirms the proposed division of the sagittal suture. The several parts are as dis-

tinctly separated as on the exterior, and, as the interior plane of the sagittal suture tends to remain open when the exterior is closed, the evidence of the disposition is here often alone available.

The side of least expansion of the parietal bones correlates with increase of thickness of the inner plate. The elevation of the inner plate of the unexpanded side is easily detected by the finger.

In No. 24 of the College of Physicians the vertex-sutures are open, the bregmal, post-bregmal, obelial, and post-obelial parts are serrated, both exteriorly and interiorly, while the intertuberal (the post-bregmal portion being here counted a separate quantity) is harmonic.

In No. 50, of the same collection, the interior view of skull is harmonic throughout, the bregmal being alone distinguished by its obliquity to the rest of the sagittal suture.

The relations of the depressions (presumably for the Pacchionian bodies) are, if of simple form, very commonly on either side of the intertuberal portion of the suture at the post-bregmal division. In thirty examinations of normal crania I have found but five where the depression was either absent or merged with a depression placed still farther back.

When the vitreous plate is thickened at the region of the former anterior fontanel and extends along the lines of the sutures so as to form a lozenge-shape figure, depressions for the Pacchionian bodies are often seen at its sides. It is rare to see depressions at the obelial or the post-obelial parts, though they may be oftener found on the frontal bones below the frontal eminences. Between the parietal tubera and the sagittal suture at the obelion an eminence is frequently found which almost equals the tuber in size. It is very commonly found in the skulls of Peruvians.

As in all other anatomical quantities, the subdivisions of the sutures of the vertex are subject to variation.

The simple statement upon which such subdivisions may be rendered tenable is one universally conceded, namely, that structures in their range of variation show traces of their origin and rates of

growth. That the bregmal and post-obelial portions of the sagittal suture are distinct from the remaining portion is probable when it is recalled that both portions are completed after birth in the process of obliteration of the fontanels. That the post-bregmal portion may be a good subdivision is also probable, since it answers pretty nearly to the position of the Pacchionian bodies and from the fact that in the parietal bone of the young subject this portion is seen to be pectinated, while the intertuberal is nearly smooth. The intertuberal portion represents the shortest distance from the tuber to the suture. The obelial portion has an admirable *raison d'etre* in being the region of the parietal foramina.

The following notes in illustration of the manner in which the foregoing statements may be employed in description of crania may be found useful: The specimens are all in the College of Physicians.

No. 114, native of Elba:

Sutures open.

Bregmal, $1^c\ 5^{mm}$; post-bregmal, $1^c\ 5^{mm}$; intertuberal, $4^c\ 5^{mm}$; obelial, 2^c; post-obelial, 1^c.

No. 30:

Acrocephalic, synostotic.

Bregmal and post-bregmal, 4^c.

Entire region elevated; not carinate; intertuberal, 4^c, slightly carinate; obelial, $2^c\ 5^{mm}$, flat; post-obelial, 2^c, carinate.

No. 92, Uskoke:

Left coronal suture closed; obelial portion lobate; post-bregmal with markedly oblique axes to the serrations, in contrast to the transversely disposed serrations of the intertuberal portions.

No. 38, Kabardine:

Both coronals obliterated; no wisdom teeth, yet the basi-cranial suture is closed; bregmal, $1^c\ 2^{mm}$; post-bregmal, $1^c\ 2^{mm}$; intertu-

beral, 5^c 5^{mm}; obelial, 2^c; post-obelial, 2^c. The obelial is serrate; post-bregmal depression is markedly developed.

No. 34, Krim:

Synostotic, forehead prominent; resembles skull of Pomeranian weaver described by B. Davis; metopic eminence conspicuous. Entire region of bregmal, post-bregmal portions, and the anterior half of the intertuberal is elevated, but broadly carinate. The posterior half of the intertuberal is smooth; the obelial and post-obelial portions carinate.

No. 98, Gypsy:

Vertex remarkably "cut off" posteriorly. Entire suture-line is carinate except the post-obelial portion.

Australian skull (Col. of Phys.):

Sagittal suture open; bregmal, 1^c; post-bregmal, 1^c; intertuberal, 6^c; obelial, 2^c; post-obelial, 2^c.

In the skulls of Esquimaux, A. N. S., the vertex is "cut off;" the intertuberal, excepting the post-bregmal part, is carinate in No. 678. The entire intertuberal is carinate in No. 279; the para-obelial eminence continuous, with a smaller ridge which extends one-half the length of the intertuberal portion of the sagittal suture in No. 677.

The right and left sides of the vertex are almost always asymmetrical. The left side at the forehead is commonly more projecting than the posterior part of the parietal bone of the same side. The reverse of these proportions is seen on the right. At the level of the occiput the left part may be projecting. Thus a circumferential measurement of the left side at the level of the frontal eminence may show the curve exaggerated anteriorly while diminished posteriorly, and a similar measurement taken from frontal eminence, so as to include the occiput above the inion, will show both anterior and posterior parts exaggerated on the left side as compared with those on the right.

Linear measurements taken in the median line from the glabella to the inion will represent more nearly the curve of the left side of the calvarium than do those taken on the right. The measurements last named may differ so widely from those of the left side as to throw the point given by Thrane for the fissure of Rolando on the right side as much as one-half inch out from that of the left.

The vertex in the space included at the sides by the temporal ridges — at the front by the corona and at the back by the lambda—is subject to local atrophic changes. Rounded depressions measuring one or two centimetres across and one to three millimetres in depth are scattered irregularly over the surface. There is no diseased action elsewhere in the skulls showing this peculiarity, and no evidence can be presented that the depressions themselves are of morbid origin. They have been seen always in crania showing early signs of advanced age, and some of them are found in distinctly senile skulls. Examples are seen in several of the skulls of Arabs (A. N. S.). A Narragansett[1] and a Chinese skull[2] also exhibit the depression.

In a cranium in the possession of the Academy of Natural Sciences the vertex has been mapped out and the localities named after the phrenological method of Gall and Spurzheim. It is interesting to note that a number of the enclosures which constitute what is known in the language of phrenology as the "organs" answer accurately to the eminences which I have named as above. Thus the para-tuberal eminence becomes the organ of "ambition," the meso-lambdoidal eminence that of "friendship," etc. The "organ" of "philoprogenitiveness" appears to be always well developed in females, and frequently so in males. I find no reference to this association of parts in the writings of phrenology, and I am, therefore, led to infer that it is a co-incidence only that the eminences which I have named happened also to have attracted the attention of the phrenologist.

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[1] No. 951. [2] No. 94.

NOTE.—H. Welcker (Wachsthum und Bau des menschlichen Schädels, 1862, Fig. 7, p. 17) divides the sagittal suture into five parts. These divisions are the same as I suggest in the text. My attention was called to Welcker's work by Dr. Frank Baker after I had delivered the lecture. Instead of naming the parts separately, Welcker includes them in the numbers 1, 2, 3, 4, and 5. It will be noticed that this writer retains the post-bregmal division, which I have included with some doubt. The reference of Welcker to the entire subject is very brief and is embraced in the following language: "For more accurate examination of the shape of these sutures I have illustrated (Plate iii, Fig. 7) five regions, of which No. 1 is on the coronalı ; No. 5 borders on the lambdoida, while No. 4, which lies between the straight parts of the parietal foramina, is a trifle smaller than the other divisions."

Rolleston (British Barrows, 1877, 623), probably influenced by the same authority, speaks of the sagittal suture as divided into fifths. The postobelial is the "posterior fifth" of this writer, and the obelial the "penultimate fifth."

REMARKS ON THE SUTURES OTHER THAN THOSE OF THE VERTEX.

Sutures often indicate the manner in which the bones have grown. As already stated, the comparatively deep serrations in the middle of the sagittal and coronal sutures correspond to the most precocious extensions of growth-force in those directions. Premature union of two opposed portions of bone, namely, at the surfaces of greatest acceleration, may lead to a suture at such portions, being raised above the plane of the adjacent surface. The carinated portion of the sagittal suture is an illustration of this peculiarity. A group of instructive examples is seen in the sutures between the maxilla and the bones adjacent; thus the malo-maxiliary at its lower part, where two obtuse processes project, the process pertaining to the maxilla being the larger; the inequality and even rugosity of the same suture, as it aids in defining the lower border of the orbit; the union of the horizontal plates of the maxillæ by means of which an upward extension results, aiding in the composition of the nasal septum; a downward extension of the same in the form of a thickening and even of an exostosis, which lies upon the roof of the mouth; and also in the nasal spine, which is formed at the intermaxillary

suture and projects from the lower anterior margin of the nasal chamber. These changes on the line of union of the maxilla with the malar bone, and with its fellow of the opposite side of the body, indicate that the direction of pressure during the growth of the bone has been greater at the sides toward the malar bone and at the median line of the face than elsewhere. It has been least between the maxillæ and the nasal bones and between the maxillæ and the palatals, which would indicate that the maxilla has grown forward and from side to side earlier and more aggressively than it has grown upward and backward. In this statement it is assumed that each nasal bone lies above the ascending process of the maxilla rather than in front of it. The backward extension of the maxilla against the palatal bone in the line of the dental arch demands special consideration, since it belongs to the means of accommodation of the molar teeth. Such as it is, however, the pressure of the extending bone in this direction leads to increased thickening of the palatal bone in all directions, and forms the pyramidal process. This process may be looked upon as an exemplification of an active suture-formation, which leads to hyperostosis of a part, although only one of the bones interested becomes entirely involved.

The maxilla in two places shows the effects of nerves and vessels in modifying suture lines. The roof of the infra-orbital canal is closed in a variable manner by the approximation of two portions of the maxilla at the inferior border of the orbit. Very commonly the border is thickened and an additional element of roughness and unevenness presented to that already noticed in the malo-maxillary suture. In like manner the maxilla as it joins the malar bone at the orbito-temporal septum exhibits one to three fissures in the immature bone (for the accommodation of minute vessels and nerves), which by the closure determine the positions of new grooves. Now, the growth in the direction of the orbito-temporal septum is variable. The maxillary process may reach the sphenoid bone or it may terminate at the malar. If it attains the bone first named, the malar bone is excluded from the spheno-maxillary fissure. If it

does not so attain, the malar enters into the composition of the fissure. (See p. 11.)

The connection which exists between nutritive processes and grooves caused by the positions of blood vessels is considered on page 70. It becomes difficult at times to decide which is the most effective in inducing the position of sutures. For example: While the masseteric ridge answers in position to the intermalar suture, it also corresponds to the position of a vessel groove. The groove is commonly seen in the immature skull. It is, however, conspicuous in the skull of an adult idiot.[1]

In illustration of the fact that nutrition of bone is apt to be influenced by the position of sutures the following may be mentioned: Nodules of a size of a millimetre, sessile in form and of hard consistence, are occasionally seen on the frontal bone near the median line. They are to be attributed to localized hyperostoses in the neighborhood of the interfrontal suture.[2]

The frontal bone directly in advance of the coronal suture is often the seat of a convexity only secondary in height to the frontal eminence. It is especially well developed in Peruvian crania. A second eminence, more generally distributed, is seen on the same bone in the temporal fossa, directly below the temporal ridge.[3]

The coronal suture is deflected forward slightly as it is crossed by the temporal ridge. In 31 out of the 64 skulls of negroes examined the suture extended parallel to the ridge for about two centimetres before it crossed it. In no other skull, save in a Seminole Indian[4] and a Carib,[5] was a similar peculiarity noticed. It thus becomes a character which should be sought for in describing the cranium of the negro. (See Vertex, p. 53.)

The borders of muscular impressions, such as the temporal ridge is to the impression for the temporal muscle, may be said to modify

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[1] No. 1190, German, A. N. S.

[2] No. 1035, Apache; 742, Mandan; 647, and three Peruvians.

[3] This is well seen in 316, a young Malay; 1029, Fiji; and 44, Menominee.

[4] No. 708, Academy of Natural Sciences. [5] 692, *ibid.*

the bone itself, and may even lead to the separation of the bone in two parts. This is apparently the case in the instance of a double parietal bone as figured by Professor Turner in the skull of an Admiralty Islander.[1] The line of origin on the inner surface of the malar bone answers to the position of the suture in two instances of double malar bone which I have studied.[2] In four crania[3] traces of a suture were seen on the maxillary portion of the hard palate extending obliquely forward and outward at or near the maxillo-palatal junction. They may unite with the junction last named at the median line or lie a little to the ectal side.

The squamosal suture (parieto-temporal) ends posteriorly at the mastoid process somewhat abruptly. A process of the suture is apt to be directed upward and backward from the hinder part of this suture on the level of the temporal vein-groove. Although small, the process practically limits the squamosal region in this direction, since the curves which are continuous with the tuber of the parietal bone here begin. The slope from the side of the skull to the occiput is also announced.[4]

THE SUTURE BETWEEN THE INFRA-ORBITAL FORAMEN AND THE INTERIOR MARGIN OF THE ORBIT, INCLUDING VARIATIONS OF THE LATTER.

An interesting region for variation is seen in the inferior border of the orbit. The border may be said to lie below a curved line which is continued across the orbit along the upper limit of the zygoma. The bones which enter into the composition of the border are the malar and the superior maxilla.

The malar comprises the outer half, nearly, of the border. As a rule, the anterior limit reaches a point about 4^{mm} from the infra-orbital canal, but in place of this it may end over the canal, or may reach the ascending process of the maxilla.

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[1] The Challenger Rep. X, 57. [2] 1255, Ostrogoth; and 130, Chinese.

[3] Nos. 20, 60, 80, 136, 139, College of Physicians.

[4] See 1482 (A. N. S.), Peruvian, right side

The maxillary portion is divided into the part over the infra-orbital foramen and the part answering to the base of the ascending process of the maxilla.

The first of these divisions is exceedingly variable. The remains of the suture at the roof of the infra-orbital foramen, usually ending at the border, may extend to the malar.[1] The entire sutural arc of the orbital border may be depressed below the rest of the curve, and a minute spicule on the median side appears to indicate that fibrous tissue had bridged or occupied the interval caused by the depression.

Negroes frequently exhibit the above-named variety. The line of the suture over the foramen is often hyperostosed, so as to assume a rounded form which may be irregularly roughened. Such a variation is often found in large, heavy crania.[2] The ascending process of the maxilla entering into the composition of the border may be sharply ridged and abruptly raised above the planes of the floor of the orbit.[3]

In No. 1516, Malay, the infra-orbital suture does not extend to the inferior border of the orbit, but reaches the malar bone. A well-defined groove is seen on the inferior orbital border in 1450, Australian; 44, Menominee; and 739, Mandan.

In the same group, with the rugose suture over the infra-orbital foramen, may be placed the rather decided ledge-like hyperostosis which marks the maxilla directly above and in front of the palatal as it lies over the spheno-palatine foramen.

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[1] 1316, Malay (A. N. S.), aged eight years.

[2] Well illustrated in a skull of Lenapé (North American Indian), No. 40, A. N. S.

[3] The suture over the infra-orbital foramen is raised or rugose in many examples of crania. In this connection see 1451, 1262, Australian; 747, Minitari; 740, Mandan. The suture is often open. Examples are seen in Nos. 1300, 1342, Sandwich Islanders; Nos. 69, 708, 707, 733, and 726, Seminole; Nos. 951 and 955, Narragansett; Nos. 1227, 745, 1233, Blackfoot; 1322, Pottawatomie; and 739, Mandan.

NOTES ON SOME OF THE FORAMINA OF THE SKULL.

The foramina of the skull are chiefly of interest in exhibiting retentions of embryonic states. The most striking of these states are seen at the base of the skull, at the region of the union of the vomer with the sphenoid bone and the sphenoidal processes of the palatal bone and pterygoid process, as already seen[1] (page 23).

The foramina may be asymmetrical; the foramen ovale less so than the others. A second group of retention—variations is seen at the surface of the sphenoid bone, where it lies against the petrosal to form the petroso-sphenoidal suture. Along the lines of this suture are found the oval foramen, the spinous foramen, and the canalis innominata. The suture widens not infrequently at the outer end to form an opening, which may receive the name of the petroso-sphenoidal foramen. The oval, spinous, and petroso-sphenoidal foramina may be confluent, or the spinous and petroso-sphenoidal may alone unite, or the oval and the spinous. The canalis innominata[2] may be large or absent. In the skull up to the fourth year the spinous and petroso-sphenoidal openings are always united. I have often remarked that the spinous foramen may be entirely absent on one side.[3] In some lower animals, as is seen in the Virginian opossum, the foramina retain throughout life the type seen in this disposition to coalescence.

The development of the tympanic bone is peculiar, for instead of uniformly extending in all its proportions a large foramen is always seen on the bone at its inferior surface. The significance of the opening is unknown.

The foramen is very variable in form and position. As a rule, it recedes with age from the aperture of the meatus, so that in adult examples the retained foramen is almost always a centimetre or more from the outer free margin. Examples of the retention of the

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[1] For a good example see No. 924, negro.

[3] The foramina ovale are at times asymmetrical.

[3] No. 142, Marquesas (A. N. S.), furnishes an example.

foramen in adult life are by no means infrequent. In fourteen skulls of Esquimaux examined eight showed the tympanic foramen of defect. I have never seen the foramen in a Sandwich Island or Tahite cranium. Extended examinations might show variable percentage of occurrences in the different races. That the foramina are factors in the distribution of pus in peri-meatal abscesses there can be no doubt.

The oval foramina of the sphenoid bone are often unequal in size and of different shapes. The form may be so slightly changed from the circular that the term oval is scarcely applicable to it. This is often seen in Esquimaux crania. The rounded shape is frequently found associated with the short skull and the oval form with the long skull. When an asymmetry of the openings exists it is rational to entertain the opinion that the side of the skull which shows the greater elongation is also the side which will retain the most elliptical foramen.

If the base of the skull were perfectly symmetrical the line of the basio-cranial suture, produced outward to the right and left, should intersect the oval foramina at a fixed point; but, in fact, the intersection is variable. This is in part owing to the differences in the shapes of the openings, as already noted, and in part to the torsion of the anterior segment of the skull. (See page 18.)

The carotid canals may be asymmetrical. The left canal, when asymmetry is present, is ordinarily the smaller.[1]

The foramen lacerum medium may be entirely absent, as is the rule with the lower animals. The union of the apex of the petrosal element against the body of the sphenoid bone is more frequently seen in long, narrow skulls than in others, but may be seen independently of skull form.

The foramina on the side of the skull are the familiar mastoid and the alisphenoid foramina. The latter are infrequently present. They are the orifices of small diploic veins which come to the sur-

[1] For good examples see 1548, Swede; 914, negro (A. N. S.).

face, probably to unite with the deep temporal veins. The spheno-palatine foramina are relatively of large size in the skull of the young subject. In an adult Tchutchi skull[1] these foramina were 6^{mm} in diameter.

The foramina of the vertex are few in number. The parietal foramina may be larger than usual, or they may disappear and abrupt openings may occur through the outer plate so as to expose the diploe along the line of the temporal ridge. They are more common on the frontal portion of the crest than elsewhere.

The variations of the front of the skull pertain to the anterior lacerated foramina, the infra-orbital foramina, and the opening along the line of the frontal suture. The differences in the anterior lacerated foramina are chiefly those of symmetry. The infra-orbital foramina vary chiefly in the manner by which the fissures of the maxilla close and the extent of the forward growth of the malar bone. Foramina occasionally appear at the median line of the forehead, and are doubtless due to the partial failure of the two halves of the frontal bone entirely to unite.

The foramina which transmit important structure are commonly modified from fissures, and in reversion easily assume again the stage of the fissure. Since they so originate, it is easy to account for their presence near the margins of fissures (as is seen in the foramen ovale and foramen spinosum, near the fissure between the sphenoidal and petrosal elements). In like manner the parietal foramina appear at the side of the sagittal suture. Exceptions to this rule are seen in a small canal (occasionally present) which transmits a vein between the squamosal and parietal bones, and in a foramen in a Peruvian skull.[2]

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[1] No. 1030, A. N. S.

[2] No. 17, from San Mateo, which exhibits an opening between the frontal and parietal bones.

THE GROOVES, OR THE INFLUENCES EXERTED BY BLOOD-VESSELS IN DETERMINING THE FORM OF THE SKULL.

Inspection of the bones of the human subject shows that the surfaces are not infrequently marked by superficial grooves which appear to be the tracks of blood-vessels. Such markings are best seen in the long bones, which exhibit the usual appearances of chronic inflammation. Assuming that the impression made upon the bones are proportionate to the amount of increase of volume of the bone, and that the vessels remain fixed, a simple problem is presented by means of which the observer can determine the significance of blood-vessel tracks in other than in inflammatory conditions.

The vessel-grooves on the periphery.—The cranium yields a number of examples of these grooves. In the forehead, especially of specimens in which the forehead is rounded, numbers of deep, narrow grooves an inch or more in length are seen extending upward and backward from near the supra-orbital foramen or from the outer side of the frontal eminence and in line with the supra-orbital foramen or supra-orbital notch. In rare instances a simple small-groove lies near the frontal portion of the temporal ridge.[1] I have seen both the above-named grooves present in a child of nine months of age. They appear earlier than the grooves described in the succeeding paragraphs.

Good examples of the frontal vessel-grooves have been found in skulls of all nationalities. They are not uncommon in the negro, when the narrow, convex forehead appears to favor their appearance.[2]

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[1] See No. 760, Copt, for a good example and many negro crania.

[2] For example see: Nos. 905, 912, negro; No. 438, Ohio Indian; No. 1035, Apache; No. 87, Peruvian; No. 1024, Fiji; No. 1214, Hamilton, Ohio, Indian; No. 1043, Pawnee; Nos. 78, 44, 35, 1222, Menominee; Nos. 749, 650, Minitari; Nos. 744, 745, Blackfoot; No. 1057, Miami; Nos. 644, 742, Mandan; Nos. 39, 1333, 1233, unnamed.

It has been found in one side of the skull only, as seen in the skull of a Sandwich Islander.[1]

In a second skull of a Sandwich Islander (No. 695) the frontal grooves are absent, but a number of foramina perforating the outer plate of the bone are directed upward. It would appear that diploic veins had passed into the frontal veins, which had in their turn failed to make any impression upon the bone itself.

Many crania show a vertically placed groove, which is more or less arborescent, and rather shallow as compared to the frontal, lying upon the squamosal, a short distance above the external auditory meatus and reaching as far as the upper limit of the bone, or even crossing the parieto-squamosal suture and describing a curve upward and forward over the parietal bone, a short distance below the temporal crest. In a few examples the track originates in the parieto-squamosal when the squamosa itself is free.

The grooves are absent on surfaces from which muscles arise, as is seen on the occiput.[2] The squamosal groove is an apparent exception to this conclusion. May it be said that the temporal muscle makes but little traction at the region of the groove?

The region of the asterion is quite commonly the seat of numerous closely disposed grooves which are deep and sharply defined. It will be observed that in the above examples the grooves are deepest where the skull is thick, as on the convex frontal bone and in the massive region of the asterion, and most shallow when the bone is the thinnest, as over the squamosal; also that they may communicate with the diploic veins, as in the forehead, or even anastomose with an intra-cranial vein, as in the parieto-squamosal suture.[3]

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[1] No. 572.

[2] I have observed a branched depression of unknown significance above the nucha-mark in the skull of a Hindoo child four years of age.

[3] For good examples of squamosal vessel-grooves see the following: 542, Miami; 670, Chinese; 741, Mandan; 1043, Pawnee; 1283, 1051, Hottentot; 59, 987, 1283, 28, unnamed.

Linear grooves of doubtful origin on the periphery.—A number of grooves are seen on the superior maxilla as it enters into the composition of the outer wall of the orbit and of the boundaries of the spheno-maxillary sinus which closely resemble those caused by vessels. They are seen as fissures in the skull of the child and as linear depression in the skull of older subjects. Should they be accepted as vessel-grooves, the interesting question is raised: May not such irregular fissures as are here seen on the maxilla as it extends upward toward the orbital wall be caused by the presence of vessels, and may not the irregular sinuate edges on the margin of a growing bone of the flat class be generally associated with such modifying causes?

The malar bone occasionally exhibits a transverse linear groove upon the middle of the inner (temporal) surface. (See page 8.) It corresponds to the division between the masseteric and the temporal surfaces as seen in the child at three years, and to the line of the suture which so rarely divides the malar into two parts.

Vessel-grooves on the encranial surface.—Among the grooves on the endocranial surface of the parietal bone which are of undoubted influence, the form of the surrounding parts, is the conspicuously broad and deep depression which lies directly back of the coronal suture. The constriction so commonly seen in the periphery in this portion of the skull cannot be disassociated with the position of these vessels. The nutritive processes appear to be at first stimulated by the presence of this line of vessels, but after union with the frontal bone it remains stationary and permits the adjacent portion of the skull to rise above it. At the antero-inferior angle of the parietal bone the groove is converted into a canal and the inner layer of the bone notably thickened. In crania which exhibit a tendency to thickening of the vitreous plate the vessel-grooves are deep, sharply defined, and resemble the tracks made by insect-larvæ in old wood and in neglected books. The diploe is often exposed at the bottom of these grooves. Doubtless the diploic vessels freely unite with the vessels.

Vessel-grooves within the nasal chamber.—The nasal bone is often marked with a groove which extends the entire length of the surface within the nasal chamber and lies near the maxilla-nasal suture. A similar groove is often found on the ascending process of the maxilla near and parallel to the same suture.

The temporal ridge, as it is crossed by the coronal suture, is occasionally depressed, or the line of the ridge may be said to exhibit a fault at the point of section of the coronal. This arrangement is seen oftener in the skulls of negroes than those of other races.[1]

The temporal ridges divide the dome of the cranium (*i. e.*, the parts included in the sides and vertex of the brain case) into the natural divisions within which the characters of the minor details are distinctive. The vertex between the ridges is almost uniformly marked by more numerous diploic openings (aperturæ emissariæ). The vessel-grooves are absent. In some examples the striæ which radiate from the tubera medianward and backward are retained and distinguish the adult cranium.[2]

In narrow "ill-filled" skulls the temporal ridge may overlie the parietal tuber, as I have observed in a cranium of a convict, or greatly underlie it, as is seen in No. 77 of the College of Physicians collection.

Among the *processes of bone* which were noticed in the course of the examination may be mentioned the following:

A number of small but stout spines, each measuring a millimetre or two millimetres in length, which were appended to the frontal portion of the temporal ridge and directed downward; the spines

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[1] The following are the numbers of negro crania in A. N. S. showing this peculiarity: No. 912, to a marked degree; also 975, 1102, 920, 994, 1094, 918, 907, 902, 913.

The ridge is well seen in No. 1300, Sandwich Islander; 1064, German; 207, Puget Sound; 133, Cossack; 89, Adrian; 99, Armenian (the four last named are in the College of Physicians).

[2] The temporal ridges often limit the distribution of morbid processes and the changes due to old age. The diameter of the vertex measured between the two temporal ridges varies greatly in individuals. In tapeinocephalic and in all long, narrow crania the distance is smaller than in other types.

were slightly curved. They were undoubtedly developed in the direction of the vertical fibres of the temporal muscle.[1]

The pneumatic process of the occipital bone was met with in six[2] instances. In six of these the process was on the left side.

The paroccipital process may be bent inward and flattened,[3] and in one instance was found to articulate on the left side with the atlas.[4]

Regions of great density of bone structure.—The disposition for some parts of the cranium to show dense ivory-like thickenings is very noticeable. The causes which induce the vascular cancellous tissue to assume greater density with diminution of blood-vessel supply would be interesting to trace. Four localities are named for the occurrence of this change—1st, the petrous portion of the temporal bone; 2d, the inner or vitreous plate of the bones entering into the composition of the vertex;[5] 3d, the margins of the jugular foramen, notably the anterior; 4th, occasionally in the interior of sinuses, as seen in the maxillary and ethmoid sinuses.

The disposition to ivory-like density is often morbid (this probably includes the third and fourth groups as given above), and may even be present in the vitreous plate of the vertex. Scarcely a cranium can be found in our dissecting-rooms in which solid nodules are not found in some part of the interior of the calvarium, especially at the frontal portion on either side of the metopic line. Many individuals exhibit dense, white, low eminences of the general internal surface at the region of the bregma. They are lozenge-shaped and measure four to six centimetres in diameter.

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[1] See No. 1271, North American Indian; No. 742, Mandan; No. 963, negro.

[2] No. 1229, Upsarooka; 20, Bengalee; 78, 35, Menominee; 204, Chenook; 707, Seminole.

[3] See skull of Alaskan in museum of Princeton College, N. J.

[4] No. 706, German.

[5] This is seen to be the case to a remarkable degree in the skull of an Esquimaux (No. 1554) in the Army Medical Museum.

The formations as they exist in the sinuses are nodular and apparently lead up by easy grades to the ivory-exostoses recognized by the physician as distinctly pathological.[1]

ON THE MANNER OF TAKING A CLINICAL NOTE OF THE CRANIUM.

It will be remembered that one of the objects in view in undertaking the study which is now completed was to ascertain the degree of correlation, if any existed, which could be traced between structural peculiarities in the region of the mouth, of the nasal chamber, of the naso-pharynx, and other portions of the cranium. A laryngologist has an opportunity of taking measurements in the mouth, throat, and adjacent parts which is withheld from the general observer. It goes without saying that for general craniological purposes it will be impossible for measurements within the nose and throat to be made. The contrast between any of these regions in patients is so great it was suggested that a series of observations might be of some importance. The following is an example of the kind of measurements which can be secured in the living subject:

In a woman aged twenty-six, suffering from chronic nasal catarrh, it was found that the distance from the axis tubercle (which is very plainly seen when the velum is lifted) to the cutting edge of the right superior incisor at the median line was $8^{c}\ 1^{mm}$; the distance from the vault of the naso-pharynx to the lower border of the anterior nasal aperture, $7^{c}\ 7^{mm}$; the distance from the glabella to the post-remal prominence, 18^{c}; the circumference of the head taken on the line of the parietal tubera was 54^{c}.

It will be noted in the above that the axo-incisorial measurement ends at the edge of the incisor. It is acknowledged that this is undesirable, since the inclination of the teeth is a variable quantity. Indeed, any point about the dental arches is subject to the same criticism, but does not apply with any greater force in this

[1] For a general essay on hyperostosis in man and animals see Gervais Journal de Zoologie, 1875, p. 421.

measurement than to other craniological lines into which the teeth may enter. It is also difficult to determine the anterior limit of the line extending from the vault of the naso-pharynx to the anterior nasal aperture (pharyngo-narial line), for the reason that the depth of the soft parts covering the nasal aperture is variable; but such an ending is not more inconstant than that of the anterior nasal spine, which is relied upon generally as a point from which measurements may be taken. The individual who furnished these measurements had a high basi-cranial angle. Indeed, it was impossible to inspect the vault of the pharynx of this subject with satisfaction, since the anterior position of the body of the axis conjoining with the acute angle of the vault made it difficult to depress the mirror so as to obtain a satisfactory image of the space.

In addition to the above the following observations were made: The lower jaw with marked outward deflection of the left angle; the antegonial depression marked; the mentum high; the bregmal depression marked; the post-coronal depression absent; the deep depression in the region of the obelion present; the para-tuberal and meso-lambdoidal eminences well developed.

It is submitted that a series of measurements made on this simple scheme might yield interesting results. The material I have collected is insufficient for study at this time.

The study of the skull in children often throws light upon the nature of morbid processes. In this connection I have special reference to minor changes, some of them, indeed, so slight as to escape notice if the standards of comparison be those which the observer is usually expected to entertain—such, for example, if the gross changes recognized as cretinic, hydrocephalic, etc., be selected as basis for study. I allude more particularly to such appearances as would follow a delayed disappearance of the anterior fontanel, the result of which is a saucer-shaped concavity at the anterior portion of the vertex. Another peculiarity is an unduly marked convexity on either side of the sagittal depression. This need not be sufficient to constitute the natiform skull (see page 56), but to suggest

with this variety a common interest, namely, a disposition to premature disappearance of the sagittal suture associated with retarded ossification of the parietals, as a result of which they become unduly convex.

The third variety is confined to the anterior cranial segment—*i. e.*, a phase of deformation in which all the peculiarities are in the frontal bone or in the bones of the face. The frontal eminences may be too near one another; the metopic suture may be here and there carinated; the muscular ridge at the anterior border of the temporal fossa may be unduly prominent; the inferior border of the orbit at the region of the union of the malar bone and the line over the infra-orbital canal may be roughened, etc. Many of these peculiarities are associated with errors in the shape of the mouth and the nasal chambers, and easily come within the range of anatomical studies which are suggested by clinical observations on catarrhal diseases of the respiratory mucous surfaces.

ADDENDUM.—The number of skulls stated on 8th line from bottom page 29 refers to others than those in the collection of the Academy of National Sciences.

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SMITHSONIAN MISCELLANEOUS COLLECTIONS.

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INDEX

TO THE

LITERATURE

OF

THERMODYNAMICS.

BY

ALFRED TUCKERMAN, PH. D.

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PREFACE.

This is similar to my Index to the Literature of the Spectroscope, published in the Miscellaneous Collections of the Smithsonian Institution, vol. XXXII, for 1888.

All of the titles are given in full in the author-index; but in the subject-index, to save useless repetition, only the authors and the places where their works are to be found are given—except in the case of books.

Applications of thermodynamics have been found, and kept, to the number of more than double the titles here given. They were omitted so as not to overload the index with matter of little or no use. But, of course, no titles have been left out which belong to the applications named in the table of contents.

The work has been brought down to the middle of the year 1889.

ALFRED TUCKERMAN.

NEWPORT, R. I.,
July, 1890.

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THIS WORK

"THE CORRECTION OF SEXTANTS for Errors of Eccentricity and Graduation."

BY

JOSEPH A. ROGERS,

FORMS

ARTICLE III

OF

VOLUME XXXIV

SMITHSONIAN MISCELLANEOUS COLLECTIONS.

SMITHSONIAN MISCELLANEOUS COLLECTIONS.

— 764 —

THE CORRECTION OF SEXTANTS

FOR ERRORS OF

ECCENTRICITY AND GRADUATION.

BY

JOSEPH A. ROGERS.

WASHINGTON:

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1890.

ADVERTISEMENT.

Improvement in the construction of instruments of precision is necessarily preceded by the development of means for detecting the errors of those already in use. The sextant, though primarily an instrument for the traveller, will, when carefully made and handled, give results of a remarkable degree of precision ; and it is a matter of great importance that even for the ordinary purposes of navigation every precaution should be taken to free it from all classes of instrumental error. Inaccuracies of mechanical construction, though very minute from the artisan's standard, are greater in effect than even experienced navigators sometimes realize.

An apparatus for investigating and determining such errors of the sextant, devised by Mr. Rogers (the theory and use of which is described in the present treatise), has been extensively employed by the officers of the United States Navy in testing sextants before their issue to the ships of the naval service.

It may be added that, in the constant re-action between advances in science and art, the reciprocal benefits of which should always be freely acknowledged, the skill of the instrument-maker (especially in the accuracy of graduation of circular measures) is still behind the delicate requirements of modern investigation ; and it is confidently believed that methods of detecting minute inaccuracies will result, as in the past, in stimulating the artisan to further refinements in instrumental construction.

S. P. Langley,
Secretary of Smithsonian Institution.

Washington, *December*, 1890.

THE CORRECTION OF SEXTANTS FOR ERRORS

OF

ECCENTRICITY AND GRADUATION.

BY JOSEPH A. ROGERS.

The sextant of reflection, in consequence of its portability and the ease and rapidity whith which it affords results of considerable accuracy in the hands of a capable observer, is so serviceable in navigation, surveying, and in determining latitude and time for astronomical purposes, that no reasonable proposition to increase the precision of observations made with an instrument of such general usefulness can be devoid of interest. One of the most obvious sources of avoidable error is a non-coincidence of the axis of rotation of the index-bar with the axis of the graduated arc. The necessary lightness of all the parts also entails a certain liability to irregularities in the rotating motion, due to flexure of the slender axis and its connections, or to wabbling when the axis is not properly supported at both ends. With good workmanship, and proper attention to the condition of the instrument when used, these irregularities may be kept within narrow limits, but there is reason to believe that they are sometimes greater than is commonly suspected. When the arc is extended into a complete circle with opposite indices, the effect of eccentricity is eliminated, and that of irregular rotation is much diminished, if not entirely compensated. These advantages of the circle have been frequently urged, and are indisputable, but they cannot be secured without some sacrifice of other desirable qualities. An instrument which is designed to be supported in the hand when used, must be light and compact; with equal bulk and weight the limb of the sextant has a greater radius than that of the circle, and consequently permits a closer reading of the angles measured, while the uncompensated effects of irregular rotation in the former may be lessened by a somewhat more massive and substantial construction of the axis and parts supporting it; moreover, the sextant is the less costly of the two, and is, perhaps, in shape, rather better adapted to convenience in manipulation. The settlement of these conflicting claims by experience has been upon the whole unfavorable to the circle, for its employment is

now quite exceptional, and there is apparently no reason to anticipate a reversal of this verdict. It is very desirable, therefore, to possess some efficient means of correcting the errors which are peculiar to the sextant.

The eccentric corrections of a sextant may be deduced from measurements made with it of three or more known angular distances. A trustworthy determination requires several angles, of such magnitudes that no part of the arc shall be very far from one of the readings obtained. The known angles may be the apparent distances between stars, computed from the positions given in the catalogues, or the angles subtended by well-marked terrestrial objects, measured by a theodolite or otherwise. One of these methods will usually suffice for an observer who wishes to obtain corrections for his own instrument, but neither of them can be regarded as generally available when large numbers of sextants are to be examined, for besides the laborious computation involved, stars are seen only at night and in clear weather, while a spot where suitable terrestrial objects at proper angles and sufficient distances are always visible is sometimes difficult to find.

During the latter part of the civil war the writer, then an assistant in the U. S. Naval Observatory, was required to superintend the repairs of sextants returned in unserviceable condition from the various fleets. Some of these instruments bore marks of violence, and it was felt to be extremely desirable that none of them should be reissued until their integrity had been verified by some adequate test. A plan for making such tests was accordingly formed, but circumstances did not permit it to be carried out at that time. Not long afterward, however, while engaged in a similar occupation at the Hydrographic Office, the apparatus about to be described was constructed. In the meantime a description was published* of Mr. T. Cooke's apparatus, which had recently been set up at the Kew Observatory for the same purpose. Disregarding minor details, the apparatus at Kew consists essentially of a series of horizontal collimators equidistant from, and directed toward, a central point where the sextant under examination is supported in a horizontal position by a small table. The collimators are firmly secured to a curved pier of substantial masonry; the angles which their axes make with each other are measured by a theodolite temporarily placed on the central table for that purpose, and are presumed to remain nearly constant. A remeasurement of these angles with any sextant affords a comparison between its graduation and that of the theodolite, at points depending upon the number and disposition of the collimators, which at Kew indicate five axial directions, including angles approximating 15°, 30°, 45°, 60°, 75°, 90°, 105°, and 120°.

* Balfour Stewart, Proc. Roy. Soc., London, 1868, XVI, 2.

The apparatus constructed at the Hydrographic Office is represented by Fig. 1, in which A is a stout walnut plank $4\frac{1}{2}$ feet long, 2 feet wide, and nearly 2 inches thick, supporting the other parts, and mounted upon legs at a convenient height above the floor. The horizontal collimator B, having an aperture of 2 inches, and a focal length of 19 inches, is firmly secured to the base-board at an elevation nearly equal to that of the telescope of a sextant in the position it occupies while undergoing examination. In the focal plane is a thin metallic plate with a minute circular hole drilled through it, which appears as a sharply defined bright disc a little less than $2'$ in diameter, when illuminated from behind and

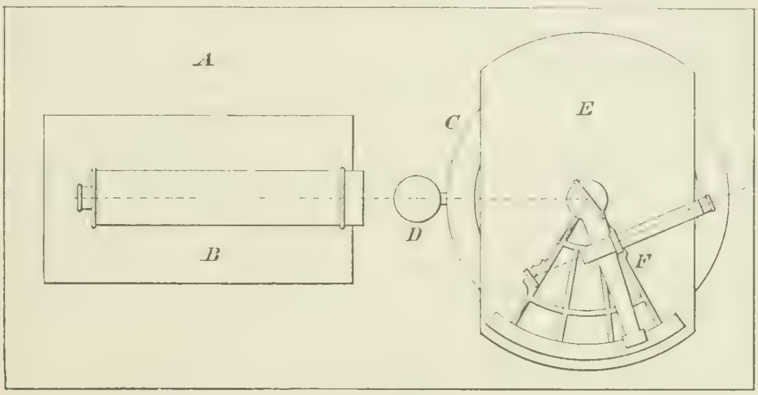

Fig. 1.

viewed with a telescope. At C is a graduated circle revolving upon a vertical axis, and supported by a tripod base after the fashion of a theodolite. One foot of the tripod, resting on its foot-screw, is directed toward the collimator as shown at D. The axis of the circle must be kept perpendicular to the line of collimation; this adjustment is made by means of the foot-screw D, and has usually been tested by placing an alidade with plane sights on the face of the circle, as extreme precision is not essential; but a reversible collimator, having collars of equal size supported by a stand laid on the table, would be a better device. The circle is divided to $5'$, and reads to $3''$ by four equidistant verniers; its diameter is about $11\frac{1}{2}$ inches. Three arms radiate from the top of the vertical axis and bear leveling screws at their extremities supporting the brass table E, which has adjustable clamps upon its upper surface to receive the legs of a sextant, and prevent them from moving in any direction. A small circular mark in the center of the table indicates the location of the vertical axis. A sextant, F, laid upon the table, with the axes of the graduated arc and circle approximately coincident, can be secured in that position by the clamps, and the

plane of the arc can then be brought parallel to that of the circle by means of the table-screws. The arc now rotates in its own plane when the table revolves, and the extent of any such motion can be accurately measured by the verniers of the circle. The index-bar being in any position desired, with the direct view through the horizon-glass cut off by interposing the colored screens, let the table be turned until the reflected image of the collimator-mark is bisected by one of the telescope wires, and let both sextant and circle be read. Now move the index-bar into any other position, bring the mark back upon the same wire by turning the table in the opposite direction, and again read the sextant and circle. In this process the angle of rotation has obviously been measured by both instruments, and the difference between the two results is, therefore, the correction of the sextant for that angle, if the readings of the circle have been duly corrected for their own errors. The circle referred to has never been thoroughly investigated, but a preliminary examination showed that its errors are probably small; a double axis permits any part of the graduation to be used, and the effect of eccentricity is, of course, always eliminated. Except the circle and its immediate appendages, which were taken from a theodolite of exquisite workmanship by Gambey, of Paris, this apparatus, intended rather to illustrate a proposed system of examination than for service, was roughly made, but it proved to be efficient, and was sent to the Naval Observatory some years afterward, where it is, I believe, still occasionally used.

As compared with Mr. Cooke's system of collimators the apparatus just described presents some important advantages. Direct reference to the circle which serves as a standard is preferable to a comparison depending upon the permanence of intermediate arrangements, even such as are believed to be of a stable character. Instead of being restricted to a few angles, which it must be inconvenient, at least, to change, any part of the graduation may be tested at pleasure; the position of every line on the arc can be verified if that is desired; and the portability of an apparatus which can be moved from one room to another by a couple of persons is a minor point in its favor. The collimators, on the other hand, conveniently dispense with all readings except those of the sextant during the examination.

Before commencing the examination of a sextant, the principal adjustments must be tested, and rectified if necessary; both mirrors must be truly parallel to the axis, and the line of collimation parallel to the plane of the limb; the state of the index correction is immaterial. The telescope of the sextant may be used when of sufficient power, but can advantageously be replaced by a special one, magnifying at least ten diameters, with a fine wire in the middle of the field crossed exactly at right angles by two other wires which inclose a central space of convenient width.

These conditions having been satisfactorily established, the course of procedure is as follows: Lay the sextant on the table of the apparatus with the central leg, or axis guard, nearly in the middle of the small circular mark, bring the clamps in opposing directions against the three legs, and secure them with just sufficient pressure to prevent any subsequent displacement. Cover the horizon-glass with all its colored screens, bring the reflected image of the collimator-mark into the field of the telescope, verify the focus, and see that the wire to be used in bisection is perpendicular to the path described by the mark when the index-bar is moved. In any position of the index-bar the mark can now be brought into the field by turning the table, and vice versa within the range of the sextant. By means of the table-screws make the axis of the sextant parallel to (and nearly coincident with) that of the circle, which will be the case when in any two, and consequently in all, positions of the index-bar the image passes through the center of the field as the table rotates. This is most readily accomplished by first turning either one of a pair of the screws which has been brought parallel to the axis of the collimator, and afterward turning the third screw when the same pair has been placed at right angles to that axis. If the index-mirror is not parallel to the axis of the sextant, the image cannot be made to pass through the center of the field in more than two positions of the index-bar, for the rotation of the axis causes that part of the ray passing through the telescope axially which lies beyond the mirror, to describe, not a plane, but a conical surface, only two elements of which can be brought parallel to the plane of the circle. It is advisable, therefore, before proceeding further, to see that the image passes through the field centrally when the index-bar is near the middle, and also when near each end, of its range.

The examination has usually been made by first setting the sextant at 0°, bisecting the collimator mark with the vertical wire by moving the tangent screw of the circle, and recording the reading of one pair of opposite verniers. After repeating this operation for each line of the graduation included in the inspection, a final reading is made at 0°, which ought to agree closely with the first one unless some displacement has intervened. With ordinary care this accident rarely happens, and if it does occur, its location should be revealed by a break in the series of readings.

The arc of a sextant, being the result of a fallible human effort to materialize a geometrical conception, must be regarded as more or less imperfect in every detail. There is everywhere, however, a tolerably close approximation to a certain supposititious graduation absolutely free from error, which may be called the mean arc, so situated that the algebraic sum of all the distances between corresponding lines of the actual and mean arcs is equal to zero. Every reading of the sextant will, therefore, require a correction consisting of two parts—one due to the eccentric posi-

tion of the axis, and the other a local correction equal to the difference between the actual reading and the corresponding reading on the mean arc. In Fig. 2 let $G\ H$ represent the mean arc of a sextant of which I is the center and G the zero of graduation, the axis of the index-mirror being at K. If the index bar is in the position $K\ H$ the reading of the vernier, γ', will be twice the angle $G\ I\ H$, whereas the true reading, γ, is

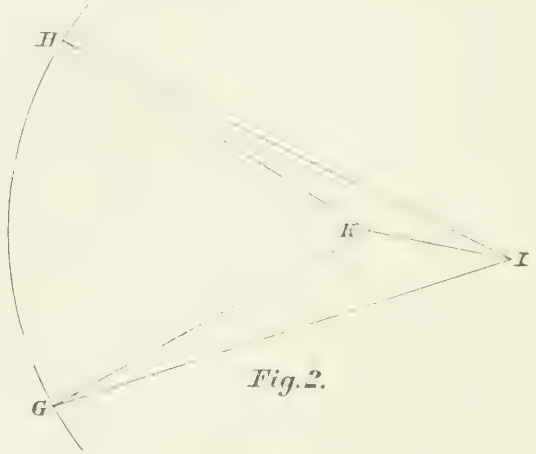

Fig. 2.

twice the angle $G\ K\ H$ by which the index is removed from 0°. Let $G\ I\ K = \varepsilon$, $I\ K = e$, and $K\ G = K\ H = L$. Now $G\ K\ H - G\ I\ H = K\ H\ I + K\ G\ I$, and the correction sought is therefore:

$$\gamma - \gamma' = 2\ K\ H\ I + 2\ K\ G\ I.$$

But $\sin K\ H\ I = \dfrac{e \sin\left(\frac{1}{2}\gamma' - \varepsilon\right)}{L} = \dfrac{e \sin \frac{1}{2}\gamma' \cos\varepsilon - e\cos\frac{1}{2}\gamma' \sin\varepsilon}{L}$, or since $K\ H\ I$ is so small as to be sensibly equal to $\dfrac{\sin K\ H\ I}{\sin 1''}$,

$$K\ H\ I = \frac{e \sin \frac{1}{2}\gamma' \cos\varepsilon - e\cos\frac{1}{2}\gamma' \sin\varepsilon}{L \sin 1''}.$$

Also, $\sin K\ G\ I = \dfrac{e \sin \varepsilon}{L}$, and

$$K\ G\ I = \frac{e \sin\varepsilon}{L \sin 1''}.$$

The substitution of these values gives:

$$\gamma - \gamma' = \frac{2\ e\cos\varepsilon}{L \sin 1''} \sin\tfrac{1}{2}\gamma' + \frac{2\ e \sin\varepsilon}{L\sin 1''}\left(1 - \cos\tfrac{1}{2}\gamma'\right),$$

and by making

$$\frac{2\ e\cos\varepsilon}{L\sin 1''} = \mathrm{A}, \qquad \frac{2\ e\sin\varepsilon}{L\sin 1''} = \mathrm{B} \tag{1}$$

the correction of any angle γ' measured from mean 0° becomes:

$$\gamma - \gamma' = A \sin\tfrac{1}{2}\gamma' + B\left(1 - \cos\tfrac{1}{2}\gamma'\right) \tag{2}$$

If we had access to the mean arc, equation (2) would enable us to find the correction for any angle whatever by making three comparisons with the standard circle, the index being set successively at 0°, and at any two other points. For the differences between the reading of the circle with the index at 0° and each of the other two readings would furnish two values of γ, in which the error of observation could be diminished by repetition to any extent desired. By substituting these values of γ and the corresponding ones of γ' in (2), two equations would be obtained determining A and B, which substituted in the same formula (2) would afford an equation giving the correction $\gamma - \gamma'$ for any value of γ'. From the definition of the mean arc it follows that the same result would be obtained from readings made with the index set successively upon every line of the actual graduation. Let R be the circle reading corresponding to any setting S of the index, and Z an assumed value of the reading when the index is at 0° of the mean arc, the true reading being $Z + X$, in which X is unknown. Then, disregarding in each case, for the reason just given, the deviation of the setting from the corresponding position on the mean arc, $\gamma' = S$, and $\gamma = R - (Z + X)$.*

Substituting these values in (2), and making

$$R - S - Z = D, \tag{3}$$

each observation will furnish an equation of the form:

$$A \sin \tfrac{1}{2} S + B (1 - \cos \tfrac{1}{2} S) + X = D. \tag{4}$$

If M is the number of observations, the normal equations will be:

$$\left.\begin{aligned}
&A\,[\sin^2 \tfrac{1}{2} S] + B\,[(1 - \cos \tfrac{1}{2} S) \sin \tfrac{1}{2} S] + X[\sin \tfrac{1}{2} S] - [D \sin \tfrac{1}{2} S] = 0\\
&A\,[\sin \tfrac{1}{2} S (1 - \cos \tfrac{1}{2} S)] + B\,[(1 - \cos \tfrac{1}{2} S)^2] + X[1 - \cos \tfrac{1}{2} S]\\
&\qquad\qquad - [D\,(1 - \cos \tfrac{1}{2} S)] = 0\\
&A\,[\sin \tfrac{1}{2} S] + B\,[1 - \cos \tfrac{1}{2} S] + M X - [D] = 0
\end{aligned}\right\} \tag{5}$$

After substituting the numerical values of the known quantities in these equations, and finding the values of A, B, and X, the correction for eccentricity of any observed reading will be given by (2). From (4) D may be obtained for any value of S, and the difference between this computed quantity and the observed value of D, for each comparison, is the local correction of the graduation, which, however, includes an unknown, and perhaps relatively large, error of observation.

An examination like that just described, embracing every line of the graduation, and repeated until the effect of errors of observation is sufficiently diminished, would afford a complete knowledge of the condition and capabilities of the instrument. For the corrections due to the position of the axis having been obtained, the local correction for each line would

* The readings of the circle are supposed to increase as the angles indicated by the sextant increase, which is actually the case in the apparatus referred to.

be the mean of the values given by the different series of comparisons, while the probable errors of the corrections, and of observations made with the sextant in question, could be deduced from the final residuals. But such a process, or even one involving only a single reading upon every line of the graduation, is far too tedious and burdensome to be practicable. We must be content in most cases with a comparatively brief and imperfect investigation, having for its object the best result that can be derived from a moderate expenditure of time and labor. With this end in view the examination must be limited to a few points equally distributed over the arc, but sufficiently numerous to warrant the assumption that they collectively represent the mean arc accurately enough for any kind of observation in which the sextant will be employed. In this way, although the attempt to secure an exact correction for every reading is abandoned, errors which similarly affect considerable portions of the arc may be corrected, and the existence of large uncorrected errors can generally be detected. The formation and solution of the normal equations usually entail a rather laborious computation, but this can be greatly abridged by a general solution, and by other convenient devices, if the examination is always made upon a uniform system of comparisons at certain invariable distances from each other. In what follows one system of this kind is presented in detail, as a type of similar systems comprising a greater or less number of comparisons.

With reference to the nature of the service expected of them, sextants may be divided into two classes, assigning to the first class instruments used in making observations for latitude and time with the artificial horizon, measuring the principal angles of surveys, etc., and to the second class those employed in the ordinary routine of navigation, and other operations of a similar grade. All the corrections of a sextant of the first class should be determined with as much precision as the capacity of such an instrument warrants, while for those of the second class it is only necessary to insure the absence of errors exceeding certain limits. In considering this subject with reference to the wants of the naval service, it was thought to be desirable that no part of the arc should be more distant in either direction from one of the points examined than the space covered by the vernier. With the usual division of the limb to 10′, reading by the vernier to 10″, this condition requires an examination at points not more than ten degrees apart. It was decided, therefore, to make circle readings with the index set successively at 0°, 10°, 20°, etc., to and including 130°. At first a comparison was also made at 140°, when the range of the sextant extended so far, but after some experience the practice was discontinued, for the definition of the collimator-mark is frequently so much impaired by extreme obliquity of the index-mirror

as to render this observation of doubtful utility. A single series of such comparisons furnishes the eccentric correction with sufficient precision for a sextant of the second class, while the residuals, containing the errors of both graduation and observation, afford a trustworthy indication of the performance to be expected of the instrument under very favorable circumstances. It was proposed to subject sextants of the first class to an examination comprising several similar series of comparisons, made with different portions of the circle, the number depending somewhat upon the circumstances of each case. These repetitions yield not only improved values of the corrections for eccentricity, but also a means of separating the local errors of graduation from the errors of observation.

The normal equations obtained by making $M = 14$, and S successively 0°, 10°, 20°, etc., 130°, in (5) are:

$$\left.\begin{array}{l} 4.72172\,A + 2.17946\,B + 7.06526\,X - [D \sin \tfrac{1}{2} S] = 0, \\ 2.17946\,A + 1.09780\,B + 2.90976\,X - [D\,(1 - \cos \tfrac{1}{2} S)] = 0, \\ 7.06526\,A + 2.90976\,B + 14\,X - [D] = 0, \end{array}\right\} \quad (6)$$

from which are deduced:

$$\left.\begin{array}{l} A = -1.5671\,[D] + 7.6467\,[D \sin \tfrac{1}{2} S] - 11.0275\,[D(1 - \cos \tfrac{1}{2} S)], \\ B = 1.8384\,[D] - 11.0275\,[D \sin \tfrac{1}{2} S] + 17.9311\,[D(1 - \cos \tfrac{1}{2} S)], \\ X = 0.4802\,[D] - 1.5671\,[D \sin \tfrac{1}{2} S] + 1.8384\,[D(1 - \cos \tfrac{1}{2} S)], \end{array}\right\} \quad (7)$$

and also:

$$X = 0.0714\,[D] - 0.5047\,A - 0.2078\,B. \quad (8)$$

From each observed value of D let the corresponding value computed by (4) be subtracted; the remainder, which may be designated $O - C$, is the sum of an observed local correction of the graduation, and an error of observation. Any single observation made under circumstances as favorable as those attending the examination, and corrected for eccentricity only, will, therefore, contain an error the most probable estimate of which is:

$$0.6745 \sqrt{\frac{[(O - C)^2]}{14 - 3}} = \sqrt{0.04136\,[(O - C)^2]}, \quad (9)$$

if the supposition is made that the errors of graduation are either small enough to be neglected, or else, like the errors of observation, devoid of any systematic arrangement. This assumption cannot always be absolutely correct, but no other is eligible when only one series of comparisons has been made.

The following example (Table 1) exhibits a convenient form of record and computation, requiring no tables except Crelle's Rechentafeln, and those contained in these pages. The entire calculation is given here. At

TABLE I.

S.	Verniers. I.	Verniers. II.	R.	D.	$D \sin \frac{1}{2} S$.	$D(1-\cos \frac{1}{2} S)$.	$A \sin \frac{1}{2} S$.	$B(1-\cos \frac{1}{2} S)$	Ecc. cor.	Ecc. cor. + X.	O — C.	(O — C)²	
°	″	″	° ′ ″	″	″	″	″	″	″	″	″		
0	57	57	0 0 56	+ 26	.00	.00	0.0	0.0	.0	31	— 5	25	Z = 0° 0′ 30″
10	57	63	10 0 60	30	+ 2.61	+ .12	— 3.5	— 0.1	— 3.6	28	+ 2	4	″
20	54	57	20 0 56	26	4.52	.39	7.0	0.5	7.5	24	2	4	— 101.9
30	51	57	30 0 54	24	6.22	.82	10.4	1.1	11.5	20	+ 4	16	— 278.3
40	36	15	40 0 40	10	3.42	.60	13.7	1.9	15.6	16	— 6	36	+ 340.1
50	45	45	50 0 45	15	6.34	1.41	17.0	3.0	20.0	12	— 3	9	— 40.1 = A
60	36	12	60 0 39	9	4.50	1.21	20.0	4.3	24.3	7	2	1	″
70	36	39	70 0 38	8	4.59	1.45	23.0	5.8	28.8	+ 3	5	25	+ 119.5
80	30	33	80 0 32	+ 2	+ 1.29	+ .47	25.8	7.5	33.3	— 2	4	16	+ 401.4
90	24	27	90 0 26	— 4	— 2.83	— 1.17	28.4	9.4	37.8	6	+ 2	4	— 553.0
100	3	6	100 0 4	26	19.92	9.28	30.7	11.5	42.2	11	— 15	225	— 32.1 = B
110	9	15	110 0 12	18	14.74	7.67	32.8	13.7	46.5	15	3	9	″
120	0	3	120 0 2	28	24.25	14.00	34.7	16.0	50.7	19	— 9	81	+ 31.2
130	18	24	130 0 21	— 9	— 8.15	— 5.19	36.3	18.5	54.8	— 23	+ 11	196	+ 57.0
140							— 37.7	— 21.1	— 58.8				— 56.7
0	54	27											
				+ 150	+ 33.49	+ 6.47					+ 38	654	31.5 = X
				— 85	— 69.89	— 37.31					— 38	27.05	
												± 5.2	
				+ 65	— 36.40	— 30.84							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)

the time of examination this sextant had just been repaired after its return from sea service.

The first column contains the settings, S, of the sextant, and the fourth gives R, the corresponding readings of the circle, the seconds being the mean of those read from the verniers, and recorded in the two preceding columns. The reading at 0° is the mean of those at the beginning and end of the examination. As the labor of computation is lessened by numerically diminishing the values of D, especially those belonging to the larger angles, Z should be either equal to the circle reading when $S = 0°$, or so chosen as to differ a few seconds therefrom, in the direction, and to an extent, indicated by the subsequent values of R. The differences D in the fifth column are found by (3). Table II is serviceable in filling out

TABLE II.

S.	Sin $\frac{1}{2}$ S.	1 — cos $\frac{1}{2}$ S.
°		
0	.000	.000
10	.087	.004
20	.174	.015
30	.259	.034
40	.342	.060
50	.423	.094
60	.500	.134
70	.574	.181
80	.643	.234
90	.707	.293
100	.766	.357
110	.819	.426
120	.866	.500
130	.906	.577
140	.940	.658
150	.966	.741

the sixth and seventh columns; it should be copied upon a slip of paper in lines spaced like those of the record, and laid upon the latter beside the fifth column, so that each value of D may closely follow its two coefficients. By opening the Rechentafeln at D, the two products can be instantly taken out and entered in the same line. The quantities in each of these three columns are next added, and the respective amounts written underneath.

No material error can be introduced by retaining only two places of decimals in these products. At first view, indeed, even the second place might seem to be superfluous, since D itself is frequently several seconds in error, but in (7) the sums of the two sets of products have coefficients with opposite signs, and the effect of an alteration in one of these sums is,

therefore, greater than that of a similar change affecting them both in the same direction. Substituting in (2) the values of A and B from (7):

$$\gamma - \gamma' = \left(-1.5671 \sin \tfrac{1}{2}\gamma' + 1.8384\,(1 - \cos \tfrac{1}{2}\gamma')\right)\left[D\right] \qquad (10)$$

$$+ \left(7.6467 \sin \tfrac{1}{2}\gamma' - 11.0275\,(1 - \cos \tfrac{1}{2}\gamma')\right)\left[D \sin \tfrac{1}{2} S\right]$$

$$+ \left(-11.0275 \sin \tfrac{1}{2}\gamma' + 17.9311\,(1 - \cos \tfrac{1}{2}\gamma')\right)\left[D(1-\cos\tfrac{1}{2}S)\right].$$

The change in the computed correction for eccentricity resulting from any given variation in $[D \sin \frac{1}{2} S]$ will be greatest when the coefficient of the latter in (10) is a maximum—that is, when:

$$7.6467 \cos. \tfrac{1}{2}\gamma' - 11.0275 \sin \tfrac{1}{2}\gamma' = 0,$$

$$\tan \tfrac{1}{2}\gamma' = \frac{7.6467}{11.0275}, \text{ and } \gamma' = 69^\circ\, 29',$$

the coefficient itself being then $+ 2.39$. By similar means it is found that the coefficient of $[D\ (1 - \cos \frac{1}{2} S)]$ attains the numerical maximum $- 3.12$ when $\gamma' = 63^\circ\, 11'$. Both coefficients reduce to 0 for $\gamma' = 0$. If the twenty-six products are correct to the second decimal place, the limit of this error in the eccentric correction is, therefore, 0 when $\gamma' = 0$, and greatest when γ' is somewhere between 63° and 69°, but everywhere less than $0.''005 \times 13 \times (2.39 + 3.12) = 0.''36$. It will be shown a little farther on that the probable error, due to errors of observation, in the eccentric correction derived from a single series of comparisons, is 0 when $\gamma' = 0$, $0.42\ t$ when $\gamma' = 20^\circ$, and still greater for all larger values of γ', t being the probable error of a single comparison. As t can seldom be much less than 2,″ this probable error is greater than the maximum error in question. It is also apparent that to increase the possible error tenfold, by retaining only one decimal digit in the products, would be unsafe.

The constants A, B, and X are computed in the last column of Table I. Each formula of (7) contains three terms which may be obtained from Table III without any greater inconvenience than that of taking out the tabular products for each digit of the argument separately and adding them together, but the table should be extended, if frequently used. The headings of the columns refer to that argument which is in the same horizontal line, *e. g.*, the third column contains both the term of B having $[D \sin \frac{1}{2} S]$ for its argument, and the term of A for which $[D\ (1 - \cos \frac{1}{2} S)]$ is the argument. The upper sign is to be applied when the argument is positive, the lower if negative. These terms

can also be taken from the Rechentafeln, by retaining only the first decimal place of sums containing more than three digits, which involves a maximum error ten-thirteenths of that referred to in connection with the products in columns 6 and 7 of Table I, and otherwise subject to the

TABLE III.

[D]				A	B	X	
[$D \sin \frac{1}{2} S$]	A	B		X			
[$D(1 - \cos \frac{1}{2} S)$]		A	B		X		
$\pm$	$\pm$	$\mp$	$\pm$	$\mp$	$\pm$	$\pm$	
$''$	$''$	$''$	$''$	$''$	$''$	$''$	
1	7.647	11.027	17.931	1.567	1.838	.480	1
2	15.293	22.055	35.862	3.134	3.677	.960	2
3	22.940	33.082	53.793	4.701	5.515	1.441	3
4	30.587	44 110	71.724	6.268	7.353	1.921	4
5	38.234	55.137	89.656	7.835	9.192	2.401	5
6	45.880	66.165	107.587	9.402	11.030	2.881	6
7	53.527	77.192	125.518	10.969	12.868	3.361	7
8	61.174	88.220	143.449	12.537	14.707	3.842	8
9	68.821	99.247	161.380	14.104	16.545	4.322	9
10	76.467	110.275	179.311	15.671	18.384	4.802	10

same conditions. Column 8 is expeditiously filled by laying the slip of paper containing Table II upon the form, as before, and opening the Rechentafeln at the value of A; then opening at B, the products in column 9 are obtained with equal facility.

The corrections for eccentricity appear in column 10, each term being the sum of those on the same line in the two preceding columns; these corrections, with the further addition of X, are entered in column 11. After subtracting the quantities in column 11 from the corresponding ones in column 5 the residuals in column 12 remain; their sum should not differ from zero by more than two or three units. Finally the squares of the residuals are entered in column 13, and the probable error of graduation and observation is computed from their sum by (9). The foregoing formulas are equally applicable to any examination consisting of fourteen comparisons at points ten degrees apart upon the arc of the sextant. If it is preferred for any reason to make the comparisons elsewhere than at 0°, 10°, etc., the eccentric corrections obtained for the points compared may be rectified by subtracting from each of them the correction computed for $\gamma' = 0°$.

As a further illustration of what is to expected in practice, the eccentric corrections, constants A and B, residuals, and probable errors, of six sextants, by makers of high repute, are given in Table IV.

TABLE IV.

S.	I.		II.		III.		IV.		V.		VI.	
	Ecc. cor.	O—C.	Ecc. cor.	O—C.	Ecc. cor.	O—C.	Ecc. cor.	O—C.	Ecc. cor.	O—C.	Ecc. cor.	O—C.
°	″	″	″	″	″	″	′ ″	″	′ ″	″	″	″
0	0	0	0	— 2	0	+ 1	0	— 3	0	+ 1	0	— 7
10	0	— 2	+ 1	— 1	+ 3	4	— 4	4	— 8	+ 8	— 7	+ 5
20	0	0	2	0	6	+ 1	9	— 5	16	— 3	13	+ 7
30	0	— 2	3	+ 3	8	— 1	14	+ 9	25	2	17	— 1
40	— 1	+ 3	4	+ 1	9	2	19	11	35	1	19	— 4
50	1	2	6	0	10	9	25	+ 7	45	— 8	20	0
60	1	1	7	+ 4	10	— 3	32	— 7	55	0	18	— 4
70	1	4	8	0	9	+ 3	38	6	1 6	— 4	15	+ 3
80	1	+ 3	9	+ 4	8	+ 8	45	— 3	1 16	— 2	10	+ 2
90	1	— 3	10	— 7	6	— 5	53	+ 6	1 27	+ 3	— 3	— 5
100	1	11	11	— 6	4	+ 11	1 0	+ 1	1 38	6	+ 5	+ 4
110	2	— 2	12	0	+ 1	— 4	1 8	— 11	1 48	+ 18	15	+ 1
120	2	+ 3	14	— 2	— 3	+ 6	1 15	— 6	1 59	— 1	27	— 5
130	2	+ 4	15	+ 6	7	— 8	1 23	+ 12	2 9	— 14	40	+ 2
140	— 2		+ 16		— 12		— 1 30		— 2 19		+ 54	
p. e.		± 3.0		± 2.7		± 4.3		± 5.5		± 5.5		± 3.2
A	— 2.1		+ 11.4		+ 40.4		— 43.3		— 86.1		— 94.6	
B	+ 0.3		+ 7.4		— 76.2		— 75.3		— 88.7		+ 217.3	

The one requiring corrections of more than 2′ is of course exceptional, but no complaint is known to have been made of it before the examination, though it had seen service at sea, possibly in the hands of some scrupulous young officer who carried out his reductions to fractions of a second. It is not the most incorrectly centered instrument in the table, however. The probable error, though derived from too small a number of observations to be regarded as precise, is a useful criterion. When much in excess of 5″ it implies that either the graduation is inaccurate, the telescopic power is too low, the mechanical or optical action of the sextant is imperfect (perhaps, through maladjustment of its parts), the observer is unskillful, or two or more of these unfavorable conditions coexist. Values of less than 3″ are frequently obtained, but this is commonly due in part to an accidental avoidance of the larger errors of observation.

Before proceeding to combine the results of several series of comparisons it is desirable to know the probable error of the corrections deduced from a single series. The eccentric correction for any reading γ', as expressed in (10), may be written:

$$\gamma - \gamma' = \Sigma \Big[D \Big\{ -1.5671 \sin \tfrac{1}{2}\gamma' + 1.8384\,(1 - \cos \tfrac{1}{2}\gamma') + \Big(7.6467 \sin \tfrac{1}{2}\gamma' - 11.0275\,(1 - \cos \tfrac{1}{2}\gamma')\Big) \sin \tfrac{1}{2} S + \Big(-11.0275 \sin \tfrac{1}{2}\gamma' + 17.9311\,(1 - \cos \tfrac{1}{2}\gamma')\Big)\,(1 - \cos \tfrac{1}{2} S)\Big\}\Big].$$

Now if t is the probable error of a single comparison, and consequently of D, the probable error of $\gamma - \gamma'$ will be:

$$t \times \sqrt{\Sigma \left[\left\{ -1.5671 \sin \tfrac{1}{2} \gamma' + 1.8384 (1 - \cos \tfrac{1}{2} \gamma') + \left(7.6467 \sin \tfrac{1}{2} \gamma' - 11.0275 (1 - \cos \tfrac{1}{2} \gamma')\right) \sin \tfrac{1}{2} S + \right.\right.}$$
$$\left.\left. \left(-11.0275 \sin \tfrac{1}{2} \gamma' + 17.9311 (1 - \cos \tfrac{1}{2} \gamma')\right) (1 - \cos \tfrac{1}{2} S) \right\}^2 \right],$$

the value of which for any given value of γ' may be taken from Table V.

TABLE V.

γ'	Prob. error of ecc. cor.
°	
0	$t \times$ 0.00
10	.23
20	.42
30	.58
40	.71
50	.81
60	.87
70	.90
80	.91
90	.89
100	.86
110	.83
120	.82
130	.85
140	0.94
150	$t \times$ 1.09

The expressions for A, B, and X, (7), are equivalent to:

$$A = \Sigma \left[D \left(-1.5671 + 7.6467 \sin \tfrac{1}{2} S - 11.0275 (1 - \cos \tfrac{1}{2} S) \right) \right],$$

$$B = \Sigma \left[D \left(1.8384 - 11.0275 \sin \tfrac{1}{2} S + 17.9311 (1 - \cos \tfrac{1}{2} S) \right) \right],$$

$$X = \Sigma \left[D \left(0.4802 - 1.5671 \sin \tfrac{1}{2} S + 1.8384 (1 - \cos \tfrac{1}{2} S) \right) \right],$$

from which it is evident that the probable error of A is:

$$t \times \sqrt{\Sigma \left[\left\{ -1.5671 + 7.6467 \sin \tfrac{1}{2} S - 11.0275 (1 - \cos \tfrac{1}{2} S) \right\}^2 \right]} = 1.90\ t,$$

that of B is:

$$t \times \sqrt{\Sigma \left[\left\{ 1.8384 - 11.0275 \sin \tfrac{1}{2} S + 17.9311 (1 - \cos \tfrac{1}{2} S) \right\}^2 \right]} = 4.23\ t,$$

and that of X is:

$$t \times \sqrt{\Sigma \left[\left\{ 0.4802 - 1.5671 \sin \tfrac{1}{2} S + 1.8384 (1 - \cos \tfrac{1}{2} S) \right\}^2 \right]} = 0.69\ t.$$

Different series of comparisons, therefore, cannot be expected to agree very closely in the values of A they furnish, still less in their values of B, but such a relation should nevertheless subsist among these apparently discordant results as to render their respective sets of eccentric corrections reasonably harmonious. The constants A and B are, in fact, as indicated by (1), rectangular coördinates of the index-axis referred to the system in which the center of the sextant arc is the origin, the radius passing through mean 0° is the axis of A, and 1″ of the arc having a radius equal to half the length of the index-bar, from axis to edge of vernier, is the linear unit. By substituting the value of X obtained from the last formula of (6) in the other two, they become the equations in A and B of two right lines, inclined to the axis of A at angles of 121° 35′, and 124° 44′, respectively, and determining by their intersection the position of the index-axis. These lines meet at an angle of but 3° 9′, and since errors of observation can only displace them laterally, without altering their direction,* it follows that, as might be expected, a straight line passing through the index-axis, and perpendicular to the radius through the middle of the arc, is located much more accurately than the position of the axis upon that line.

Let D' be the observed value of D obtained by any one setting S' of the sextant, all the other values of S and D which furnished equations of condition retaining those designations; the local correction of the graduation at S' will then be the difference between D' and the value of D computed by (4) for $S = S'$, or:

$$D' - \left(A \sin \tfrac{1}{2} S' + B\,(1 - \cos \tfrac{1}{2} S') + X\right),$$

which, by substituting the values of A, B, and X, in (7), becomes:

$$\left(1.5671 \sin \tfrac{1}{2} S' - 1.8384\,(1 - \cos \tfrac{1}{2} S') - 0.4802 + 1\right) D' +$$

$$\left(-\,7.6467 \sin \tfrac{1}{2} S' + 11.0275\,(1 \cos \tfrac{1}{2} S') + 1.5671\right) D' \sin \tfrac{1}{2} S' +$$

$$\left(11.0275 \sin \tfrac{1}{2} S' - 17.9311\,(1 - \cos \tfrac{1}{2} S') - 1.8384\right) D'(1 - \cos \tfrac{1}{2} S') +$$

$$\left(1.5671 \sin \tfrac{1}{2} S' - 1.8384\,(1 - \cos \tfrac{1}{2} S') - 0.4802\right) \Sigma \left[D\right] +$$

$$\left(-\,7.6467 \sin \tfrac{1}{2} S' + 11.0275\,(1 - \cos \tfrac{1}{2} S') + 1.5671\right) \Sigma \left[D \sin \tfrac{1}{2} S\right] +$$

$$\left(11.0275 \sin \tfrac{1}{2} S' - 17.9311\,(1 - \cos \tfrac{1}{2} S') - 1.8384\right) \Sigma \left[D(1 - \cos \tfrac{1}{2} S)\right].$$

*The probable displacement of the line represented by the normal equation in A, which is the one making an angle of 121° 35′ with the radius through 0°, is $t \times 0.79$; the probable displacement of the other line is $t \times 0.92$.

The probable error of this correction is therefore:

$$t \times \sqrt{\left\{\begin{array}{l} 1.5671 \sin \tfrac{1}{2} S' - 1.8384\,(1 - \cos \tfrac{1}{2} S') - 0.4802 + 1 + \\ \left(-7.6467 \sin \tfrac{1}{2} S' + 11.0275\,(1 - \cos \tfrac{1}{2} S') + 1.5671\right) \sin \tfrac{1}{2} S' + \\ \left(11.0275 \sin \tfrac{1}{2} S' - 17.9311\,(1 - \cos \tfrac{1}{2} S') - 1.8384\right)(1 - \cos \tfrac{1}{2} S') \end{array}\right\}^2 + \Sigma \left[\left\{\begin{array}{l} 1.5671 \sin \tfrac{1}{2} S' - 1.8384\,(1 - \cos \tfrac{1}{2} S') - 0.4802 + \\ \left(-7.6467 \sin \tfrac{1}{2} S' + 11.0275\,(1 - \cos \tfrac{1}{2} S') + 1.5671\right) \sin \tfrac{1}{2} S + \\ \left(11.0275 \sin \tfrac{1}{2} S' - 17.9311\,(1 - \cos \tfrac{1}{2} S') - 1.8384\right)(1 - \cos \tfrac{1}{2} S) \end{array}\right\}^2\right]},$$

which will be found in the second column of Table VI for all values of S'. When D' belongs not to one of the fourteen comparisons from which A, B, and X were derived, but to an additional comparison at any point whatever, the quantity inclosed by the first parenthesis in the preceding expression retains only the term $+ 1$, and the resulting values are given in the third column of Table VI. In all the foregoing expressions of probable errors $\frac{t}{\sqrt{N}}$ must be substituted for t, if N is the number of series of comparisons constituting the examination.

Table VI.

S'	Probable errors of local corrections.	
°		
0	$t \times 0.72$	$t \times 1.22$
10	.85	1.13
20	.91	1.08
30	.93	1.06
40	.93	1.06
50	.92	1.07
60	.92	1.08
70	.92	1.08
80	.92	1.07
90	.93	1.06
100	.93	1.06
110	.91	1.08
120	.85	1.13
130	$t \times 0.72$	1.22
140	----------	1.35
150	----------	$t \times 1.54$

When several series of comparisons are made, they are first reduced separately in the manner already described. It is not absolutely necessary to find the value of X at this stage, or to fill out columns 8 to 13 of the form represented by Table I, but only a little time is saved by these omissions, and useful verifications of the work are lost. In Table VII are given the constants, eccentric corrections, residuals, etc., obtained by the preliminary reduction of three successive examinations of the same sextant.

The means of the three values of A, and B, and the three values of X computed therefrom by (8) are:

$$\begin{array}{ll} A = + \;\; 5''.3 & X' \;\; = - 24.16 \\ & X'' \;= - 24.94 \\ B = + 68''.0 & X''' = - 31.37 \end{array}$$

TABLE VII.

S	First series.		Second series.		Third series.	
	Ecc. cor.	O — C.	Ecc. cor.	O — C.	Ecc. cor.	O — C.
°	//	//	//	//	//	//
0	0.0	+ 10	0.0	+ 7	0.0	+ 7
10	+ 0.8	— 5	0.0	— 3	+ 1.3	— 4
20	2.0	5	+ 0.6	1	3.1	+ 5
30	3.8	— 7	1.9	2	5.4	— 6
40	5.9	+ 6	3.7	2	8.1	1
50	8.5	— 3	6.2	5	11.2	6
60	11.4	— 10	9.1	— 7	14.7	4
70	14.8	+ 3	12.8	0	18.5	— 1
80	18.4	9	16.8	+ 8	22.7	+ 9
90	22.4	2	21.4	8	27.1	+ 2
100	26.7	1	26 4	3	31.9	0
110	31.2	+ 9	31.9	+ 3	36.8	+ 12
120	35.9	— 6	37.8	— 4	42.0	— 5
130	40.9	— 5	43.9	— 5	47.3	— 6
140	46.0	(— 4)	50.4	(— 15)	52.7	(— 12)
150	+ 51.2		+ 57.1		+ 58.3	
Prob. error		± 4.9		± 3.7		± 4.4
	A_1	= + 6.6	A_2	= — 3.5	A_3	= + 12.8
	B_1	= + 60.5	B_2	= — 81.6	B_3	= + 61.8
	X_1	= — 23.2	X_2	= — 23.3	X_3	= — 34.0
	$[D_1]$	= — 103.	$[D_2]$	= — 114.	$[D_3]$	= — 204.
	$[D_1 \sin \frac{1}{2} S]$	= — 1.31	$[D_2 \sin \frac{1}{2} S]$	= — 3.57	$[D_3 \sin \frac{1}{2} S]$	= — 44.12
	$[D_1 (1 - \cos \frac{1}{2} S)]$	= + 13.13	$[D_2 (1 - \cos \frac{1}{2} S)]$	= + 14.04	$[D_3 (1 - \cos \frac{1}{2} S)]$	= — 2.77

The reduction is completed in Table VIII, which is so similar to the corresponding portion of Table I as to require but little explanation.

Table VIII.

S	$A \sin \frac{1}{2} S$	$B (1 - \cos \frac{1}{2} S)$	Ecc. correction.		D, comp.	D, obs.	$O - C$.	Local cor.	Diff. from mean.	(Diff.)^2
°	″	″	″	″	″	″	″	″	″	
0 ----------	0.0	0.0	0.0	0.0	— 24	— 13	+ 11	+ 8	+ 3	9
					25	16	+ 9		+ 1	1
					31	27	+ 4		— 4	16
10 ----------	+ 0.5	+ 0.2	+ 0.7	± 0.2	23	27	— 4	— 4	0	0
					24	26	— 2		+ 2	4
					31	37	— 6		— 2	4
20 ----------	0.9	1.0	1.9	0.4	22	26	— 4	1	— 3	9
					23	24	— 1		0	0
					29	26	+ 3		+ 4	16
30 ----------	1.4	2.3	3.7	0.6	20	26	— 6	— 5	— 1	1
					21	23	— 2		+ 3	9
					28	35	— 7		— 2	4
40 ----------	1.8	4.1	5.9	0.7	18	11	+ 7	+ 1	+ 6	36
					19	22	— 3		— 4	16
					25	27	— 2		— 3	9
50 ----------	2.2	6.4	8.6	0.8	16	18	— 2	— 5	+ 3	9
					16	22	— 6		— 1	1
					23	29	— 6		— 1	1
60 ----------	2.6	9.1	11.7	0.9	12	22	— 10	— 7	— 3	9
					13	21	— 8		— 1	1
					20	23	— 3		+ 4	16
70 ----------	3.0	12.3	15.3	0.9	9	6	+ 3	+ 1	+ 2	4
					10	11	— 1		— 2	4
					16	— 16	0		— 1	1
80 ----------	3.4	15.9	19.3	0.9	5	+ 4	+ 9	9	0	0
					6	+ 1	+ 7		— 2	4
					12	— 2	+ 10		+ 1	1

90	3.7	19.9	23.6 0.9	1 1 − 8	+ 1 + 6 − 5	+ 2 + 7 + 3	4	− 2 + 3 − 1	4 9 1
100	4.1	24.3	28.4 0.9	+ 4 + 3 − 3	+ 4 + 6 − 2	0 + 3 + 1	1	− 1 + 2 0	1 4 0
110	4.3	29.0	33.3 0.9	+ 9 8 2	+ 17 12 15	+ 8 + 4 + 13	+ 8	0 − 4 + 5	0 16 25
120	4.6	34.0	38.6 0.8	14 14 7	7 10 3	− 7 − 4 − 4	− 5	− 2 + 1 + 1	4 1 1
130	4.8	39.2	44.0 0.9	20 19 13	13 16 7	− 7 − 3 − 6	− 5	− 2 + 2 − 1	4 4 1
140	5.0	44.7	49.7 1.0	26 25 + 18	19 12 + 7	− 7 − 13 − 11	(− 10)	(+ 3) (− 3) (− 1)	
150	+ 5.1	+ 50.4	+ 55.5 ± 1.1						
							+ 32	+ 43	260 ″
							− 32	− 43	± 1.79

The three terms of each group in the fifth column are obtained by successively adding X', X'', and X''', to the eccentric correction in the preceding column. The three values of D following in the sixth column are those which were observed in the first, second, and third, series of comparisons respectively. By subtracting from each of the latter the computed value in the fifth column, the three values of the local correction in the seventh column are found; their mean is given in the eighth column.

The residuals in the ninth column are obtained by subtracting this mean successively from its three constituents, and the tenth column contains the squares of the residuals.

There are 42 residuals, and the sum of their squares is 260; the probable error of a single observation is therefore:

$$t = 0.6745 \sqrt{\frac{260''}{42 - 5}} = \pm 1''.79.$$

The probable errors of the eccentric corrections in the fourth column of Table VIII are taken from Table V with this value of t, and divided by $\sqrt{3}$, since three independent determinations of A and B have been made. The maximum probable error of a local correction deduced from a single series of comparisons, as given in the second column of Table VI, is $t \times 0.93$; the probable error of the local corrections in this example (excepting that derived from the additional comparison at 140°) is therefore not greater than $\frac{\pm 1''.79 \times 0.93}{\sqrt{3}} = \pm 1''.0$, which is small enough to justify some degree of confidence in them. It should be mentioned here that the three series of comparisons were all made with the same portion of the circle in this instance, and that the effect of errors in the circle is consequently but little diminished by the repetition. In a mere illustration of the capabilities of the method this uniformity is preferable, since it affords a value of t nearly identical with that which would be obtained if the circle were faultless, while the absolute verity of the corrections is of minor importance. But charging the sextant with the imperfections of both instruments, and ignoring also the error of observation, which cannot be inappreciable, none of these corrections imply an error of circular division exceeding 5″, one that is certainly to be expected in all graduation except that of the very highest class.

The probable error of observation in this example, $t = \pm 1''.8$, is very small, as it ought to be, for the sextant was firmly supported in a convenient position, the pointing was deliberate, and directed upon a singularly well-defined object, the index was set in a definite position always referred to the same lines of the vernier, and the observer was perhaps somewhat expert at that time. This error will ordinarily be larger, indeed

an observer who contents himself with the least count of a 10″ vernier cannot reduce his probable error below 2″.5; but under some circumstances—as, for instance, in measuring circummeridian altitudes of Polaris with the sextant mounted upon a stand—the precision attained in this examination should be closely approached. With practice the vernier may be read as closely as it can be set, for so long as the direction of the necessary movement is recognized, the distance can also be estimated.

The 42 residuals are distributed as follows:

Between	Number of Errors	
	By theory.	Found.
″ ″		
0.0 and 0.5	6	5
0.5 " 1.5	12	12
1.5 " 2.5	9	11
2.5 " 3.5	7	7
3.5 " 4.5	4	5
4.5 " 5.5	2	1
5.5 " 6.5	1	1
6.5 " ∞	1	0

When the preliminary reductions have been carried out in full, as in Table I, the eccentric corrections may also be found by taking the means of the corrections in the different series for the same values of S, and the local corrections by similarly taking the means of the residuals $O - C$; but the final residuals must be obtained as in Table VIII, with values of X', X'', etc., calculated by (8) from the mean values of A and B. Unless this is done the computed probable errors will, in general, be somewhat too small. The effect of local errors upon the determination of eccentricity is usually unimportant. If the eccentric corrections in Table VIII are recomputed after applying the local corrections to the observed values of D, there will be no change amounting to half a second.

Some test of the general trustworthiness of the examination is always desirable. A sextant may be in such condition as to operate correctly under the delicate manipulation it receives upon the table of the apparatus, yet when removed therefrom and handled less cautiously, or returned to its case, a slight displacement of the axis may occur; so that if compared again the two sets of eccentric corrections will differ considerably from each other, although a small probable error is found for each. Any great change of this sort may be detected by comparing the differences D in the successive series of comparisons, which should always

be scrutinized for that purpose before beginning the reductions. It is not possible, however, to decide in this way whether small abnormal variations exist or not. The agreement or disagreement between the different pairs of values of A and B is also an insufficient test, for a reason already given, but if the preliminary reductions are carried out far enough to determine the eccentric corrections for each series separately, the probable error of this correction at any point, as deduced from the differences between the corrections furnishedby each of the series and their mean, maybe compared with the same probable error taken from Table V. The two values can scarcely be expected to agree exactly, but the difference between them should not be too great. The following results were obtained from the data in Tables VII and VIII.

γ'.	By diffs.	By Table V.
°	″	″
0	0.0	0.0
20	± 0.9	± 0.8
40	1.5	1.3
70	2.0	1.6
100	2.1	1.5
130	2.2	1.5
150	± 2.6	± 2.0

In reading a sextant it is not merely the coincidence of a single line of the vernier with one of the limb that is noted, but the relative positions of several adjacent lines are taken into account, or ought to be; the effect of errors peculiar to individual lines is thereby rendered comparatively innocuous, for such errors cannot be large without being visible. The most pernicious errors of graduation are progressive displacements in alternating directions, extending throughout the arc in waves more or less regular, but of considerable length. The existence of systematic errors having a period long enough to embrace several of the points which have been examined is indicated by a succession of local corrections with the same algebraic sign. It is sometimes advisable to attempt the correction of such errors, especially when they are large, and when many series of comparisons have been obtained. A convenient process is to plot the values of S as abscissas, and the computed local corrections as ordinates, to draw a fair curve approximating the points thus laid down, and lastly to measure and tabulate the ordinates of the curve as mean local corrections. This method is a rather rough one, but it is useful when the corrections to be adjusted are small, as the local

corrections always are. Much exercise of good judgment is, however, essential: if the curve were drawn through all the points, or, what is the same thing, if the computed local corrections were adopted without any adjustment, the error resulting from sporadic defects in the graduation would apparently vanish; but it is not certain that actual errors would always be diminished, for any single local correction may be considerably in error, and may also refer to a point not impartially representing the general state of the graduation in its vicinity. The number of points of contrary flexure in the curve must be very small as compared with the number of given points, and in every doubtful case it is safer to err on the side of proximity to the axis of *S*. The local corrections in the eighth column of Table VIII are not large, but they show unmistakable signs of systematic arrangement. The mean local corrections in Table IX were accordingly obtained from them by the process just referred to. There is one point of contrary flexure in the curve not far from $S = 68°$.

TABLE IX.

S.	Loc. cor.	Mean loc. cor.	Residuals.
°	″	″	″
0	+ 8	+ 3	+ 5
10	— 4	+ 1	— 5
20	1	— 1	0
30	— 5	3	— 2
40	+ 1	4	+ 5
50	— 5	5	0
60	— 7	— 5	— 2
70	+ 1	+ 1	0
80	9	6	+ 3
90	4	6	— 2
100	1	5	— 4
110	+ 8	+ 2	+ 6
120	— 5	— 1	— 4
130	5	5	0
140	(— 10)	— 10	(0)
			+ 19
			— 19

As this series of supplementary corrections is a somewhat typical one, its significance should be recognized. Disregarding the abberrations of individual lines, the actual and mean arcs are coincident at points near 15°, 68°, and 117°. The mean arc overlaps the actual arc at both ends; the mean length of divisions of the latter is, therefore, too small; they are actually too small between 0° and about 55°, too large from 55° to

85°, and again too small from 85° onward. From a point near 68°, where it is greatest, their length decreases in both directions. These systematic irregularities may have been produced by inequalities in the operation of the dividing engine, but they can also be accounted for by supposing an almost infinitesimal distortion to have occurred after the graduation was executed, the middle of the arc approaching the center, and the ends receding therefrom, the greatest change being at the end opposite 0°. The apparent difference between the two ends may, however, be partially due to a slight deviation from parallelism in the surfaces of the index glass.

Table X.

γ'	Total correction.
°	″
0	0
10	— 1
20	2
30	2
40	— 1
50	+ 1
60	4
70	13
80	22
90	27
100	30
110	32
120	35
130	36
140	+ 37

The total correction of this sextant, or sum of the eccentric and mean local corrections, is given in Table X. For the sake of convenience in use the correction at 0° has been reduced to 0 by adding — 3″ throughout.

The correction applied to any angle measured with the sextant is always the difference between two tabular corrections—that of the observed reading, and that of the reading made in determining the index correction. Let D_1', D_2', etc., be the observed values of D corresponding to the setting S' in the different series of comparisons, the computed local correction for this reading is then:

$$\left.\begin{array}{l} D_1' - A \sin \tfrac{1}{2} S' - B\,(1 - \cos \tfrac{1}{2} S') - X_1 + \\ D_2' - A \sin \tfrac{1}{2} S' - B\,(1 - \cos \tfrac{1}{2} S') - X_2 + \\ \quad \text{etc.}, + \text{etc.}, + \\ D_n' - A \sin \tfrac{1}{2} S' - B\,(1 - \cos \tfrac{1}{2} S') - X_n \end{array}\right\} \div N =$$

$$\frac{\Sigma D' - \Sigma X}{N} - A \sin \tfrac{1}{2} S' - B\,(1 - \cos \tfrac{1}{2} S'),$$

and the sum of the eccentric and local corrections is: $\frac{\Sigma D' - \Sigma X}{N}$. For any other setting S'' this sum is: $\frac{\Sigma D'' - \Sigma X}{N}$, which subtracted from the preceding expression leaves: $\frac{\Sigma D' - \Sigma D''}{N}$.

Now the probable error of each of the N differences D_1', D_1'', D_2', D_2'', etc., is t; the probable error of $\frac{\Sigma D' - \Sigma D''}{N}$ is therefore:

$$\frac{\sqrt{2\,N\,t^2}}{N} = 1.41\,\frac{t}{\sqrt{N}}.$$

But each point of the curve from which the mean local corrections were obtained depends upon two or more of the computed corrections: the probable error of the correction applied to any angular measurement is consequently less than $1.41 \frac{t}{\sqrt{N}}$ in some proportion depending upon the skill with which the curve was traced. This error is still further diminished when the two readings differ so little that their mean local corrections depend in part upon the same comparisons, and it vanishes when that difference is very small. For the series of total corrections in Table X, $1.41 \frac{t}{\sqrt{N}} = 1.41 \frac{\pm 1''.79}{\sqrt{3}} = \pm 1''.45$.

The numerical mean of the residuals in Table IX is $\pm 2''.7$, which is so much greater than $\frac{t}{\sqrt{N}} = \pm 1''.03$, the probable variation due to error of observation, as to excite a suspicion that the sporadic errors of graduation, including systematic errors of short period, are rather large in this instrument.

The vernier is liable to errors as well as the limb, and they are sometimes large enough to require correction. If the initial and terminal lines of the vernier do not simultaneously coincide with lines of the limb the former is usually blamed, though not always justly; for the vernier of Sextant V in Table IV, if of the right length, will afford nearly simultaneous coincidences at the initial line, and at the additional line next the terminal one, on all parts of the arc; while the vernier of Sextant VI in the same table, should apparently be correct near 50°, too long at 0°, and considerably too short at 140°. When the corrections of the limb are known, a better judgment can be formed, but any examination by comparisons of this sort is necessarily limited to the extremities of the vernier, and gives no indication of the state of affairs between them. An examination may be made with the sextant apparatus, however, at any number of points, equidistant or otherwise, by bringing them in succession to the same division of the limb and comparing with the circle.

Let l' be the actual length of a vernier, whose proper or intended length is l,* and whose nominal length, equal to one division of the limb, is i; also let s be the reading at which the vernier is set in making a comparison, r being the corresponding reading of the circle, and z an

* The lengths l' and l are to be accounted positive when the readings of the limb and vernier increase in the same direction—negative when they increase in opposite directions.

assumed value of that reading when $s = 0$, the true value being $z + x$, in which x is unknown. Then

$$\frac{s}{i}(l - l') = \frac{s}{i}\,l - (z + x - r),^{*}$$

or if $l - l' = c$, and $\frac{l}{i} = p,^{\dagger}$

$$\frac{s}{i}\,c = ps - z - x + r,$$

and finally by making $r + ps - z = d,^{\ddagger}$ we obtain:

$$\frac{s}{i}\,c + x = d.$$

Each comparison furnishes an equation of this form, and if m comparisons are made, the normal equations are:

$$\left.\begin{aligned} \left[\left(\frac{s}{i}\right)^2\right] c + \left[\frac{s}{i}\right] X - \left[\frac{s}{i}d\right] &= 0, \\ \left[\frac{s}{i}\right] c \quad + \quad m\,x \quad - \left[d\right] &= 0. \end{aligned}\right\} \qquad (11)$$

The true reading of a vernier is the product of the actual difference between one of its divisions and a division of the limb, multiplied by the number of divisions embraced in the reading; if then δ' be any reading, δ its corrected or true value, and q the number of divisions in the vernier:

$$\delta = \frac{\delta'}{i}\,q\left(\pm\, i - \frac{l'}{q}\right),$$

the upper and lower signs of i pertaining to the "short" and "long" forms of vernier respectively. The correction is therefore:

$$\delta - \delta' = \frac{\delta'}{i}\left(i\,(\pm\, q - 1) - l'\right),$$

or since $i\,(\pm\, q - 1) = l$, and $l - l' = c$,

$$\delta - \delta' = \frac{\delta'}{i}\,c, \qquad (12)$$

in which c is positive for either a "short" vernier which is too short, or a "long" vernier which is too long, and vice versa.

Although the normal equations are easily solved for any values of s, time can be saved, even in this case, by adopting a uniform system of

* In the apparatus here referred to the circle readings diminish as the readings of the usual or "short" form of vernier increase.

† For most sextants $p = 59$.

‡ For a reversed or "long" vernier s is essentially negative.

TABLE XI.

s	VERNIERS. I	VERNIERS. II	r			ps	d	$\frac{s}{i}d$	Correction, $\frac{s}{i}c$		$\frac{s}{i}c+x$	$O-C$	$(O-C)^2$	
′	″	″	°	′	″	″	″	″	″	″	″	″		
0	51	54	9	59	54	0	— 6	0.0	0.0	0.0	— 3.8	— 2	4	$Z = 10° 0' 00''$.
1	57	0	9	0	58	0	2	— 0.2	—0.9	± 0.2	4.7	+ 3	9	
2	57	0	8	1	58	0	2	0.4	1.8	0.4	5.6	4	16	+ 42.32
3	54	54	7	2	54	0	6	1.8	2.7	0.6	6.5	+ 1	1	— 51.45
4	48	51	6	3	50	0	10	4.0	3.7	0.8	7.5	— 2	4	— 9.13 = C.
5	48	54	5	4	51	0	9	4.5	4.6	1.0	8.4	1	1	
6	45	48	4	5	46	0	14	8.4	5.5	1.2	9.3	— 5	25	— 29.57
7	51	51	3	6	51	0	9	6.3	6.4	1.4	10.2	1	1	25.75
8	45	48	2	7	46	0	14	11.2	7.3	1.6	11.1	— 3	9	— 3.82 = X.
9	48	48	1	8	48	0	12	10.8	8.2	1.8	12.0	0	0	
10	51	51	0	9	51	0	— 9	— 9.0	—9.1	± 2.0	— 12.9	+ 4	16	
0	54	57												
							— 93	— 56.6				+ 13 — 13	86	

examination and computation. If comparisons are made at the two ends of the vernier, and at nine equidistant points between them, $\frac{s}{i}$ is successively 0, 0.1, 0.2, etc., 1.0; $m = 11$, and the normal equations (11) become:

$$\left.\begin{aligned} 3.85\, c + 5.5\, x - \left[\tfrac{s}{i} d\right] &= 0, \\ 5.5\, c + 11\, x - \left[d\right] &= 0. \end{aligned}\right\} \qquad (13)$$

Their solution gives:

$$\left.\begin{aligned} c &= -\,0.455 \left[d\right] + 0.909 \left[\tfrac{s}{i} d\right], \\ x &= \quad 0.318 \left[d\right] - 0.455 \left[\tfrac{s}{i} d\right], \end{aligned}\right\} \qquad (14)$$

and the weight of c is 1.1.

Table XI is an example of the record of examination and form of computation. It refers to a sextant divided to 10′, and reading to 10″ by a "short" vernier, for which, therefore, $i = 10'$ and $p = 59$. The calculation scarcely requires any tables except one of squares for extracting the square root in finding the probable errors, but it is convenient to take from Crelle's table the four products employed in computing the values of c and x by (14).

The first three columns contain the record of examination, and the fourth receives the products ps, when they are expressed. For the sake of clearness the values of r have been given in full, as well as the seconds of ps, but it is unnecessary to write down more than the seconds of r, and the seconds of ps when they vary, as in the case of a sextant divided to 15′ and reading to 15″. A sufficient explanation of the remaining columns is to be found in their headings. The sum of the squares of the residuals being 86, the probable error of a single comparison is $0.6745 \sqrt{\frac{86}{11 - 2}} = \pm 2''.09$, and that of c is $\frac{\pm 2''.09}{\sqrt{1.1}} = \pm 2''.0$. The corrections in the seventh column are computed by making $\delta' = s$, and $i = 10'$, in (12); the appended probable errors are therefore $\pm 2''.0 \frac{s}{10'}$.

In the probable error of a single comparison is included the probable error of graduation, and that of observation, and the latter is presumably equal to t if the limb and vernier were examined under similar circumstances. If, therefore, the probable error of one comparison is notably

greater than t, there is reason to suspect that the graduation is sensibly imperfect. The probable error of any angular measurement, due to the errors of the vernier corrections, is simply the difference between the probable errors of the two tabular corrections applied, being, in fact, that proportional part of the probable error of c corresponding to the fraction of the vernier actually used. If the residuals indicate the existence of a systematic error, a supplementary correction may be obtained by the graphic process which has been described, though such an adjustment will rarely be required, except when valuable observations have been made with a sextant which had previously received some injury. If the index-bar were bent, for instance, so that the two ends of the vernier are unequally distant from the axis, it will be found that the divisions are longest at the nearer end, and shortest at the more distant one. The vernier corrections, like those pertaining to the limb, may be determined with increased accuracy by several series of comparisons, preferably made with different parts of the circle, and combined by methods too obvious to require special explanation.

For any given sextant reading the argument of the correction is not the reading itself, but that of the point where a line of the vernier coincides with one of the limb*. The readings $8° 59' 50''$ and $9° 0' 10''$ differ only twenty seconds, but upon a sextant divided to $10'$ and reading to $10''$ they refer to positions on the limb nearly ten degrees apart. This circumstance, which is not invariably mentioned in the text-books, is also of considerable importance in determining eccentric corrections by the methods commonly recommended. When extreme precision is desired, the accuracy of an observation already made may sometimes be increased by a device applicable to any sextant, whether its errors have been investigated or not. Find the point of coincidence of the recorded reading, and, after setting the zero of the vernier exactly upon that position, read the vernier at the other end; then, setting the terminal line of the vernier at the point previously occupied by the initial line, read at the zero end. Subtract the nominal length of the vernier from the sum of the two vernier readings and divide the remainder by three; the quotient is a correction to be applied to the original sextant reading. An error in

* Every sextant reading is the sum of the limb reading and the vernier reading, and may be readily separated into these two parts when the scheme of graduation is known. If the vernier is of the direct or "short" form, almost universally applied to sextants, the point of coincidence may be found by the following rule: To the limb reading add the vernier reading multiplied by the number of divisions in the vernier. Thus on the arc of a sextant divided to $10'$, and reading to $10''$, and which, therefore, has a vernier of 60 divisions, the reading $6° 49' 50''$ is made at $6° 40' + 9' 50'' \times 60 = 16° 30'$. For a reversed or "long" vernier, the same product is to be subtracted from the limb reading.

the length of the vernier does not affect this result, since one reading is always as much augmented thereby as the other is decreased; but if another limb correction is applied, it should be the mean of the corrections for the three points of coincidence. By this artifice each observation is referred to three distinct positions on the arc, with a corresponding diminution in the effect of purely local errors.

When the presence of large errors upon any small portion of the limb is suspected, in consequence of a local injury or otherwise, as many of the lines in this tract may be examined as the nature of the case requires. These comparisons are not to be employed in finding the eccentricity, but must be reduced separately to determine the local errors.

If it should become generally known that sextants purchased in considerable numbers were inspected only at certain points of the graduation, unscrupulous makers, especially those using copying engines, might be tempted to bestow greater care upon the critical divisions than upon the rest of the arc. But the points to be examined may be selected at pleasure, if their number and distance from each other remain unchanged, as has been shown. In arranging a system of examination for any given service, the number of comparisons in a series will naturally depend on the special requirements of the case. A greater number affords a better representation of the entire graduation, while, on the other hand, it extends the time during which errors may be introduced by gradual or sudden changes in the apparatus, or in the sextant itself. Perhaps there can seldom be any sufficient reason for spacing the comparisons more closely than at points 5° apart.

An alternative form of the apparatus has been devised, having two collimators—one fixed, the other carried by an arm attached to the circle and directed toward the sextant, which is supported by a fixed table immediately over the circle. The two collimator-marks—one seen direct, the other by reflection—are brought into coincidence in the field of the sextant telescope, as in the ordinary use of that instrument. No apparatus of this form has been constructed, but the details have been worked out far enough to show that no serious practical difficulty is to be apprehended. The principal feature of improvement is that nothing will depend upon immobility of the sextant, which lies freely upon its table during the examination and may be removed for inspection at any time. It may be, however, that this advantage is rather apparent than real, for in either case the sextant must be handled with the utmost caution, and experience has not shown that there is any difficulty in preventing an appreciable displacement. On the other hand, the proposed form must necessarily be more expensive than the existing one, the illumination is not so easily effected, and probably the observations will not be quite so good

with two collimator images as with one image bisected by a wire. In either form the circle should be provided with microscopes, which are more expeditiously read, and less fatiguing to the eyes, than verniers.

The examples which have been discussed in the preceding pages are sufficient to show that, under favorable circumstances, observations can be made with the sextant leaving very little to be desired as regards accuracy. In practice, however, such precision is not always, perhaps not commonly, attained. The most insidious source of error is an unstable condition of the eccentricity—a fault clearly traceable to defective construction when due attention has been paid to the care and preservation of the sextant. A judicious observer always endeavors to distribute his observations so as to neutralize the unknown errors of his instrument as nearly as possible; but variations in the eccentricity, which may occur at any moment, cannot be evaded by this means. If the conical axis is so improperly fitted as to be circumferentially supported only at the smaller end, its position will probably be maintained by friction, and the viscosity of the wax-like lubricant used for this bearing, until some extraneous force is applied, when displacement into a new position of temporary quiescence may be expected to ensue. Such a movement, to the extent of one-thousandth of an inch in a sextant of seven inches radius, may produce errors of 50″ and upward. Any unavoidable defect in fitting should evidently subsist in the direction opposite to that here supposed, and possibly some changes in the usual dimensions and materials of construction might be advantageous. But whatever the requisite alteration in existing practice may be, its discovery and adoption can safely be intrusted to the instrument-makers if a sufficient inducement to persevere in the search for improvement is offered to them. The mechanician who expends time and money in striving after a perfection which observers do not demand, or appreciate, unwisely impairs his ability to compete with his rivals. When sextants are so generally and adequately tested that the reputation of each maker rests on the actual merits of his work, a remedy for the evils of injudicious design and inferior workmanship will soon be found; until that time it cannot reasonably be expected.

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BIBLIOGRAPHY

OF THE

CHEMICAL INFLUENCE OF LIGHT.

BY

ALFRED TUCKERMAN, Ph. D.

WASHINGTON CITY:
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1891.

PREFACE.

This bibliography, having for its object the scientific aspects of the chemical influence of light, the practical applications, including that of Photography, are nearly all omitted, as has been the case in my previous works of this kind.

An index to the literature of Photography is being prepared under the auspices of the Committee for Indexing Chemical Literature, of the American Association for the Advancement of Science.

ALFRED TUCKERMAN.

NEWPORT, R. I.,
April, 189[illegible]

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SMITHSONIAN MISCELLANEOUS COLLECTIONS.

— 843 —

THE MECHANICS

OF THE

EARTH'S ATMOSPHERE.

A COLLECTION OF TRANSLATIONS

BY

CLEVELAND ABBE.

CITY OF WASHINGTON:
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1893.

CONTENTS.

THE MECHANICS OF THE EARTH'S ATMOSPHERE:

A COLLECTION OF TRANSLATIONS.

By Cleveland Abbe.

INTRODUCTION.

The complexity of the phenomena of the atmosphere has rendered it necessary to delay their mathematical treatment until our knowledge of hydro-dynamics and thermo-dynamics could attain the perfection which it began to acquire about the middle of this present century at the hands of Helmholtz, Clausius, Sir William Thomson, and their disciples. During the past few years some of the fundamental problems of meteorology have been treated analytically and graphically with great success. The present collection of translations presents some of the best memoirs that have lately been published on the respective subjects by European investigators; a few earlier memoirs of great excellence are included in the collection because of the references subsequently made to them. Other mathematical memoirs by Guldberg and Mohn, Marchi and Diro Kitao have been omitted because their length would have made this collection too large for the present mode of publication.

There is a crying need for more profound researches into the mechanics of the atmosphere, and believing as I do that meteorology can only be advanced beyond its present stage by the devotion to it of the highest talent in mathematical and experimental physics, I earnestly commend these memoirs to such students in our universities as are seeking new fields of applied science.

I have taken a very few liberties in translating the language and notation of the distinguished authors whose works are here collected. I have frequently used the word *liquid* instead of "Wasser," "Tropfbar-Flussigkeit," "Inkompressible Flussigkeit," and the word *gas* or *vapor* as equivalent to compressible or elastic fluid, and have used the word *fluid* when the more general term including liquids, vapors, and gases is needed. As the ideal or "perfect" liquid is absolutely incompressible and devoid of all resistance to mere change of shape, having neither elasticity nor viscosity, namely, internal friction, it seems more proper

to use the general terms liquid, gas, and fluid when neglecting the resistance, compressibility, elasticity, and viscosity as in dealing with these ideal substances, and to reserve the terms air, water, etc., for use when dealing with actual natural fluid phenomena where slight compressions and expansions and resistances occur.

The relation between elastic pressure, volume, and temperature, as deduced by Boyle, Mariotte, Gay-Lussac, and Charles, that characterizes a gas, and the equation for which the Germans call the "Zustands-Gleichung" in common with other equations of condition, I have preferred to speak of as the *equation of elasticity* or the characteristic equation of a perfect gas.

In view of the remarkable want of uniformity existing in English and American works in respect to the notation for total and partial differentials I have decided to make such alterations in the original notations of these papers as shall make the whole series consistent with the elegant and classical notation that is rapidly being adopted in Germany, and that will, I hope, eventually be accepted by all English and French writers. In accordance with this I shall always express the *total differential* by d, as first introduced into geometry by Leibnitz for the infinitesimal difference; the small increment or *variation* by δ, as introduced by Lagrange; the large *finite difference* by Δ, first used by Euler; the *partial differential* by ∂, ("the round d,") as used by Jacobi. Occasionally the dotted variable $\dot{x}$ will indicate the rate of variation with regard to the time, or the fluxion as first introduced into mathematical physics by Sir Isaac Newton, a notation which has lately been extensively revived in England by those devoted to classic authority.

Evidently the problems here treated by elegant mathematical methods are not always precisely the problems of nature. The differences between the conclusions of Rayleigh, Margules, and Ferrel as to the diurnal and semi-diurnal tides due to heat, or the differences between Ferrel, Oberbeck, and Siemens on the one hand and nature on the other as to the general circulation, show that by the omission of apparently minor local and periodical irregularities we have constructed for ourselves problems that still differ from the case of the earth's atmosphere, although they may more closely represent the conditions of such a planet as Jupiter.

I have to acknowledge the assistance of my friend, Mr. G. E. Curtis, in copying a portion of the formulæ for these translations, and renew the expression of my hope that a coming generation of American meteorologists may prosecute to further conquests the mathematical studies begun by Ferrel and perfected by our European colleagues.

CLEVELAND ABBE.

FEBRUARY, 1891.

THE MEASUREMENT OF THE RESISTANCES EXPERIENCED BY PLANE PLATES WHEN THEY ARE MOVED THROUGH THE AIR IN A DIRECTION NORMAL TO THEIR PLANES.*

By Professor G. H. L. Hagen.

Some time since I submitted to the Academy the results of a series of observations that I had instituted upon the motions of air and of water when the uniform flow of these fluids is interrupted by means of interposed planes.† By means of small bits of paper or tin foil floating from the tips of needles the direction of the motion could be perceived at every point. The velocities were indeed too feeble to be capable of direct measurement, but the disposition of particles of pulverized amber that were strewn over the water showed the limits of the strongest current, and when the coarser particles came to rest before the finer ones it was to be inferred that there was a gradual diminution of velocity at such points.

In general it was concluded that air and water alike swerve in curved paths in front of such obstacles and flow towards the free openings. In the latter and directly adjoining the outer ends of the obstacle the strongest current is formed which here retains its direction unaltered, therefore free from all variations. The deviation in front of the obstacle does not take place at any definite distance from it, but rather extends up to the obstacle itself and even when the plate faces the current it is seen that a feeble motion still exists immediately adjoining it.

Behind the obstacle the fluid by no means remains at rest, but rather there is always formed here a counter current whose length is equal to four or five times the distance of the head of the obstacle from the neighboring side wall of the channel, which counter current, however, is not only fed at its rear end, but principally also at two intermediate points by the steadily broadening main current. The latter immediately behind the head of the cross-wall meets the outcoming counter-current

*Read before the Academy of Sciences, Berlin, January 22, February 16, and April 20, 1874. (Translated from The Mathematical Memoirs [Abhandlungen] of the Royal Academy of Sciences at Berlin for the year 1874, pp. 1 to 31.)

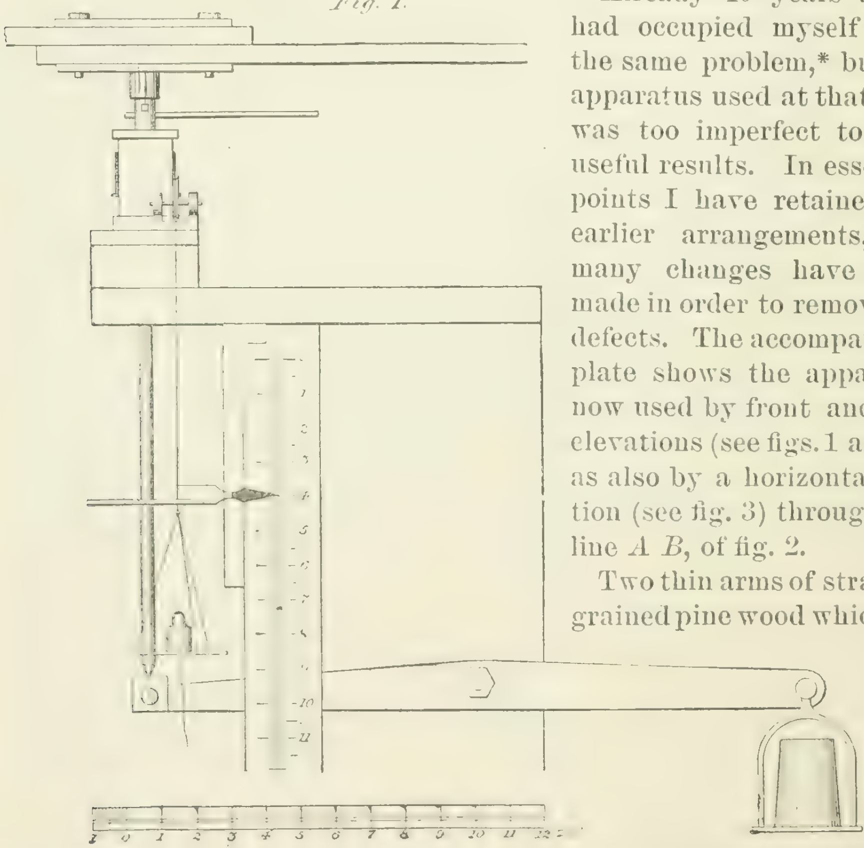
†See the Monats-Berichte for 1872, p. 861.

and here, as also at the two intermediate places just mentioned, whirls are formed which set in rotation the little vanes placed there. The phenomena agree with those that one observes in streams and rivers in front of and behind sharp protruding rocks or piers.

It must still be mentioned that neither water nor air rebounds like elastic spheres from the obstacle against which it strikes, as is frequently assumed. Even strong streams of water that I allowed to play against the plates did not rebound, but continued their onward path close to the obstacle, producing a strong current there.

I had instituted these experiments in order to see in what manner the resistances originate that the liquid experiences in such deviations and which cause the pressure against the opposing plate. However, I thought it was allowable to assume that when the plate is itself moved through stationary water or air the ratios remain nearly the same and that similar currents of the fluid occur in its neighborhood. The pressure that the plate experiences in this latter case is the object of the following investigation which is moreover confined to plane disks moved through the air in a direction perpendicular to their planes.

Fig. 1.



Already 40 years ago I had occupied myself with the same problem,* but the apparatus used at that time was too imperfect to give useful results. In essential points I have retained the earlier arrangements, but many changes have been made in order to remove the defects. The accompanying plate shows the apparatus now used by front and side elevations (see figs. 1 and 2), as also by a horizontal section (see fig. 3) through the line A B, of fig. 2.

Two thin arms of straight-grained pine wood which are

*Some of the series of observations made at that time are communicated as examples of the application of the method of least squares in the first edition of the "Grundzüge der Wahrscheinlichkeits-Rechnung."

bevelled on the sides that cut through the air, rest upon a vertical metal axis which communicates the rotary motion to them. Each of these arms is 8 feet or 96 Rhenish inches long and on its end the disk is fastened whose resistance is to be measured. In order to prevent the bending of the arms they are held not far from their ends by small wires which pass over a support 18 inches high vertically above the vertical axis. The drawing presents only the connection of the two arms between themselves and with the axis. The latter is in its upper portion turned slightly conical and carries the corresponding hollow hub which is screwed to the brass plate under the arms.

Fig. 2.

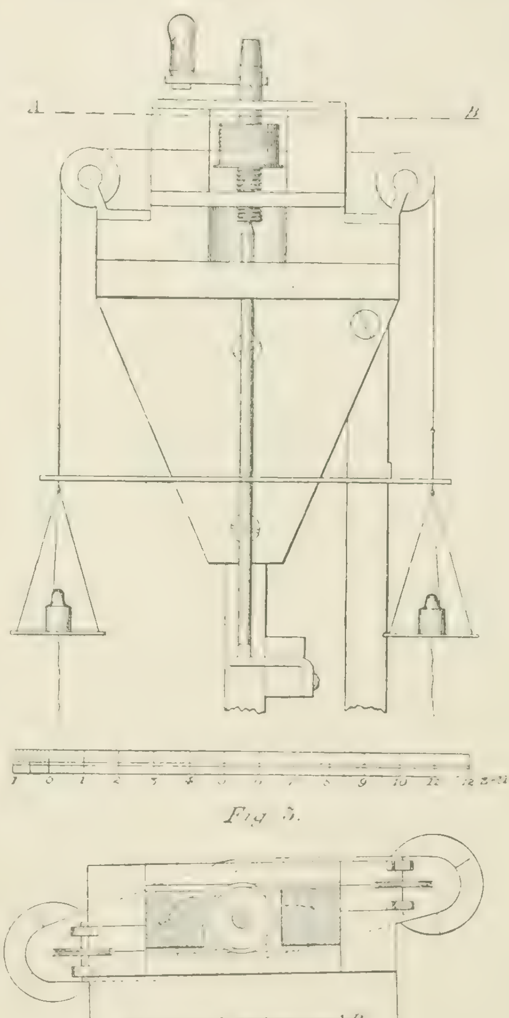
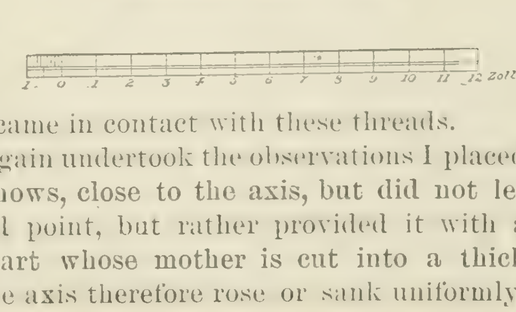


Fig. 3.



The rotation is brought about by the tension of two small threads which are wound in the same direction around the ivory spindle that is fastened to the axis, and are then drawn in opposite directions over two rollers and drawn taut by light scale pans with weights therein. These rollers I had formerly fastened at the greatest possible distance on the opposite walls of the room in order that when winding up the weights the threads might lie uniformly alongside of and not over each other, but this design was by no means certainly attained and the far-stretched threads materially increased the labor of the observation, especially since the arms and the disks fastened to them occasionally came in contact with these threads.

When in the past summer I again undertook the observations I placed the rollers, as the drawing shows, close to the axis, but did not let the latter stand upon a fixed point, but rather provided it with a screw thread on its lower part whose mother is cut into a thick plate of brass. By rotation the axis therefore rose or sank uniformly,

and the threads simultaneously arranged themselves alongside of each other on the spindle from which they were drawn always in a horizontal direction. Underneath the roller I connected both threads by means of a light rod, and on this hung the scale-pans for weights; I also fastened thereon a pointer which slid close to the graduated scale and served to measure the velocity.

Notwithstanding the great convenience of this change it introduced the troublesome consideration that the friction became disproportionately great and varied so much during the observation that its magnitude and its influence on the measured velocity could not be determined with the necessary accuracy. This great evil I removed in that I allowed a steel point to work in the conical depression already formed by the turning lathe at the lower end of the axis, which point exerted an upward pressure equal to the weight of the arms, the discs, and the axis. The axis is therefore completely supported by the steel point, and the screw serves only as a guide in order to raise and lower the spindle corresponding to the windings of the thread. This steel point forms the upper end of a stout wire 12 inches high, whose lower end, ground to a wedge shape, stands in a metallic groove that is fastened at the end of a lever whose equal arms are 19 inches long. This lever, whose center of gravity lies in its axis of rotation, was so formed that its axis lay in a straight line with the metal groove and the point of suspension of the scale-pan, and was equally distant from both. This pan, with the counterpoise, corresponded exactly to the pressure of the axis on the wire when no resisting discs were placed upon the arms, but as soon as the latter occurred the counterpoise was always increased in a corresponding degree by an appended light cup with shot. Before attaching the discs these were laid upon a balance and the cup was partly filled with shot until brought into equilibrium with it.

Since the lever changes its position during the rotation of the axis the steel wire deviates somewhat from the vertical position, but, as will be shown in the following, so slightly that this may be overlooked. The result of these changes in the apparatus proved to be very favorable, for whereas before at least 3 Prussian loths had to be placed in each scale in order to set the arms in permanent motion, now, the weight of the rod and the scale-pan, which together weighed 3.3 loths, sufficed without any additional weight to produce a uniform motion.

At the ends of the arms pieces of perforated cork are glued, and in these the stems of the various discs find their support. The discs were always pushed so far on that they closely touched the ends of the arms. The distance of the disc from the axis of rotation is found from the known lengths of the arms; the stems of the discs did not extend through the corks, therefore the resistance of the air against the arms was only increased by that which the discs themselves experienced. Therefore, after the resistance which the arms experienced at each velocity had been determined by observation of the rotation of the

arms under various loads, this could be subtracted each time from the resistance observed with the disc in place and thereby the resistance of the various discs for various velocities be determined.

The ivory spindle around which the threads wind was, like the axis, very carefully turned cylindrical, and is 1.1 inches high and 1.6 inches in diameter. The portion of the axis extending above the spindle is also turned cylindrical so that for any position of the upper perforated brass plate it is securely held with very little play. Under its slightly conical flat head are found, as the figure shows, two openings perpendicular to each other, one square and the other circular. The first serves for the introduction of a small crank handle by means of which the axis is turned backwards when the weights are being raised. Before taking off the arms a wire is put through the circular opening which prevents the axis from turning forward while the observer is taking off the crank and putting on the arms and discs. Moreover, at a distance of 12 inches from the axis there is placed a bent lever, one arm of which stands upright and hinders the turning of the arms that carry the discs when the weight fastened to the other arm of the bent lever hangs freely. While the arm carrying the discs is thus held by the bent lever the stout wire is withdrawn and the air is allowed to come to rest. If the weight be placed on the neighboring table then the bent-lever arm falls and the apparatus starts in motion.

The pitch of the screw of the axis below the spindle is 0.05 inch, and this distance corresponds to the width of both threads so that the latter lie regularly close to each other on the surface of the spindle. This always occurred very regularly even when the axis was turned very rapidly by means of the crank handle. The threads, the so-called "iron twine," were so strong that each with safety carried 4 pounds, which weight, however, was never even distantly approached in practice. The threads were so light that 40 feet weighed only 0.1 loth, so that the fall of the index by 6 feet increased the driving power by only 0.03 loth. Nevertheless, for very feeble loads in the scale-pans a slight increase in the velocity was apparent during the descent, and in order to prevent this the small increase in the weight was annulled by means of two equal threads suspended from the scale-pans to the floor.

Since the two former or driving threads were fastened to the rod they were thereby prevented from turning and unwinding, which I had been able to avoid in my earlier work only by guiding the scale-pans by means of taut wires. Even if, however, the threads by this method of fastening did not materially change, still it remained to be proved whether perhaps they lengthened sensibly with greater tension, in which case the relation between the path of the index and the rotation of the arms could not remain constant. Such an extension could not be mistaken when I laid a weight of 1 pound on the empty scale-pans when they were at their lowest position. The index then sank at once 0.2 of an inch. A further extension, however, did not follow; at least

it was not to be observed in the short interval occupied by each separate observation. In consequence of this extension of the threads it was incumbent to lay those weights that were to be used to set the axis in motion during the next observation upon the scale-pan while the latter was at its lowest position. The threads were therefore always wound up under the same tension with which they were to do the work.

The question now arose whether with stronger tensions the spiral windings of the threads perhaps lay flatter on the spindle than with weaker tensions, and whether therefore the length of a winding or the path that the index described for one turn of the arms became shorter. This point was decided in that with various loads in the scale-pan I measured the path that the index described during a certain number of revolutions. The above-mentioned bent lever offered the opportunity of always stopping the arms at the same point, but it was necessary to bring them to rest by gentle pressure, because with a strong blow against the upright standing arm the horizontal arms carrying the discs could easily turn somewhat on the conical head of the axis. After the position of the index was read off I allowed the arms to make five complete turns and again read off the position of the index on the scale, estimating only to the hundredth part of an inch.

The lengths of the paths for the corresponding weights in each scale-pan are as follows:

Weight.*	Path.†
0	25.69
4	.67
8	.68
16	.66
24	.67
28	.65

* Prussian loths. † Rhenish inches.

A very slight shortening of the path appears from this to occur for the heavier loads, but if it actually exists it is so small that it is far less than the accuracy of the measurement of the path of the index on the divided scale. It may therefore be assumed that the velocity of the index stands in a constant ratio to that of the arms or disks.

The lengths of the individual windings of the thread around the spindle as resulting from the above measures do not correspond in all accuracy to the circumference of a circle that is normal to the axis of the spindle, and at a distance therefrom equal to that of the central axis of the threads, inasmuch as the threads lie spirally around the spindle. Now the pitch of the screw measures 0.05 inch; therefore the threads on the surface of the spindle make an angle with the horizon $0° 33' 29''$. Since the average length of one winding of the thread is 5.134 inches, therefore the equivalent thread encircling the normal is somewhat smaller, namely, 5.1338. Hence the resulting distance of the center of the threads from the axis of rotation or the length of the lever arm by

which the weight acts is equal to 0.81705 inch. This figure is adopted in the following computations, where it is represented by the letter a.

It remains still to investigate whether the steel wire that carries the axis may perhaps depart so far from the vertical direction by the movement of the lever on which it rests that it occasionally may exert an appreciable side pressure and thereby in an injurious way increase the friction in the screw threads. The lever is, as was mentioned, not only perfectly balanced, but the point that carries the counterpoise is also situated in the prolongation of the straight line drawn through the supporting point of the wire and the rotation axis of the lever. Therefore for every position of the lever the foot of the wire is pressed upwards vertically with equal force, but it rises only 0.8 of an inch, while the weight that drives the disks around in the extreme case sinks 80 inches. Therefore the deviation of the foot of the steel wire from a mean position amounts only to 0.4 of an inch, or in angle 2° 24′ 48″, for a length of the lever arm of 9.5 inches. Therefore the deviation from the initial verticality is limited to 0.0086 inch, and consequently the wire 12 inches long is inclined 0° 2′ 38″ to the vertical. Even this small inclination can be reduced by one-half if we place the axis of the wire or its upper point in the vertical line that bisects the deviation of its lower end, but such accuracy in the establishment of the apparatus must not be anticipated. It is evident from this that there can be no sensible increase of the friction in consequence of the movement of the lever.

As regards the execution of the observations the remark must be prefaced that the Rhenish inch, or the twelfth part of the Prussian foot according to the earlier determination of the standard, and the old Prussian loth, of which 32 make 1 Prussian pfund, have been adopted as units of length and weight.* The divided scale over which the index glides is divided into tenths of inches, but this subdivision is only used for determining the length of one winding of the thread, as previously described. In all other cases only the transit of the index over the heavier division marks for each 10 inches was observed by the beating of the seconds clock and the corresponding whole or half seconds noted.

Since at the beginning of an observation the arms do not immediately assume that velocity for which the resistance in connection with the friction balances the acceleration, therefore the significant observations began only when the weight had fallen 20 inches or the index had passed over the twentieth inch mark. At the seventieth inch the weight-scale pan had approached the floor, and therefore here the measures must be stopped. When, however, the rotation of the arms was observed without disks and the weights employed were very slight, then the speed continued increasing somewhat longer and the time of transit over the twentieth inch could not be used in the calculations.

[* One Rhenish inch = 1.0297217 English inch = 26.15446 millimetres. One Prussian loth = 0.032226 pounds avoirdupois = 14.616 grammes. (See Barnard's Weights and Measures, C. A.]

In order to determine with the greatest accuracy the resistance of the air against each separate pair of disks it certainly would have been advantageous to employ very different weights and thereby attain very different velocities. This intention, however, could not be carried out by reason of the moderate length of the arms, which was limited by the dimensions of the room. If I loaded each scale pan with more than 1 pfund then the whole mass of air in the room, especially when using larger disks, assumed a rotatory motion, in which case the resistance during the individual observation is always less or the velocity is always greater. Even with a load of 1 pfund the light paper vanes that floated at the tips of the needles already showed a feeble continuous rotation, although the flame of a candle did not allow of its recognition. In all the following observations therefore in the extreme cases only 28 loth was placed in each scale pan. To this it is to be added also that the measurements for very large velocities lose in accuracy on account of the relative magnitude of the unavoidable error. According to this the index should not move faster than an inch in 1.8 seconds. On the other hand, however, on account of the excessive influence of the very variable friction, the movement became highly irregular, when more than 8 seconds elapsed while the index described 1 inch. Within these limits the times in which 10 inches were described did not easily deviate more than half a second from the average value. The velocities of the disks were therefore not greater than 66 and not less than 17 inches per second.*

In order to attain a uniform tension with reference to the axis the weights placed in the two scale pans were always equal and since on each occasion the disks attached to the arms were also always of equal magnitude, therefore each of these weights corresponded to the resistance of one disk. To this indeed should still be added one-half of the weight of the rod and the two scale pans but this may be disregarded since for each individual observation the value of the constant term which indicates the friction has to be especially computed. This constant term will then be the sum total of these weights less the friction, and presented itself always with the negative sign because the friction remained less than the weight of the rod and the scales.

In order to simplify the computation I have at first referred not to the velocity of the disks, but only to that of the index, whence as above mentioned the velocity of the rotation can be easily deduced. In this way the opportunity was offered at each observation with disks to take into consideration that resistance which the arms alone experienced for the corresponding velocity of rotation.

Before and after each series of observations, which generally occupied 3 or 4 hours, the barometer and thermometer were read off, the latter being at the same altitude above the floor as that at which the arms revolved. The computed coefficients of resistance were re-

* Between 3.8 miles and 0.9 mile per hour.

duced to the barometric pressure of 28 Paris inches and the temperature 12° Réaumur or 15° C. Assuming that the resistance of the air is proportional to its density I formed a table of the logarithms of this correction whereby the separate reduction is very easy. In case the temperature sensibly changes during the time of observation it must be assumed that this change occurred gradually and therefore for each individual observation the correction corresponding to the time is adopted. When especially large variations occurred readings were also made in the intervals; still, in such cases very large deviations were sometimes apparent, and it was repeatedly remarked that then the movement of the arms steadily increased or that the times in which the index sank 10 inches became smaller the lower its position was, which never occurred with uniform temperature. The reason of this is certainly nothing else but this, that the equilibrium of the warmer and colder air in the room gave rise to special currents that were combined with the movement of the disks. When the temperature during a series of observations changed by two degrees or more, the results deduced became so discrepant that they had to be rejected as entirely useless. For this reason the room before and during the observation could not be heated warm. On the contrary, the oven used for heating the room must be cooled down completely. Even when the sun shone on the window whose shutters could not hinder the warming, nothing remained but to stop the observations.

Almost equally troublesome was the friction in the various parts of the apparatus. This varied perpetually, wherefore its value for each individual observation had to be especially determined. Of course it diminished when fresh oil was introduced between the rubbing surfaces, but then the variations became of such magnitude and were often so sudden that the observations were again useless. Only after many days and after the arms had remained for a long time continuously in motion there was established a greater regularity. When this, however, became evident from the measures immediately following each other, then again on the next day the conditions would be remarkably changed. It was therefore necessary that the whole of any series of observations that were to be compared among themselves should be made in immediate succession. In order to render this possible it was necessary to reduce the number of measures as much as was any way allowable, namely, to the number of the desired constants. Such a course is defensible also because the individual readings, in a long series of observations, accord much more closely with the law deduced therefrom than with the similar measures repeated at other times.

These preliminary remarks are the result of the great number of observations that I have executed during a half year. These were, especially at the first, extremely unreliable, and only gradually were all the circumstances perceived that come into consideration. The following observations, which are the only ones serving as a basis for the sub-

sequent computations, were made at recent dates with the greatest possible care and under quite favorable external conditions.

The resistance that the arms alone experience for different velocities must first be determined because this must be subtracted every time from the total resistance of the disc and the arms. The following table contains the measures made on this point. G is the weight [in loths] that is placed in each scale-pan, and t the number of seconds occupied by the index in passing over 1 inch. The velocity of the index is therefore equal to $\frac{1}{t}$ according to the adopted unit of measure. The observations were made twice for each load in the scale-pan, and in the second column of the table the two values t_1 and t_2 thus found are given separately, while the third column contains the mean value (t) adopted in the succeeding computation.

G.	t_1.	t_2.	t.	A.	Diff.	B.	Diff.
0.0	5.725	5.725	5.725	0.040	+0.040	−0.009	−0.009
0.5	4.238	4.225	4.2315	0.514	+0.014	+0.498	−0.002
1.0	3.488	3.500	3.494	1.001	+0.001	1.007	+0.007
2.0	2.725	2.735	2.730	1.979	−0.021	2.006	+ 6
3.0	2.300	2.312	2.306	2.986	−0.014	3.018	+ 18
4.0	2.038	2.038	2.038	3 972	−0.028	4.001	+ 1
6.0	1.700	1.700	1 700	5.941	−0.059	5.946	− 054
8.0	1.475	1.675	1.475	8.066	+0.066	8, 029	+ .029

Earlier observations had shown that the resistances could be expressed by the simple formula

$$G = z + \frac{1}{t^2} S$$

On attempting to introduce a third term containing as factor the first power of the velocity the constant coefficient corresponding had a very slight value and even sometimes a negative one. Therefore I now first chose the preceding expression, and by the method of least squares found

$$z = - \ 0.531$$
$$S = + 18.703$$

By the introduction of these constants I obtained the values for G, which are given in the column headed A. The next following column shows the error or the differences (A—G) for each of the weights actually used. We notice that these errors progress very regularly in that both for the smallest and largest values of G they attain the largest positive values while between these they become negative. From this circumstance it may be inferred that the form of the formula has not been appropriately chosen, and I therefore repeated the computation using the expression

$$G = z + \frac{1}{t} p + \frac{1}{t^2} s$$

This then gave,

$$z = -\ 0.724$$
$$p = +\ 1.034$$
$$s = +15.518$$

According to this last we obtain for G the values given in the column headed B, whose errors $B - G$ are shown in the last column. We remark that these latter do not occur regularly, owing to the change of the signs for the heavier weights, and therefore can be looked upon as accidental errors of observation. The sum of the squares of the errors amounts in the last case to 0.004252, whereas in the first case it was 0.011055, therefore more than twice as great.

There is still another reason that favors the introduction of the first power of the velocity. So long as I neglected this term there occurred without exception the inexplicable phenomenon that for observations with disks the numerical value of the constant z after the negative sign was always greater, therefore the friction was always smaller, the larger and heavier the disks were. This anomaly disappeared upon the introduction of such a second term.

There is, moreover, as the observations show, a peculiar condition in connection with the second term. The coefficient p assumes a very small value or entirely disappears when the screw on the axis is freshly oiled. From this we may conclude something as to its significance. *i. e.*, it indicates the resistance that arises from the viscosity of the oil and which is proportional to the velocity.

When disks are attached, the resistance peculiar to them is found when we subtract from the observed resistance that which the arms experience for equal velocities. This latter, however, is so variable that we must measure it anew every time, and since it assumes various values within even short intervals, therefore there remains only one method to determine the value of the three constants z, p, and s, namely, to allow the arms to revolve alone with three different velocities both before and after each observation. When, however, as usually happened, a second measure again gave somewhat different values, then the appropriate mean value corresponding to the intervening time should be used in the computation.

In the resistances of the disks found in this manner the second term proportional to the velocity is no longer contained, because the influence of the viscosity of the oil has already been allowed for in the resistances of the arms. The constant z is, on the other hand, so variable that it must be specially deduced from each series of observations.

The following observations were made with two square disks of 6 inches on each side.* G' indicates the weight placed in each scale pan, and this changes to G when we subtract the weight required to overcome the resistance of the arms for equal velocities. The second column contains as before the times during which the index sinks by 1 inch, as found from the two measurements respectively.

G'	t'	t''	t	G	A	*Diff.*
loth.	*sec.*	*sec.*	*sec.*	*loth.*	*loth.*	*loth.*
1	9.42	....	9.42	1.117	1.064	−0.053
2	7.32	....	7.32	1.986	1.983	− .003
3	6.22	....	6.22	2.860	2.875	+ .015
4	[illegible]	5.54	[illegible]	[illegible]	[illegible]	[illegible]
6	4.62	4.63	4.625	5.492	5.472	− .020
8	4.02	4.04	4.03	7.231	7.314	+ .083
12	3.35	3.33	3.34	10.737	10.798	+ .061
16	2.92	2.92	2.92	14.251	14.235	− .016
20	2.62	2.64	2.63	17.770	17.625	− .145
24	2.39	2.40	2.395	21.247	21.323	+ .076
28	2.23	2.22	2.225	24.760	24.760	.000

Adopting the expression,

$$G = z + \frac{1}{t^2}\, r$$

I find as most probable values

$$z = - \quad 0.335$$
$$r = + 124.24$$

From this the values of G given in the column marked A are deduced for the respective times. The errors of these, as contained in the following column, vary so much in sign that we can consider them as accidental and there is no reason to introduce still another term in the above expression. In this connection it must still be mentioned that when in the computation of the earlier observations I have assumed the coefficient p equal to zero, a satisfactory agreement of the resistances appears for larger disks as soon as I set the resistance proportional to the square of the velocity. This is explained by the fact that the value of the term $\frac{p}{t}$ is very small in comparison with the stronger resistances which the disks experience.

[*In all that follows it is to be understood that before and after each series for the determination of the resistance of the disks a special series has been made with the arms without disks for the determination of the combined correction for arms plus friction, and that thence the correction for the resistance due to the arms has been computed. G' is the weight required to overcome the friction plus the resistance of the disks and arms; G is the weight required to overcome the friction plus resistance of the air to the motion of the disks; z is the weight required to overcome the friction; $\frac{r}{t^2}$ is the weight required to overcome the resistance to the disks.—C. A.]

The resistance of the air against the disks is therefore proportional to the square of the velocity, and a single observation would suffice to give the coefficient r if the value of z were known, but since this is so very variable, therefore at least two observations at two different velocities are necessary. The further extension of the measures is unnecessary, as already before mentioned, because the greater accuracy attained surpasses the other inevitable errors; but for greater security and especially to avoid possible mistakes I have always repeated these two measures, and in such a way that beginning with the less velocity I then execute the two measures with the greater velocity and finally return again to the less.

From the values of r found in this manner the pressure that the disk experiences for various velocities is directly given. Let a be the known distance of the axis of rotation from the center of the threads wound round the spindle and R the distance of the same axis from the center of pressure of the air against the disk, then this pressure becomes

$$D=\frac{a}{R}(G-z)=\frac{a}{t^2R}r$$

But $\frac{1}{t}$ is the velocity of the thread, hence the velocity of the center of pressure of the disk is

$$c=\frac{R}{at}$$

and

$$D=\frac{a^3}{R^3}r\,c^2.$$

if we introduce the pressure on a unit of surface, since F is the whole surface of the disk, we have

$$\frac{D}{F}=\frac{a^3}{R^3}\cdot\frac{r}{F}c^2=kc^2$$

In order to reduce the constant r to the barometric pressure of 28 inches or 336 Paris lines, and to reduce the temperature to 15° C., we have for an observed pressure, λ, in Paris lines, and an observed temperature τ in centigrade degrees during the observations the reduced r

$$=\frac{336}{\lambda}(0.9480+0.00347\tau)\,r.$$

The distances R, on account of the great lengths of the arms in comparison with the width of the disks, agree quite nearly with the distances of their centers of gravity from the axis of rotation, but they are always somewhat larger and there is no reason to omit this correction, which is easily executed.

We consider first a rectangular disk whose height is h and width b. As the origin of abscissas we may take its center of gravity whose distance from the axis of rotation is A, and consider the disk divided into

elementary portions, the area of any one of which is hdx, and the pressure that it experiences is

$$dD=\frac{Kh}{a^2t^2}(A+x)^2\,dx$$

consequently the pressure against the whole disk, found by taking the integral from $x=-\frac{1}{2}b$ to $x=+\frac{1}{2}b$, is

$$D=\frac{Khb}{a^2t^2}(A^2+\tfrac{1}{12}b^2)$$

or the average normal pressure on a unit of surface is

$$\frac{D}{F}=\frac{K}{a^2t^2}(A^2+\tfrac{1}{12}b^2).$$

If now I seek that value of x which belongs to the elementary area that experiences a pressure the same as this average, then it represents the center of pressure for the whole disk. The result is,

$$A+x=R=\sqrt{A^2+\tfrac{1}{12}b^2}$$

For circular disks we again take A as the distance of the center from the axis of rotation, while the radius of the disk is ρ. In the division of the disk into elementary vertical sections I indicate the limits of these by the angle φ which is measured from the horizontal diameter. The area of such a section is then

$$2\,\varphi\sin\varphi^2\,d\,\varphi$$

and the pressure that it experiences is

$$dD=\frac{2\,K\,\rho^2}{t^2}\left(\frac{A+\rho\cos\varphi}{a}\right)^2\sin\varphi^2\,d\varphi$$

By expanding the binomial and converting the $\cos^2\varphi$ and $\cos^4\varphi$ into the sines of the multiple angle, the integration becomes very simple and the greater part of the terms disappear since the integral is taken from $\cos\varphi=-1$ to $\cos\varphi=+1$. We obtain

$$D=\frac{K\,\rho^2}{t^2\,a^2}(A^2+\tfrac{1}{4}\rho^2)\,\pi$$

or

$$\frac{D}{F}=\frac{K}{t^2\,a^2}(A^2+\tfrac{1}{4}\rho^2)$$

and the section that experiences this average pressure is that whose φ satisfies the equation—

$$A+\rho\cos\varphi=R=\sqrt{A^2+\frac{1}{4}\rho^2}$$

It follows that in both kinds of disks the difference between A and the desired R remains very small when, as in my apparatus, A is very large compared with b and ρ.

Next, a series of observations will be communicated, made with five pairs of circular disks, whose diameters were 2.5, 3.5, 4.5, 5.5, 6.5 inches. Each time only two different weights were laid in the scale-pan; with

these, however, as above mentioned, the measures were executed twice. The resulting values of z and r are given in the last columns. The other letters correspond to those above given:

ρ	G^1	t_1	t_2	t	G	z	r
1.25	0.75	5.42	5.42	5.42	—0.041		
	9.0	2.00	1.988	1.994	+4.058	—0.683	18.850
1.75	1.5	5.31	5.30	5.305	0.690		
	14	2.00	1.98	1.990	9.054	—0.679	38.545
2.25	2	5.76	5.68	5.720	1.302		
	20	2.04	2.03	2.035	15.270	—0.722	66.243
2.75	3	5.89	5.86	5.875	2.345		
	24	2.24	2.24	2.240	20.079	—0.671	104.117
3.25	3	6.97	6.89	6.930	2.521		
	28	2.43	2.44	2.435	24.670	—0.599	149.827

In order from this to find the pressure on the unit of surface, or k, we have to assume the lever arm $a=0.81705$ inch, as already shown above. The following table contains the values of R, as well as the reduced r, and the surfaces of the disks F, as to which latter it is to be noticed that after more careful measurements the radii of the second and third disks resulted 1.745 and 2.245:

TABLE I.

ρ	1.25	1.745	2.245	2.75	3.25
R	97.252	97.754	98.256	96.760	99.260
* Reduced r	18.791	38,463	66 165	104.095	149.942
F	4.909	9.566	15.834	23.758	33.182
k	2.2760	2.3476	2.4028	2.4810	2.5199

* i. e., Reduced to standard density of air.

In order to avoid too small numbers these values of k are given too large, and must be divided by one million in order to present the desired constant factors, which, multiplied by the squares of the velocities in inches, will give the pressures in loths for each square inch of the disk. This same multiplication of k is also continued in the following paragraphs.

Many days later I repeated these observations with the same disks. The results were—

ρ	G^1	t_1	t_2	t	G	z	r
1.25	1	5.00	5.02	5.01	0.168	—0.592	19.091
	10	1.91	1.91	1.91	4.641		
1.75	1.5	5.21	5.22	5.215	0.721		
	16	1.87	1.87	1.87	10.405	—0.708	38.861
2.25	2	5.67	5.70	5.685	1.329		
	20	2.05	2.04	2.045	15.291	—0.746	67.066
2.75	3	5.74	5.79	5.765	2.338		
	24	2.23	2.24	2.235	20.025	—0.790	103.983
3.25	4	6.06	6.09	6.075	3.391		
	28	2.45	2.43	2.44	24.633	—0.694	150.786

The values of R and F are the same as in the first series. The following values of k are computed from the reduced r:

TABLE II.

ρ	1.25	1.745	2.245	2.75	3.25
Reduced r	18.952	38.576	66.575	103.221	149.683
k	2.2894	2.3549	2.4176	2.4602	2.5154

It evidently results that k becomes larger as soon as the surface of the disk increases, as also that the differences are proportional, not to the increase of the surfaces, but to the increase of the radii.

Measures were also made with square disks whose sides measured $b = 2$, 3, 4, 5, 6 inches, respectively. These gave—

b	G^1	t_1	t_2	t	G	z	r
2	0.5	5.80	5.86	5.830	—0.188		
	10	1.84	1.83	1.835	+4.104	—0.66[illegible]	16.042
3	1	6.00	5.95	5.975	+0.346		
	14	1.97	1.96	1.965	8.840	—0.684	36.774
4	2	6.06	6.03	6.045	1.364		
	20	2.08	2.08	2.080	15.383	—0.519	68.798
5	3	5.99	6.06	6.025	2.364		
	24	2.30	2.28	2.290	20.168	—0.643	109.135
6	4	6.50	6.43	6.465	3.443		
	28	2.55	2.54	2.545	24.874	—0.488	164.270

The closer investigation showed again that the surfaces of the disks in part needed some small corrections, as in the following Table III:

TABLE III.

b	2	3	4	5	6
R	97.002	97.504	98.008	98.512	99.015
Reduced r	15.607	35.810	67.053	106.455	160.522
F	4.000	8.977	16.000	24.958	36.000
k	2.3317	2.3472	2.4281	2.4338	2.5055

The following results were given by a subsequent repetition of the same observations:

b	G^1	t_1	t_2	t	G	z	r
2	0.5	5.76	5.79	5.775	—0.149	—0.630	16.020
	10	1.84	1.83	1.835	+4.128		
3	1	5.96	5.94	5.950	+0.397	—0.641	36.744
	14	1.96	1.97	1.965	8.876		
4	2	5.74	5.78	5.760	1.371	—0.608	68,976
	20	2.07	2.07	2.070	15.387		
5	3	5.92	5.93	5.925	2.415	—0.714	109.855
	24	2.29	2.29	2.290	20.233		
6	4	6.26	6.26	6.260	3.485	—0.700	164.000
	28	2.53	2.53	2.530	24.922		

According to this, the values of k are:

TABLE IV.

b	2	3	4	5	6
Reduced r ..	15.704	35.998	67.524	107.493	160.378
k ..	2.3461	2.3595	2.4452	2.4574	2.5032

By connecting among themselves the two first, as also the two last series of observations, the law according to which the value of k depends on the size of the disk may be approximately recognized, but the relation between the two forms of disks does not appear clearly. In order to discover this I tried allowing circular and square disks to run one immediately after the other, the radius of the first being 0.5 greater than the side of the latter. From this, however, it could only be inferred that for equal areas the resistance of the square disk is the greater.

In order to recognize the influence of the shape I tried also disks which formed equilateral triangles of 7.6 inches on each side, which were fastened in such a way that one of the sides stood vertically at the end of an arm. The area of each disk measured 25 square inches, agreeing, therefore, to within a very small quantity, which subsequent accurate measures showed, with that of the square disk of 5 inches on a side. As I observed these two pair of disks one immediately after the other under the same load, it appeared that the square disk revolved somewhat more rapidly. This result, however, was not decisive, in that the distances of the centers of pressure from the axis of rotation, or R, did not remain the same. In this respect it may be mentioned that when the side of the equilateral triangle $=b$ and its altitude $=h=b \cos 30°$ and the distance of the center of the surface from the axis of rotation is A, we then find

$$R=\sqrt{(A^2+\tfrac{1}{18}h^2)}$$

A complete series of observations, together with the preliminary and the concluding determinations of the value of p and s, gave the following:

G^1	t	G	z	r
3	5.91	2.220	....	....
6	4.35	4.715	....	....
10	3.43	8.081	....	....
28	2.12	23.525	−0.875	+108.640

After the computation of $R=98.204$, as also after the reduction of F and r there is found

$$K=2.5026.$$

Directly following the above, the same observations were repeated with the square disk of 5 inches on a side with the following results:

G^1	t	G	z	r
3	5.96	2.234	....	....
6	4.40	4.739	....	....
10	3.46	8.110	....	....
28	2.10	23.448	−0.875	+107.390

From these latter there finally resulted

$$k = +2.4491.$$

The results thus far obtained warrant the suspicion that for equal areas of the disks, the resistance becomes smaller the shorter is the deviated path that the air must describe in order to pass around the disk. Hence it is to be expected that the resistance would become especially small for long and narrow disks. Consequently I took a pair of disks 1 inch broad and 16 inches high, which therefore had the same area as the square disks of 4 inches on a side. These I allowed to run interchangeably with the square disks and under equal loads, but most unexpectedly the velocity of the square disks was always somewhat greater than the narrow ones. This was so much the more remarkable as the square ones, on account of the greater distances from the axis of rotation, were expected to show a greater resistance.

As at first I allowed these long disks to run under only two different loadings, I found

G^1	t	t	t	G	z	r
2	6.33	6.69	6.51	1.514	....	....
20	2.07	2.09	2.08	15.488	−0.075	67.332

For the feeble load the velocity had shown very discrepant values. Therefore the repetition of the observation was important, and for greater security this was done on the following day for six different loads.

G^1	t	G	A	Diff.
1	8.51	0.748	0.743	−0.005
2	6.28	1.538	1.508	−0.030
4	4.48	3.049	3.127	+0.078
8	3.23	6.254	6.174	−0.080
16	2.28	12.495	12.564	+0.069
24	1.87	18.790	18.760	−0.030

From this there results as the most probable values

$$z = -\ 0.171$$
$$r = +66.199$$

If these constants are introduced into the expression for G, the latter assumes the values given the column headed A, whose departures from the observed G are given in the last column.

The surfaces of these disks measured very accurately 16 square inches, and the distance of the center of pressure was 96.500 inches. After reduction to the adopted normal density of the air the constants r for the two series of observations became respectively

66.65 and 66.373

whence

$$k = 2.5286 \text{ and } k = 2.5178 \text{ respectively.}$$

The constant coefficient of the square of the velocity resulted therefore in this case as great as the series of observations III and IV would have led us to conclude would have been found for square disks of about 7 inches on each side; consequently the suspicion arises that the increase in the value of k is not proportional to any linear dimension, but to the circumference of the disk. A simple consideration leads to the same result.

All previously given observations show that a disk of an area F moving with a velocity c through the air in. a direction normal to its plane experiences a resistance

$$D = k\ Fc^2.$$

If we analyze k into two terms

$$k = \alpha + p\ \beta$$

where p expresses the circumference of the disk, then the first part of D, namely, $\alpha\ Fc^2$, corresponds to the ordinary assumption. The second part

$$p\ Fc^2\ \beta = Fc.\ p.\ c.\ \beta$$

contains, as a factor, the mass of the passing air, which is proportional to Fc, also p, or the circumference of the disk, which the air touches, and finally the velocity c, under which this contact takes place. It appears therefore that the cause of the increase of the resistance can be none other than the friction of the air against the edge of the disk. However, as the experiments already mentioned in the preface have shown, the air immediately adjacent to the edge of the disk flows perfectly regularly past it, without taking up any whirling motion, which latter first forms behind, where the air protected by the obstacle is touched. Friction is therefore (in accordance with the experience* with water) proportional to the first power of the velocity.

Before I computed the appropriate constants by the combination of all of the observations, I made an attempt to compare among them-

* "On the Influence of the Temperature on the Movement of Water in Tubes." Hagen, *Math. Abh. Akad. Wiss. Berlin*, 1854, p. 69.

selves the twenty-one observations made with circular and square disks, in order to convince myself as to what assumed value of p presented the greatest agreement.

If I assumed for p the circumference of the disk, there resulted

$$\alpha = 2.210$$
$$\beta = 0.0132$$

and the sum of the squares of the outstanding errors was

$$[xx] = 0.01425$$

By introducing the square root of the surface I obtained

$$\alpha = 2.200$$
$$\beta = 0.0526$$
$$[xx] = 0.00976$$

I then put p, equal to three different transverse lines drawn through the center of the disk. First, the smallest transversals, for which of course the sides of the square and the diameters of the circles were directly introduced. This gave

$$\alpha = 2.204$$
$$\beta = 0.0487$$
$$[xx] = 0.01282$$

For the greatest transversals, namely, the diagonals of the squares and diameters of the circles, I obtained

$$\alpha = 2.230$$
$$\beta = 0.0354$$
$$[xx] = 0.02221$$

Finally, for the average transversals which I drew [centrally] across the disks at distances apart of every 3 degrees, and took the arithmetical mean of all, I found

$$\alpha = 2.200$$
$$\beta = 0.04675$$
$$[xx] = 0.00966$$

It is evident that this latter method must lead to very nearly the same result as the introduction of the square root of the surface since β diminishes in the same ratio as the coefficient of β increases.

Judging by the sums of the squares of the errors it would, according to this, be advisable to introduce the square roots of the surfaces as factors, but this is impossible, even although the results of the observations made with the long disk should be included under this same law. There only remains to introduce the circumference as a factor, even although in this case notable departures still remain. These are in nowise however errors of observation, but result principally from the inevitable variations in friction. An error of 1 per cent. in the time could scarcely have been made, but still such discrepancies and even larger ones show themselves very frequently since the friction induced now faster and now slower motion. Nevertheless, from the following collection of all the observations it results that these have led to a quite trustworthy result.

Radii and sides.		k	p	A	Diff.	Squares.
Circle $\rho=$	1.25	2.270	7.854	2.338	+0.068	0.004624
	1.75	2.348	10.996	2.368	+0.020	0400
	2.25	2.403	14.137	2.397	−0.006	0036
	2.75	2.481	17.279	2.427	−0.054	2916
	3.25	2.520	20.420	2.456	−0.064	4096
Circle $\rho=$	1.25	2.289	7.854	2.338	+0.049	2401
	1.75	2.355	10.996	2.368	+0.013	0169
	2.25	2.418	14.137	2.397	−0.021	0441
	2.75	2.460	17.279	2.427	−0.033	1089
	3.25	2.515	20.420	2.456	−0.059	3481
Square $b=$	2.	2.332	8.0	2.339	+0.007	0049
	3.	2.347	12.0	2.377	+0.030	0900
	5.	2.428	16.0	2.415	−0.013	0169
	5.	2.434	20.0	2.452	+0.018	0324
	6.	2.505	24.0	2.490	−0.015	0225
Square $b=$	2.	2.346	8.0	2.339	−0.007	0049
	3.	2.360	12.0	2.377	+0.017	0289
	4.	2.445	16.9	2.415	−0.030	0900
	5.	2.457	20.0	2.452	−0.007	0049
	6.	2 503	24.0	2.490	−0 013	0169
Triangle..........		2.503	22.795	2.479	−0.024	0576
Square $b=$	5.	2.449	20.0	2.452	+0.003	0009
Parallelogram		2.529	34.0	2.584	+0.055	3025
Parallelogram		2.518	34.0	2.584	+0.066	4356
						0.030742

From this table there results as the most probable values

$$\alpha = 2.2639$$

$$\beta = 0.009416$$

The values of k computed from this are given in the column A; from the differences in the next column, with reference to the observed values of k, there results the probable error 0.0252, and we find the probable error of α equal to 0.01338, or about $\frac{1}{2}$ per cent., and of β equal to 0.000719, or about $7\frac{1}{2}$ per cent.

Although the reliability of these results, especially in their application to still larger surfaces and greater velocities, leaves much to be desired, still scarcely any important higher degree of accuracy is to be attained with apparatus that is similar to that above described. On the other hand the concluded law of resistance would be in an important degree confirmed or corrected, if on a firm rod in front of a locomotive, disks are fastened, whose pressure could be measured by the tension of a spring, while the milestones on the roadside would serve very conveniently for the determination of the velocity.*

* [This experiment has been carried on recently by Wild and others, but the resulting value of k is not so reliable as that deduced from observations with large whirling machines.—*C. A.*]

From the preceding it results that the pressure of the air against a plane disk turned normally towards it is

$$D = \frac{2.264 + 0.00942 \times p}{1,000,000} F c^2$$

Where D is expressed in old Prussian loths and p, F, and c in [Rhenish] inches. According to the above, the pressure against a square disk of 1 square foot area, moving with a velocity of 50 feet per second, would for example be 140.8 loths, or nearly 4.4 pfund.

For reduction to metric measures and weights I take not the metre itself but the decimetre as the unit of measure for lengths and surfaces, in order to remain within the limits of the observations. Therefore the resistance of the air for a temperature of 15° C. and a barometric pressure of 28 Paris inches,* expressed in grammes, amounts to

$$(0.00707 + 0.0001125\,p)\, F c^2,$$

Where p represents the circumference of the disk, F the sectional area, and c the velocity expressed in decimetres.

The pressure that very small disks experience when struck normally by a current of air is also given by another simple consideration, whose correctness has in general been confirmed by many experiments. These experiments indeed are limited, so far as known, to streams of water; but the expansibility of the air is certainly in this case without influence, since the observations mentioned in the preface, upon the direction and strength of currents deviated in front of opposing disks, showed identical results with water and with air.

Imagine a vessel filled to the height h with a fluid of which one unit of volume or 1 cubic inch weighs γ loths. The bottom of the vessel therefore experiences on each square inch a pressure equal to γh, when no side pressure exists. If there is suddenly made therein an opening of 1 square inch, the outflow of the fluid through it begins with the velocity $c = 2\sqrt{gh}$†, and if we catch the stream by an equally large surface directed normally against it, then the pressure D upon this is again equally as great as before upon the bottom of the vessel, namely, γh. From this we have

$$D = \gamma h = \frac{\gamma}{4g} c$$

For the density of the air above adopted its specific weight is 0.001223; therefore a cubic inch weighs 0.001495 loth, and g is equal to 187.6, if the semi-acceleration due to gravity is expressed in inches. From this we have these results:

$$D = 0.000001992 = 1.992 \text{ millionths of a loth.}$$

* The density is that of air at 15° C. and 28 Paris inches or 757.96^{mm} under gravity at Berlin (52° 30′), but strictly speaking the pressure should be stated in standard measure as 758.47^{mm} under gravity at 45° and sea level.

† g is the height fallen through in 1 second, or one-half the acceleration due to gravity.

As the first term of the above value of k comes out 2.264 or larger than this by nearly 14 per cent., the stronger resistance deduced from the observations is explained by the rarefaction of the air occurring at the rear of the disk, which rarefaction in the case of an assumed outflow into empty space does not take place.

Although the present investigation is confined only to those positions in which the disks are turned normal to the direction of their motion, still it was important to be convinced that slight and unavoidable deviations from this normal position had no important influence. The pins by means of which the disks were fastened to the arms were directed radial'y towards the axis of rotation. Thus the disks could be given any desired inclination to the direction of their motion. One such experience however showed this arrangement to be entirely unallowable in the observations, in that the simple relation between the resistance and the velocity of the disk completely disappeared. The reason for this irregularity is apparent. According as the two disks were inclined downwards or upwards they were pressed up or down by the impinging air, and by so much the more the greater their velocity was. The arms with the inclined disks and with the axis of rotation therefore pressed variably upon the steel point on which the axis rested, and accordingly the screw threads on the axis were variably pressed up or down, whereby the friction each time experienced an important change. When however I inclined one disk upwards and the opposite disk downwards, the axis was pressed to one side, and by so much the more, the greater the velocity was.

In order not to change the simple arrangements for fastening the disks, I provided the two 5-inch square disks with roof shaped piece. in addition, so that in front of the lower half of the disk the inclined plane was turned upwards, and in front of the lower half an equal plane with the same inclination was turned downwards. Each of the two disks thus changed was thus both raised and depressed by equal forces for all velocities, so that the injurious effect upon the axis of rotation disappeared.

A complete series of observations (wherein both at the beginning and at the end the arms were set in motion without disks in order to determine the resistance) gave—

(a) When the roof surface was inclined 40° to the vertical or to the plane disk,

$$r = 83.92.$$

(b) For an inclination of 20° to the vertical,

$$r = 101.16.$$

(c) And for the plane disk itself, therefore, after removing the additions

$$r = 110.93.$$

If we divide these values by the cosines of 40, 20, and 0 degrees, respectively, there results

109.55, 107.65, and 110.93.

The resistances are therefore in accordance with the ordinary assumption, proportional to the cosine of the inclination.

In case the plane of the plane disk does not include the axis of rotation, we should also have to consider the diminution of the surface opposed to the impinging air in consequence of the projection upon the direction of motion, and for both reasons the resistance diminishes in the ratio of the square of the cosine of the deviation. Since the disks were always adjusted by the plumb line, therefore an error of 2 degrees, by which the resistance would only be diminished by its thousandth part, could not easily remain unnoticed.

Finally, it still remains to be investigated whether the nature of the surface of the disks, according as they were smooth or rough, had any influence on the resistance. To this end I took two disks, each of which was covered on one side with very smooth paper but on the other with very coarse sandpaper. I allowed these to run with various velocities, exposing each time first the smooth and then the rough side to the impinging air. In both cases the times in which the index described 10 inches remained very nearly the same. The differences were very irregular, and not larger than occurred in repeated experiments with equal pairs of disks. Hence the nature of the surface of a plane disk has no influence on the resistance of the air when the surfaces are normal to the direction of motion.

II.

ON THE INTEGRALS OF THE HYDRO-DYNAMIC EQUATIONS THAT REPRESENT VORTEX-MOTIONS.*

By Prof. HERMANN VON HELMHOLTZ.

Hitherto the integrals of the hydro-dynamic equations have been sought almost exclusively under the assumption that the rectangular components of the velocity of every particle of liquid can be put equal to the differential quotients in the corresponding directions of a certain definite function that we will call the velocity potential.

On the one hand Lagrange† had proven that this assumption is allowable whenever the movement of the mass of water has arisen and is maintained under the influence of forces that can be expressed as the differential quotients of a force potential, and even that the influence of moving solid bodies that come in contact with the liquid do not affect the applicability of the assumption. Since now most of the forces of nature that are easily expressed mathematically can be presented as the differential quotients of a force potential, therefore also by far the majority of the cases of fluid motion that are treated mathematically fall into the category of those for which a velocity potential exists.

On the other hand, even Euler‡ had called attention to the fact that there are cases of fluid motion where no velocity potential exists; *e. g.*, the rotation of a fluid with equal angular velocities in all its parts about an axis. The magnetic forces that act upon a fluid permeated by electric currents, and especially the friction of fluid particles on each other and on solid bodies, belong to the forces that can give rise to such forms of motion. The influence of friction on fluids could not hitherto be mathematically defined, and yet it is very large in all cases where we are not treating of infinitely small vibrations, and causes the most important deviations between theory and nature. The difficulty of defining this influence and of finding methods for its measurement certainly lay

* Crelle's *Journal für die reine und angewandte Mathematik*, 1858, vol. LV, p. 25–85. Helmholtz, *Wissenschaftliche Abhandlungen*, 1882, vol. I, pp. 101–134. *London, Edinburgh, and Dublin Philosophical Magazine*, June, 1867 (4), XXIII, pp. 485–510

† *Mécanique Analytique*, Paris, 1815, vol. II, p. 304.

‡ *Histoire de l'Académie des Sciences de Berlin*, anno 1755, p. 292.

mostly in the fact that we had no idea of the forms of motion that friction produces in the fluid. Therefore in this respect an investigation of those forms of motion in which no velocity potential exists seems to me to be of importance.

The following investigation will now show that in those cases in which a velocity potential does exist the smallest particles of liquid have no motion of rotation, but that when no velocity potential exists then a part at least of the liquid particles are in the act of rotation.

By *vortex lines* (Wirbellinien) I designate lines that are so drawn through the mass of liquid that their directions everywhere coincide with the direction of the instantaneous axis of rotation of the liquid particles at that point of the line.

By *vortex filament* (Wirbelfäden) I designate the portion of the mass of liquid that is cut out when we construct the corresponding vortex lines passing through every point of the circumference of an infinitely small element of the surface.

The following investigation shows that when a force potential exists for all the forces that act upon the fluid then:

(1) No particle of liquid acquires rotation that was not in rotation from the beginning.

(2) The particles of liquid that at any moment belong to the same vortex line remain belonging to the same vortex line, even although they have a motion of translation.

(3) The product of the sectional area by the velocity of rotation of an infinitely slender vortex filament is constant along the whole length of the filament and also retains the same value during the translatory motion of the filament. Therefore the vortex filaments must return into themselves within the liquid or can only have their ends at the boundaries of the fluid.

This last proposition makes it possible to determine the velocities of rotation when the form of a particular vortex filament is given at different moments of time. Further we solve the problem to determine the velocity of the particles of liquid for a given moment of time when the velocities of rotation are given for this moment, but in the solution there remains undetermined one arbitrary function that must be utilized to satisfy the boundary conditions.

This last problem leads to a remarkable analogy between the vortex motions of liquids and the electro-magnetic actions of electric currents.

When in a simply connected space* filled with moving liquid a velocity potential exists, the velocities of the liquid particles are equal to and in the same direction as the forces that a certain distribution of

* I use this expression (einfach zusammenhängenden Raume) in the same sense in which Riemann (*Journal für die reine und angewandte Mathematik,* 1857, LIV, p. 108) speaks of simple and multiple-connected surfaces. A space that is n-times connected is therefore one such that n—1 but not more intersecting surfaces can pass through it without cutting the space into two completely separate portions. A ring is therefore in this sense a doubly-connected space. The intersecting surfaces must be completely surrounded by the lines in which they cut the surface of the space.

magnetic masses on the surface of the space would exert upon a magnetic particle in the interior.

On the other hand, when vortex threads exist in any such space the velocities of the liquid particles are equal to the forces exerted upon a magnetic particle by a closed electric current that flows partly through the vortex filaments in the interior of the mass and partly in the boundary surface, and whose intensity is proportional to the product of the sectional area of the vortex filament by its velocity of rotation.

I shall therefore in the following lines often allow myself to hypothecate the presence of magnetic masses or of electric currents, simply in order thereby to obtain shorter and more perspicuous expressions for the nature of functions that are just the same functions of the coördinates as the potential functions, or the attractive forces for a magnetic particle, are of the magnetic masses or electric currents.

By these propositions the forms of motion concealed in that class of integrals of the hydro-dynamic equations not hitherto treated of become accessible at least to the imagination even although it be possible to execute the complete integration only in a few of the simplest cases where only one or two rectilinear or circular vortex filaments are present in masses of liquid that are either unlimited or partially bounded by one infinite plane.

It can be demonstrated that rectilinear parallel vortex filaments in a mass of water that is bounded only by planes perpendicular to such filaments, rotate about their common center of gravity, when in the determination of this center we consider the velocity of rotation as equivalent to the density of a mass. In this rotation the location of the center of gravity remains unchanged. On the other hand, for circular vortex filaments, all standing perpendicular to a common axis, the center of gravity of their cross-section advances parallel to the axis.

I. DEFINITION OF ROTATION.

At a point within a liquid whose position is defined by the rectangular coördinates x, y, z, and at the time t, let the pressure be p, the three components of the velocity u, v, w, the three components of the external forces acting on the unit mass of the liquid X, Y, Z, and h be the density whose changes can be considered as negligible; the established equations of motion for an interior point of the fluid are:

$$\left.\begin{aligned}
X-\frac{1}{h}\frac{\partial p}{\partial x}&=\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\\
Y-\frac{1}{h}\frac{\partial p}{\partial y}&=\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\\
Z-\frac{1}{h}\frac{\partial p}{\partial z}&=\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\\
0&=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Hitherto, almost exclusively, only those cases have been treated where not only the forces X, Y, Z, have a potential V so that they can be expressed in the form,

$$X=\frac{\partial V}{\partial x}, Y=\frac{\partial V}{\partial y}, Z=\frac{\partial V}{\partial z}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

but also where a velocity potential φ can be found so that

$$u=\frac{\partial \varphi}{\partial x},\ v=\frac{\partial \varphi}{\partial y},\ w=\frac{\partial \varphi}{\partial z}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1b)$$

The problem is thereby greatly simplified since the first three of equations (1) give a common integral equation from which to find p after we have determined φ in accordance with the fourth equation which in this case takes the form

$$\frac{\partial^2 \varphi}{\partial x^2}+\frac{\partial^2 \varphi}{\partial y^2}+\frac{\partial^2 \varphi}{\partial z^2}=0,$$

and which therefore agrees with the established differential equation for the potential of magnetic masses that lie outside the space within which this equation holds good. Moreover, it is known that every function φ that satisfies this last differential equation within a simply connected space,* can be expressed as the potential of a definite distribution of magnetic masses on the boundary surface of the space as I have stated already in the introduction.

In order that we may be able to make the substitution required in the equation (1b) we must have

$$\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}=0,\ \frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}=0,\ \frac{\partial w}{\partial x}-\frac{\partial v}{\partial z}=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1c.)$$

In order to understand the mechanical significance of these last three conditions, we may imagine the change that any infinitely small volume of water experiences in the elementary time dt to be compounded of three different motions: (1) a motion of transference of the whole through space: (2) an expansion or contraction of the particle along the axis of dilatation, whereby every rectangular parallelopipedon of water whose sides are parallel to the principal axis of dilatation remains rectangular while its sides change their lengths but remain parallel to their original directions: (3) a rotation about some temporary axis of rotation having any given direction, which rotation can by a well-known proposition be always considered as the resultant of three rotations about the three coördinate axes.

*In manifold-connected-spaces φ can have several values, but for such many valued functions as satisfy the above differential equations the fundamental proposition of Green's theory of electricity no longer holds good (see Crelle *Journal*, XLIV, p. 360, or "The Mathematical Papers of the late George Green"), and therefore fail also a greater part of the propositions resulting from this which Gauss and Green have demonstrated for the magnetic potential functions, which functions are in their very nature always uni-valued.

If the conditions (1c) are satisfied at the point whose coördinates are $\mathfrak{x}$, $\mathfrak{y}$, $\mathfrak{z}$, and if we designate the values of u, v, w, and their differential quotients as follows:

$$u=A, \quad \frac{\partial u}{\partial x}=a, \quad \frac{\partial w}{\partial y}=\frac{\partial v}{\partial z}=\alpha.$$

$$v=B, \quad \frac{\partial v}{\partial y}=b, \quad \frac{\partial u}{\partial z}=\frac{\partial w}{\partial x}=\beta.$$

$$w=C, \quad \frac{\partial w}{\partial z}=c, \quad \frac{\partial v}{\partial x}=\frac{\partial u}{\partial y}=\gamma.$$

We obtain for the point whose coördinates x, y, z, differ differentially from $\mathfrak{x}$, $\mathfrak{y}$, $\mathfrak{z}$:

$$u=A+a\,(x-\mathfrak{x})+\gamma\,(y-\mathfrak{y})+\beta\,(z-\mathfrak{z}),$$

$$v=B+\gamma\,(x-\mathfrak{x})+b\,(y-\mathfrak{y})+\alpha\,(z-\mathfrak{z}),$$

$$w=C+\beta\,(x-\mathfrak{x})+\alpha\,(y-\mathfrak{y})+c\,(z-\mathfrak{z}),$$

or when we put:

$$\begin{aligned}\varphi=&A\,(x-\mathfrak{x})+B\,(y-\mathfrak{y})+C\,(z-\mathfrak{z})\\&+\tfrac{1}{2}\,a\,(x-\mathfrak{x})^2+\tfrac{1}{2}b\,(y-\mathfrak{y})^2+\tfrac{1}{2}c\,(z-\mathfrak{z})^2\\&+\alpha(y-\mathfrak{y})\,(z-\mathfrak{z})+\beta\,(x-\mathfrak{x})\,(z-\mathfrak{z})+\gamma\,(x-\mathfrak{x})\,(y-\mathfrak{y}),\end{aligned}$$

there results:

$$u=\frac{\partial\varphi}{\partial x},\; v=\frac{\partial\varphi}{\partial y},\; w=\frac{\partial\varphi}{\partial z}$$

It is well known that by a proper selection of another system of rectangular coördinates x_1, y_1, z_1, whose origin is at the point $\mathfrak{x}$, $\mathfrak{y}$, $\mathfrak{z}$, the expression for φ can be brought into the form:

$$\varphi=A_1\,x_1+B_1\,y_1+C_1\,z_1+\tfrac{1}{2}\,a_1\,x_1^2+\tfrac{1}{2}b_1\,y_1^2+\tfrac{1}{2}\,c_1\,z_1^2$$

where the component velocities u_1, v_1, w_1, along these new coördinate axes have the values:

$$u_1=A_1+a_1\,x_1, \quad v_1=B_1+b_1\,y_1, \quad w_1=C_1+c_1\,z_1.$$

The velocity u_1 parallel to the axis of x_1 is therefore alike for all liquid particles that have the same value of x_1. therefore particles that at the beginning of the elementary time dt lie in a plane parallel to that of $y_1\,z_1$ are also still in that plane at the end of the elementary time dt. This same proposition is true for the planes $x_1\,y_1$ and $x_1\,z_1$. Therefore when we imagine a parallelopipedon bounded by three planes parallel to the last named coördinate planes and infinitely near to them, the liquid particles inclosed therein still form at the end of the time dt a rectangular parallelopipedon whose surfaces are parallel to the same coördinate planes. Therefore the whole motion of such an indefinitely small parallelopipedon is, under the assumption expressed

in (1*c*) compounded only of a motion of translation in space and an expansion and contraction of its edges and it has no rotation.

We return now to the first system of coördinates, that of x, y, z, and imagine added to the hitherto existing motion of the infinitely small mass of liquid surrounding the point $\mathfrak{x}, \mathfrak{y}, \mathfrak{z}$, a system of rotatory motions about axes that are parallel to those of x, y, z, and that pass through the point $\mathfrak{x}, \mathfrak{y}, \mathfrak{z}$, and whose angular velocities of rotation may be ξ, η, ζ, thus then the component velocities parallel to the coördinate axes of x, y, z, as resulting from such rotations are respectively:

Parallel to x:	Parallel to y:	Parallel to z:
$0,$	$(z-\mathfrak{z})\,\xi,$	$-(y-\mathfrak{y})\,\xi,$
$-(z-\mathfrak{z})\,\eta,$	$0,$	$(x-\mathfrak{x})\,\eta,$
$(y-\mathfrak{y})\,\zeta,$	$-(x-\mathfrak{x})\,\zeta,$	$0,$

Therefore the velocities of the particles whose coördinates are x, y, z, become:

$$u=A+a(x-\mathfrak{x})+(\gamma+\zeta)(y-\mathfrak{y})+(\beta-\eta)(z-\mathfrak{z}),$$

$$v=B+(\gamma-\zeta)(x-\mathfrak{x})+b(\mathfrak{x}-y)+(\alpha+\xi)(z-\mathfrak{z}),$$

$$w=C+(\beta+\eta)(x-\mathfrak{x})+(\alpha-\xi)(y-\mathfrak{y})+c(z-\mathfrak{z}),$$

whence by differentiation there results:

$$\left.\begin{aligned}\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}&=2\xi.\\ \frac{\partial w}{\partial x}-\frac{\partial u}{\partial z}&=2\eta.\\ \frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}&=2\zeta.\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (2)$$

Therefore the quantities on the left-hand side, which according to equation (1*c*) must be equal to zero in order that a velocity potential may exist, are equal to double the velocity of rotation about the three coördinate axes of the liquid particles under consideration. The existence of a velocity potential excludes the existence of a rotary motion of the particles of liquid.

As a further characteristic peculiarity of fluid motions that have a velocity potential, it may be further stated that in a simply-connected space S, entirely inclosed within rigid walls and wholly filled with fluid, no such motion can occur; for when we indicate by n the normal directed inwards to the surface of such space then the component velocity $\frac{\partial \varphi}{\partial n}$ directed perpendicular to the wall must be everywhere

equal to zero. Therefore, according to the well-known Green's theorem,*

$$\iiint \left[\left(\frac{\partial \varphi}{\partial x} \right)^2 + \left(\frac{\partial \varphi}{\partial y} \right)^2 + \left(\frac{\partial \varphi}{\partial z} \right)^2 \right] dx\, dy\, dz = - \int \varphi \frac{\partial \varphi}{\partial n} d\omega,$$

where, on the left hand, the integration is to be extended over the whole of the volume S, but on the right hand over the whole surface S whose elementary surface is designated by $d\omega$. If, now, $\frac{\partial \varphi}{\partial n}$ is to be equal to zero for the whole surface, then the integral on the left hand must also be zero, which can only be true when for the whole volume S

$$\frac{\partial \varphi}{\partial x} = \frac{\partial \varphi}{\partial y} = \frac{\partial \varphi}{\partial z} = 0,$$

that is to say, when there exists no motion whatever of the liquid. Every motion within a simply connected space of a limited mass of fluid that has a velocity potential is therefore necessarily connected with a motion of the surface of the fluid. If this motion of the surface, *i. e.*, $\frac{\partial \varphi}{\partial n}$, is known completely, then the whole movement of the inclosed fluid mass is also thereby definitely determined. For suppose there are two functions, $\varphi_{\prime}$ and $\varphi_{\prime\prime}$, that simultaneously satisfy the equation

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0$$

in the interior of the space S, and also the condition

$$\frac{\partial \varphi}{\partial n} = \psi$$

for the surface of S, where ψ indicates the value of $\frac{\partial \varphi}{\partial n}$ deduced from the assumed motion of the surface, then would the function $(\varphi_{\prime} - \varphi_{\prime\prime})$ also satisfy the first condition for the interior of the space S, but for the surface this function would give

$$\frac{\partial (\varphi_{\prime} - \varphi_{\prime\prime})}{\partial n} = 0;$$

whence, as just shown, it would follow that for the whole interior of S we would have

$$\frac{\partial (\varphi_{\prime} - \varphi_{\prime\prime})}{\partial x} = \frac{\partial (\varphi_{\prime} - \varphi_{\prime\prime})}{\partial y} = \frac{\partial (\varphi_{\prime} - \varphi_{\prime\prime})}{\partial z} = 0.$$

Therefore both functions would also correspond to exactly the same velocities throughout the whole interior of S.

Therefore rotations of liquid particles and circulatory motions within simply-connected wholly inclosed spaces can only occur when no velocity potential exists. We can therefore in general characterize the motions in which a velocity potential does not exist, as vortex motions.

*This is the proposition in Crelle *Journal*, vol. LIV, p. 108, already alluded to, and which does not hold good for complex or manifold-connected space.

II. PERMANENCE OF THE VORTEX MOTION.

We will next determine the variations of the velocities of rotation ξ, η, ζ during the movement (of the surface) when only such forces are effective as have a force potential.

I note first in general that when ψ is a function of x, y, z, t, and increases by the quantity $\delta\psi$, while the last four quantities increase by δx, δy, δz, and δt, respectively, we have:

$$\delta\psi = \frac{\partial\psi}{\partial t}\delta t + \frac{\partial\psi}{\partial x}\delta x + \frac{\partial\psi}{\partial y}\delta y + \frac{\partial\psi}{\partial z}\delta z.$$

If now the variation of ψ during the short time δt is to be determined for one and the same particle of liquid, we must give the quantities δx, δy, δz the same values that they would have for the moving particle of liquid, namely:

$$\delta x = u\,\delta t, \qquad \delta y = v\,\delta t, \qquad \delta z = w\,\delta t,$$

and obtain:

$$\frac{\delta\psi}{\delta t} = \frac{d\psi}{dt} + u\frac{\partial\psi}{\partial x} + v\frac{\partial\psi}{\partial y} + w\frac{\partial\psi}{\partial z}$$

I shall in the following always use the notation $\frac{\delta\psi}{\delta t}$ only in the sense that $\frac{\delta\psi}{\delta t}dt$ indicates the variation of ψ during the element of time $d\,t$ for the same particle of water whose coördinates at the beginning of the time ∂t were x, y, and z.

If by differentiation we eliminate the quantity p from the first of the equations (1) and introduce the notation of equations (2) and substitute for the forces X, Y, Z the expressions in equation (1a), we obtain the following three equations:

$$\left.\begin{aligned}
\frac{\delta\xi}{\delta t} &= \xi\frac{\partial u}{\partial x} + \eta\frac{\partial u}{\partial y} + \zeta\frac{\partial u}{\partial z} \\
\frac{\delta\eta}{\delta t} &= \xi\frac{\partial v}{\partial x} + \eta\frac{\partial v}{\partial y} + \zeta\frac{\partial v}{\partial z} \\
\frac{\delta\zeta}{\delta t} &= \xi\frac{\partial w}{\partial x} + \eta\frac{\partial w}{\partial y} + \zeta\frac{\partial w}{\partial z}
\end{aligned}\right\} \quad \ldots\ldots \quad (3)$$

or

$$\left.\begin{aligned}
\frac{\delta\xi}{\delta t} &= \xi\frac{\partial u}{\partial x} + \eta\frac{\partial v}{\partial x} + \zeta\frac{\partial w}{\partial x} \\
\frac{\delta\eta}{\delta t} &= \xi\frac{\partial u}{\partial y} + \eta\frac{\partial v}{\partial y} + \zeta\frac{\partial w}{\partial y} \\
\frac{\delta\zeta}{\delta t} &= \xi\frac{\partial u}{\partial z} + \eta\frac{\partial v}{\partial z} + \zeta\frac{\partial w}{\partial z}
\end{aligned}\right\} \quad \ldots\ldots \quad (3a)$$

If ξ, η, and ζ for any particle of water are simultaneously zero then also—

$$\frac{\delta\xi}{\delta t} = \frac{\delta\eta}{\delta t} = \frac{\delta\zeta}{\delta t} = 0.$$

Therefore those particles of water that do not already have a rotatory motion will receive none in the subsequent time.

As is well known, we can combine rotations together after the method of parallelograms of forces. If ξ, η, ζ are the velocities of the rotations about the coördinate axes, then the velocity of rotation (q) about the instantaneous axis of rotation is

$$q=\sqrt{\xi^2+\eta^2+\zeta^2}$$

and the cosines of the angles that this axis makes with the coördinate axes are respectively $\frac{\xi}{q}$, $\frac{\eta}{q}$, and $\frac{\zeta}{q}$.

If now we consider an infinitely small distance $q\varepsilon$ in the direction of the instantaneous axis of rotation, then the projections of this distance on the three coördinate axes are respectively $\varepsilon\xi$, $\varepsilon\eta$, and $\varepsilon\zeta$. While at the point $x\ y\ z$ [at one end of $q\varepsilon$] the components of the velocity are u. v, w, they are at the other end of $q\varepsilon$ respectively

$$u_1=u+\varepsilon\xi\frac{\partial u}{\partial x}+\varepsilon\eta\frac{\partial u}{\partial y}+\varepsilon\zeta\frac{\partial u}{\partial z},$$

$$v_1=v+\varepsilon\xi\frac{\partial v}{\partial x}+\varepsilon\eta\frac{\partial v}{\partial y}+\varepsilon\zeta\frac{\partial v}{\partial z},$$

$$w_1=w+\varepsilon\xi\frac{\partial w}{\partial x}+\varepsilon\eta\frac{\partial w}{\partial y}+\varepsilon\zeta\frac{\partial w}{\partial z}.$$

Therefore in the course of the elementary time dt the projection of the distance of the two particles of liquid that at the beginning of dt were distant by the quantity $q\varepsilon$ has attained a value that, considering the equation (3), can be written as follows:

$$\varepsilon\xi+(u_1-u)\,dt=\varepsilon\left(\xi+\frac{\delta\xi}{\delta t}dt\right),$$

$$\varepsilon\eta+(v_1-v)\,dt=\varepsilon\left(\eta+\frac{\delta\eta}{\delta t}dt\right),$$

$$\varepsilon\zeta+(w_1-w)\,dt=\varepsilon\left(\zeta+\frac{\delta\zeta}{\delta t}dt\right).$$

On the left are the projections of the new location of the connecting line $q\varepsilon$; on the right are the projections multiplied by the constant factor ε of the new velocity of rotation. It follows from these equations that the line connecting the two liquid particles that at the beginning of the time dt limited the portion $q\varepsilon$ of the instantaneous axis of rotation will also after the lapse of the time dt still coincide with the now changed axis of rotation.

When we, as above agreed on, call a line whose direction throughout its whole length agrees with the direction of the instantaneous axis of rotation of the particle of liquid at each point, a *vortex line*, we can express the proposition just found as follows: *Every vortex line remains permanently composed of the same particles of liquid while it progresses with these particles through the liquid.*

The rectangular components of the velocity of rotation increase in the same ratio as the projections of the portion $q\varepsilon$ of the axis of rotation; hence it follows that *the magnitude of the resulting velocity of rotation varies for a given particle of liquid in the same ratio as the distance of this particle from its neighbors in the axis of rotation.*

Imagine vortex lines drawn through all points of the circumference of an indefinite small surface. Then will a thread of infinitely small section, which is called the "vortex filament," be thereby cut out of the liquid. The volume of a portion of such a filament included between two given particles of liquid, which volume according to the propositions just proven always remains filled by the same particles, must remain constant during its progressive motion; therefore its section must vary in the inverse ratio of its length. Hence we can express the last proposition thus: *In a portion of a vortex filament, consisting of the same particles of liquid, the product of the velocity of rotation by the section ever remains constant during its translatory motion.*

From equation (2) it directly follows that—

$$\frac{\partial \xi}{\partial x}+\frac{\partial \eta}{\partial y}+\frac{\partial \zeta}{\partial z}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

Hence it further follows that—

$$\int\int\int\left(\frac{\partial \xi}{\partial x}+\frac{\partial \eta}{\partial y}+\frac{\partial \zeta}{\partial z}\right)dx\, dy\, dz=0$$

where the integration can be extended over any arbitrary portion S of the mass of liquid. When we partially integrate this there results:

$$\int\int\xi\, dy\, dz+\int\int\eta\, dx\, dz+\int\int\zeta\, dx\, dy=0$$

where the integrations are to be extended over the whole surface of the volume of S. If we let $d\omega$ be an element of this surface and α, β, γ the three angles that the normal to $d\omega$ drawn outwards makes with the coördinate axes, then—

$$dy\, dz=\cos\alpha\, d\omega, \qquad dx\, dz=\cos\beta\omega, \qquad dx\, dy=\cos\gamma\, d\omega;$$

therefore

$$\int\int(\xi\cos\alpha+\eta\cos\beta+\zeta\cos\gamma)d\omega=0,$$

or when we let σ be the resulting velocity of rotation and θ the angle between this velocity and the normal

$$\int\int\sigma\cos\theta\, d\omega=0,$$

where the integration is to be extended over the whole surface of S.

Let S be a portion of a vortex filament bounded by two infinitely small planes $\omega_{,}$ and $\omega_{,,}$ perpendicular to the axis of the filament, then will $\cos\theta=+1$ for one of these planes and $\cos\theta=-1$ for the other, but $\cos\theta=0$ for the whole of the remaining surface of the filament; consequently, if $\sigma_{,}$ and $\sigma_{,,}$ are the velocities of rotation at $\omega_{,}$ and $\omega_{,,}$, respectively, the last equation reduces to

$$\sigma_{,}\,\omega_{,}=\sigma_{,,}\,\omega_{,,}$$

whence it follows that *the product of the velocity of rotation by the area of the section is constant throughout the whole length of the vortex filament.* It has already been shown that this product does not change during the progressive motion of the filament.

It follows from this that a vortex filament can not possibly end anywhere within the fluid, but must either return into itself, like a ring within the fluid, or must continue on to the boundaries of the fluid. For in case a filament ended anywhere within the fluid it would be possible to construct a closed surface for which the integral $\int \sigma \cos \theta \, d\omega$ is not zero.

III. INTEGRATION BY VOLUME.

When we can determine the motions of the vortex filaments present in the fluid we can, by means of the above established propositions, also determine completely the quantities ξ, η, and ζ. We will now consider the problem to determine the velocities u, v, and w from the quantities ξ, η, and ζ.

Within a mass of liquid that fills the region S let values of ξ, η, and ζ be given, which quantities should satisfy the condition that—

$$\frac{\partial \xi}{\partial x}+\frac{\partial \eta}{\partial y}+\frac{\partial \zeta}{\partial z}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a).$$

Such values of u, v, and w are to be found as may, throughout the whole region S, satisfy the conditions [of Eq. (1_4) and (2), viz.]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1_4)$$

$$\left.\begin{aligned} \frac{\partial v}{\partial z}-\frac{\partial u}{\partial y} &= 2\xi \\ \frac{\partial w}{\partial x}-\frac{\partial u}{\partial z} &= 2\eta \\ \frac{\partial u}{\partial y}-\frac{\partial v}{\partial x} &= 2\zeta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

to which is still to be added the condition demanded by the boundary of the region S, according to the nature of the specific problem in hand.

According to the distribution of ξ, η, ζ, as above specifically given, there can occur on the one hand such vortex lines as shall return into themselves within the limits of the region S and on the other hand such as extend to the boundary and there suddenly break off. When this latter is the case then we can certainly prolong these [fragments of] vortex lines either along the surface of S or beyond S until they return into themselves, so that a larger space S_1 exists that contains only closed vortex lines and for whose whole surface both ξ, η, ζ and their resultant σ itself are all equal to zero or at least

$$\xi \cos \alpha+\eta \cos \beta+\zeta \cos \gamma=\sigma \cos \theta=0.$$

Here, as before, α, β, γ indicate the angles between the coördinate axes and the normal to the appropriate portion of the surface of S_1.

θ indicates the angle between the normal and the resulting axis of rotation.

We now obtain the values of u, v, w, that satisfy the equations (1_4) and (2) by putting

$$\left.\begin{aligned} u &= \frac{\partial P}{\partial x} + \frac{\partial N}{\partial y} - \frac{\partial M}{\partial z} \\ v &= \frac{\partial P}{\partial y} + \frac{\partial L}{\partial z} - \frac{\partial N}{\partial x} \\ w &= \frac{\partial P}{\partial z} + \frac{\partial M}{\partial x} - \frac{\partial L}{\partial y} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

and determine the quantities L, M, N, P by the conditions that within the region S_1 we must have

$$\left.\begin{aligned} \frac{\partial^2 L}{\partial x^2} + \frac{\partial^2 L}{\partial y^2} + \frac{\partial^2 L}{\partial z^2} &= 2\xi, \\ \frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} + \frac{\partial^2 M}{\partial z^2} &= 2\eta, \\ \frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2} + \frac{\partial^2 N}{\partial z^2} &= 2\zeta, \\ \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} &= 0, \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \quad (5)$$

The method of integrating these last equations is well known. L, M, N are the potential functions of imaginary magnetic masses distributed through the space S_1 with the densities $\frac{-\xi}{2\,\pi}, \frac{-\eta}{2\,\pi}, \frac{-\zeta}{2\,\pi}$; P is the potential function for masses that lie outside of the region S. If we indicate by r the distance from the point x, y, z to the point whose coördinates are a, b, c; and by ξ_a, η_a, ζ_a the values of ξ, η, ζ at the point a, b, c, then

$$\left.\begin{aligned} L &= -\frac{1}{2\pi}\int\!\!\int\!\!\int \frac{\xi_a}{r}\,da\,db\,dc, \\ M &= -\frac{1}{2\pi}\int\!\!\int\!\!\int \frac{\eta_a}{r}\,da\,db\,dc, \\ N &= -\frac{1}{2\pi}\int\!\!\int\!\!\int \frac{\zeta_a}{r}\,da\,db\,dc, \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \quad (5a)$$

where the integration is extended over the space S_1 and

$$P = \int\!\!\int\!\!\int \frac{k}{r}\,da\,db\,dc,$$

where k is an arbitrary function of a, b, c and the integration is to be extended over the exterior space S_1, that includes the region S. The

arbitrary function k must be so determined that the boundary conditions are satisfied, a problem whose difficulty is similar to those [difficulties that are met with in problems] on the distribution of electricity and magnetism.

That the values of u, v, and w, given in equation (4), satisfy the condition (1_4), is seen at once by differentiation and by considering the fourth of equations (5).

Further, we find by differentiation of equations (4), and considering the first three of equations (5) that:

$$\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} = 2\xi - \frac{\partial}{\partial x}\left[\frac{\partial L}{\partial x} + \frac{\partial M}{\partial y} + \frac{\partial N}{\partial z}\right]$$

$$\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} = 2\eta - \frac{\partial}{\partial y}\left[\frac{\partial L}{\partial x} + \frac{\partial M}{\partial y} + \frac{\partial N}{\partial z}\right]$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial z} = 2\zeta - \frac{\partial}{\partial z}\left[\frac{\partial L}{\partial x} + \frac{\partial M}{\partial y} + \frac{\partial N}{\partial z}\right]$$

The equations (2) are also equally satisfied when it can be shown that throughout the whole region S_1 we have

$$\frac{\partial L}{\partial x} + \frac{\partial M}{\partial y} + \frac{\partial N}{\partial z} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5b)$$

That this is the case results from the equations $(5a)$ which give

$$\frac{\partial L}{\partial x} = \frac{1}{2\pi}\int\int\int \frac{\xi_a(x-a)}{r^3}\,da\,db\,dc,$$

or after partial integration

$$\frac{\partial L}{\partial x} = \frac{1}{2\pi}\int\int \frac{\xi_a}{r}\,db\,dc - \frac{1}{2\pi}\int\int\int \frac{1}{r}\cdot\frac{\partial \xi_a}{\partial a}\,da\,db\,dc$$

$$\frac{\partial M}{\partial y} = \frac{1}{2\pi}\int\int \frac{\eta_a}{r}\,da\,dc - \frac{1}{2\pi}\int\int\int \frac{1}{r}\cdot\frac{\partial \eta_a}{\partial b}\,da\,db\,dc$$

$$\frac{\partial N}{\partial z} = \frac{1}{2\pi}\int\int \frac{\zeta_a}{r}\,da\,db - \frac{1}{2\eta}\int\int\int \frac{1}{r}\cdot\frac{\partial \zeta_a}{\partial c}\,da\,db\,dc.$$

If we add these three equations and again indicate by $d\omega$ the element of the surface of S, we obtain:

$$\frac{\partial L}{\partial x} + \frac{\partial M}{\partial y} + \frac{\partial N}{\partial z} = \frac{1}{2\pi}\int (\xi_a \cos\alpha + \eta_a \cos\beta + \zeta_a \cos\gamma)\frac{1}{r}\,d\omega$$
$$- \frac{1}{2\pi}\int\int\int \frac{1}{r}\left(\frac{\partial \xi_a}{\partial a} + \frac{\partial \eta_a}{\partial b} + \frac{\partial \zeta_a}{\partial c}\right) da\,db\,dc.$$

But since throughout the whole interior of the space S_1 we have

$$\frac{\partial \xi_a}{\partial a} + \frac{\partial \eta_a}{\partial b} + \frac{\partial \zeta_a}{\partial c} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

and since for the whole surface we have

$$\xi_a \cos\alpha + \eta_a \cos\beta + \zeta_a \cos\gamma = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2b)$$

therefore both integrals are equal to zero and the equation (5*b*) as well as the equations (2) are satisfied. The equations (4) and (5) or (5*a*) are thus true integrals of the equations (1_4) and (2).

The analogy mentioned in the introduction between the action at a distance of vortex filaments and the electro-magnetic action at a distance of conducting wires, which analogy affords a very good means of making visible the form of the vortex motion, results from this proposition.

When we substitute in the equation (4) the values of L, M, N, from the equation (5*a*) and designate by Δu, Δv, Δw the infinitely small portions of the velocities u, v and w in the integral which depend on the material elements da, db, dc and designate their resultant by Δp, we obtain

$$\Delta u = \frac{1}{2\pi}\,\frac{(y-b)\zeta_a-(z-c)\eta_a}{r^3}da\;db\;dc,$$

$$\Delta v = \frac{1}{2\pi}\,\frac{(z-c)\xi_a-(x-a)\zeta_a}{r^3}da\;db\;dc,$$

$$\Delta w = \frac{1}{2\pi}\,\frac{(x-a)\eta_a-(y-b)\xi_a}{r^3}da\;db\;dc.$$

From these equations it follows that,

$$\Delta u(x-a)+\Delta v(y-b)+\Delta w(z-c)=0,$$

that is to say, Δp, the resultant of Δu, Δv, Δw, is at right angles to r. Further,

$$\xi_a\Delta u+\eta_a\Delta v+\zeta_a\Delta w=0,$$

that is to say, this same resultant, Δp, also makes a right angle with the resulting axis of rotation at the point a, b, c. Finally,

$$\Delta p=\sqrt{(\Delta u)^2+(\Delta v)^2+(\Delta w)^2}=\frac{da\;db\;dc}{2\pi r^2}\;\sigma \sin \nu,$$

where σ is the resultant of [the elementary velocities of rotation] ξ_a, η_a, ζ_a, and ν is the angle between this resultant and r, as determined by the equation,

$$\sigma r \cos \nu=(x-a)\xi_a+(y-b)\eta_a+(z-c)\zeta_a$$

Therefore every rotating particle of liquid a *causes in every other particle* b *of the same mass of liquid a velocity that is directed perpendicularly to the plane passing through the axis of rotation of the particles* a *and* b. *The magnitude of this velocity is directly proportional to the volume of* a, *to its velocity of rotation, and to the sine of the angle between the line* ab *and the axis of rotation, and inversely proportional to the square of the distance of the two particles.*

The force that an electric current, moving parallel to the axis of rotation at the point a, would exert upon a magnetic particle at b, follows exactly the same law as above.

The mathematical relationship of both classes of natural phenomena

consists in the fact that in the case of liquid vortices there exists in those parts of the liquid that have no rotation a velocity potential φ, which satisfies the equation:

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=0,$$

which equation fails to hold good only within the vortex filaments themselves. But when we imagine the vortex filaments as closed, either within or without the mass of liquid, then the region in which the above differential equation for φ holds good is a manifold-connected space, for it remains still connected when we imagine intersecting planes passing through it, each of which is completely bounded by a vortex filament. In such manifold-connected spaces a function φ that satisfies the above differential equation becomes many-valued, and it must be many-valued if it is to represent re-entering currents: for since the velocities [u, v, w,] of the liquid particles outside of the vortex filaments are proportional to the [partial] differential coëfficients of φ [with reference to x, y, z], therefore, following the liquid particle in its motion one would find the values of φ steadily increasing. Therefore, if the current returns into itself, and if one by following it comes finally back to the place where he before was, he will find for this place a second value of φ larger than before. Since we can repeat this process indefinitely therefore for every point of such a manifold-connected space, there must be an infinite number of different values of φ, which differ from each other by equal differences, like the different values of

$$\text{tang}^{-1}\left(\frac{x}{y}\right)$$

which is such a many-valued function as satisfies the above differential equation.

The electro-magnetic effects of a closed electric current have relations similar to the preceding. The current acts at a distance as would a certain distribution of magnetic masses over a surface bounded by the conductor. Therefore, outside of such a current the forces that it exerts upon a magnetic particle can be considered as the differential quotients of a potential function V which satisfies the equation

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0.$$

Here also the space that surrounds the closed conductor and throughout which this equation holds good, is manifold-connected, and V is many-valued.

Therefore in the vortex motions of liquids, as in the electro-magnetic actions, velocities or forces respectively external to the space occupied by the vortex filaments or the electric currents depend upon many-valued potential functions which moreover satisfy the general differential equations of the magnetic potential function, while on the other hand *within* the space occupied by the vortex filaments or electric cur-

rents, instead of potential functions which can not exist here, there occur other common functions such as are expressed in the equations (4), (5), and (5a). On the other hand, for simple progressive movements of liquids and for the magnetic forces, just as for gravitation, for electric attractions and for the steady flow of electricity and heat, we have to do with single-valued potential functions.

The integrals of the hydro-dynamic equations, for which a single-valued velocity potential exists, we can call *integrals of the first class.* Those on the other hand for which there are rotations in one portion of the liquid particles, and correspondingly a many-valued velocity potential for the non-rotating particles we call *integrals of the second class.* It can happen that in the latter case only such portions of the space are to be considered in the problem as contain no rotatory particles of liquid, *e. g.*, in the case of the movements of liquid in a ring-shaped vessel, where a vortex filament can be imagined traversing the axis of the vessel, and where notwithstanding this the problem belongs to those that can be resolved by means of the assumption of a velocity potential.

In the hydro-dynamic integrals of the first class the velocities of the liquid particles have the same direction as, and are proportional to the forces that would be produced by a certain distribution of the magnetic masses outside of the liquid acting on a magnetic particle at the location of the particle of liquid.

In the hydro-dynamic integrals of the second class the velocities of the liquid particles have the same direction as, and are proportional to forces acting on the magnetic particle such as would be produced by a closed electric current flowing through the vortex filament and having a density proportional to the velocity of rotation of this filament, combined with the action of magnetic masses entirely outside the liquid. The electric currents within the liquid would flow forward with the respective vortex filaments, and must retain a constant intensity. The adopted distribution of magnetic masses outside of the liquid or on its surface must be so defined that the boundary conditions are satisfied. Every magnetic mass can also, as is well known, be replaced by electric currents. Therefore instead of introducing into the values u, v, and w, the potential function P of an exterior mass k, we can obtain an equally general solution if we give to the quantities ξ, η, and ζ external to the fluid or even only on its surface, such arbitrary values that only closed current filaments arise, and then extend the integration of the equations (5a) over the whole region for which ξ, η, and ζ differ from zero.

IV. VORTEX SHEETS AND THE ENERGY OF THE VORTEX FILAMENTS.

In the hydro-dynamic integrals of the first class it suffices, as I have already shown, to know the movement of the surface; the movement in the interior is then entirely determined. For the integrals of the second class, on the other hand, the movements of the vortex filaments

located within the fluid are to be determined, taking account of their mutual influences and of the boundary conditions whereby the problem becomes much more complicated. However, for certain simple cases, even this problem can be solved, especially in those cases where the rotations of the liquid particles take place only on certain surfaces or lines and the forms of these surfaces and lines remain unchanged during the translatory motions.

The properties of surfaces that adjoin an indefinitely thin layer of rotating fluid particles are easily seen from the equations (5a). When ξ, η, and ζ differ from zero only within an infinitely thin layer, then, according to well-known propositions, the potential functions L, M, and N will have equal values on both sides of the layer,* but the partial differential coefficients of these functions for the direction normal to the layer will be different on the two sides of the layer. Imagine the coördinate axes so placed that at the point of the vortex sheet under consideration the axis of z corresponds to the normal to the sheet, the axis of x to the axis of rotation of the liquid particles situated in the sheet, so that at this point we have $\eta=\zeta=0$; then will the potentials M and N, as also their partial differential coefficients, have the same values on both sides of the sheet, similarly L and $\frac{\partial L}{\partial x}$ and $\frac{\partial L}{\partial y}$; but $\frac{\partial L}{\partial z}$ will have two different values whose difference is equal to $2\xi\varepsilon$, when ε indicates the thickness of the stratum. Corresponding to this the equation (4) shows that u and w have the same values on each side of the vortex sheet, but v has values that differ from each other by $2\xi\varepsilon$. Therefore, that component of the velocity that is perpendicular to the vortex line and tangent to the vortex sheet has different values on either side of the vortex sheet. Within the layer of rotating liquid particles we must imagine the respective components of the velocity as uniformly increasing from the value that obtains on one side of the surface to that which obtains on the other side. For when, as here, ξ is constant through the whole thickness of the layer, and we indicate by α a proper fraction, by v^1 the value of v on one side, by v_1 its value on the other side, by v_α its value within the layer itself at a distance $\alpha\varepsilon$ from the former side; then, as we saw before,

$$v^1-v_1=2\xi\varepsilon$$

because a layer of the thickness ε and the rotatory velocity ξ lies between the two sides. For the same reasons we must have

$$v^1-v_\alpha=2\xi\varepsilon\alpha=\alpha\,(v^1-v_1),$$

which covers the proposition just enunciated. Since we must think of the rotating liquid particles as themselves moving forward and since the change of distribution on the surface depends on their own motion, therefore we must, through the whole thickness of the layer, attribute

* [This is the "vortex sheet" of English writers.]

to these particles such a mean velocity of progression parallel to the surface as corresponds to the arithmetical mean of the velocities [v^1 and v_1] prevailing on the two sides of the layer.

For instance such a vortex sheet would be formed when two fluid masses previously separated and in motion come into contact with each other. At the surface of contact the velocities perpendicular thereto must necessarily balance each other. In general the velocities tangent to this surface will, however, be different from each other in the two fluids. Therefore the surface of contact will have the properties of a vortex sheet.

On the other hand, we should not in general think of individual vortex filaments as infinitely slender, because otherwise the velocities on opposite sides of the filament would have infinite values and opposite signs, and therefore the velocity proper of the filament would be indeterminate. In order now to draw certain general conclusions as to the movement of very slender filaments of any sectional area, the principle of the conservation of living force will be made use of.

Therefore before we pass to individual examples, we must first write the equation for the living force K of the moving mass of water, or

$$K=\tfrac{1}{2}h\int\int\int(u^2+v^2+w^2)dx\,dy\,dz. \qquad (6)$$

In this integral I substitute from equation (4):

$$u^2=u\left(\frac{\partial P}{\partial x}+\frac{\partial N}{\partial y}-\frac{\partial M}{\partial z}\right)$$

$$v^2=v\left(\frac{\partial P}{\partial y}+\frac{\partial L}{\partial z}-\frac{\partial N}{\partial x}\right)$$

$$w^2=w\left(\frac{\partial P}{\partial z}+\frac{\partial M}{\partial x}-\frac{\partial L}{\partial y}\right)$$

and integrate by parts; then I indicate by cos α, cos β, cos γ, and cos θ the angles made by the coördinate axes and the resulting velocity, q, respectively, with the interior normal to the element $d\omega$ of the mass of liquid and having regard to equations (2) and (1_4) I obtain:

$$K=-\frac{h}{2}\int d\omega[Pq\cos\theta+\mathrm{L}(v\cos\gamma-w\cos\beta)$$

$$+\mathrm{M}(w\cos\alpha-u\cos\gamma)+\mathrm{N}(u\cos\beta-v\cos\alpha)] \qquad (6a)$$

$$-h\int\int\int(\mathrm{L}\xi+\mathrm{M}\eta+\mathrm{N}\zeta)dx\,dy\,dz.$$

The value of

$$\frac{dK}{dt}$$

is obtained from the equation (1) by multiplying the first by u, the second by v, the third by w, and adding; whence results:

$$h\left(u\frac{du}{dt}+v\frac{dv}{dt}+w\frac{dw}{dt}\right)=-\left(u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}\right)$$
$$+h\left(u\frac{\partial V}{\partial x}+v\frac{\partial V}{\partial y}+w\frac{\partial V}{\partial z}\right)$$
$$-\frac{h}{2}\left(u\frac{\partial(q^2)}{\partial x}+v\frac{\partial(q^2)}{\partial y}+w\frac{\partial(q^2)}{\partial z}\right)$$

When we multiply both sides by $dx\,dy\,dz$, then integrate over the whole volume of the liquid mass, and recall that because of (1_4)

$$\iiint\left(u\frac{\partial\psi}{\partial x}+v\frac{\partial\psi}{\partial y}+w\frac{\partial\psi}{\partial z}\right)dx\,dy\,dz=-\int\psi\,q\cos\theta\,d\omega,$$

where ψ denotes a function that is continuous and univalued throughout the interior of the liquid mass, we obtain,

$$\frac{dK}{dt}=\int d\omega\,(p-hU+\tfrac{1}{2}hq^2)\,q\cos\theta \quad . \; . \; . \; . \; . \; . \; . \; . \quad (6b)$$

When the liquid mass is entirely inclosed within rigid walls then at all points of the surface $q\cos\theta$ must be zero, therefore then will $\frac{dK}{dt}=0$, or K become constant.

If we imagine this rigid wall to be at an infinite distance from the origin of coördinates and all vortex filaments that may be present to be at an infinite distance from this origin, then will the potential functions L, M, N [of imaginary magnetic matter], whose masses ξ, η, ζ, $\left[\text{or densities } \frac{-\xi}{2\pi}, \frac{-\eta}{2\pi}, \frac{-\zeta}{2\pi}\right]$, each and all are equal to zero, diminish at the infinite distance $\Re$ as $\frac{1}{\Re^2}$ and the velocities [which are the partial differential coefficients of L, M, N], will vary as $\frac{1}{\Re^3}$, but the elementary surface $d\omega$, if it is always to correspond to the same solid angle at the origin of the coördinates, will increase as $\Re^2$. The first integral in the expression for K, equation $(6a)$, which is extended over the surface of the liquid mass, will diminish as $\frac{1}{\Re^3}$ and therefore will be zero for $\Re$ equal to infinity.

The value of K then reduces to the expression,

$$K=-h\iiint(L\xi+M\eta+N\zeta)\,dx\,dy\,dz \quad . \; . \; . \; . \; . \; . \quad (6c)$$

and this quantity is unchanged during the movement.

V. RECTILINEAR PARALLEL VORTEX FILAMENTS.

We will first investigate the case where only rectilinear vortex threads exist parallel to the axis of z, either within a liquid mass of infinite extent or which comes to the same thing, in one that is bounded by two infinite planes perpendicular to the vortex filaments. In this case

all motions take place in planes that are perpendicular to the axis of z and are precisely the same in all such planes.

Therefore we put

$$w = \frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = \frac{\partial p}{\partial z} = \frac{\partial V}{\partial z} = 0.$$

Then equations (2) reduce to

$$\xi = 0,\ \eta = 0,\ 2\zeta = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x},$$

the equations (3) become

$$\frac{\delta\zeta}{\delta t} = 0$$

Therefore the vortex threads, in so far as they have constant sectional areas, have also constant velocities of rotation.

The equations (4) reduce to,

$$u = \frac{\partial N}{\partial y}, \quad v = \frac{\partial N}{\partial x}, \quad \frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2} = 2\zeta.$$

In this I have put $P = 0$ in accord with the remark in Sect. III. Therefore the equation of the streamline is $N =$ constant.

In this case N is the potential function of infinitely long lines; this function itself is infinitely large, but its differential coefficients are finite. Let a and b be the coördinates of a vortex filament the area of whose cross-section is $da\,db$, then is

$$-v = \frac{\partial N}{\partial x} = \frac{\zeta\,da\,db}{\pi} \cdot \frac{x-a}{r^2},$$

$$u = \frac{\partial N}{\partial y} = \frac{\zeta\,da\,db}{\pi} \cdot \frac{y-b}{r^2}.$$

Hence it follows that the resultant velocity q is perpendicular to the r drawn perpendicular to the vortex filament and its value is

$$q = \frac{\zeta\,da\,db}{\pi r}.$$

If within a liquid mass of indefinite extent in the direction x and y we have many vortex filaments whose coördinates are respectively x_1, y_1; x_2, y_2, etc., while the products of rotatory velocity by the sectional area are for each distinguished by m_1, m_2, etc., and if we form the sums,

$$U = m_1\,u_1 + m_2\,u_2 + m_3\,u_3, \text{ etc.,}$$

$$V = m_1\,v_1 + m_2\,v_2 + m_3\,v_3, \text{ etc.,}$$

then these sums are each equal to zero, because that part of each sum that is due to the action of the second vortex filament on the first is counterbalanced by the action of the first vortex filament on the second. That is to say, the two effects are, respectively,

$$m_1 \cdot \frac{m_2}{\pi} \cdot \frac{x_1 - x_2}{r^2} \text{ and } m_2 \cdot \frac{m_1}{\pi} \cdot \frac{x_2 - x_1}{r^2},$$

and so on through all the other pairs of sums. Now U is the velocity in the direction of x, of the center of gravity of the masses m_1, m_2, etc., multiplied by the sum of these masses; similarly V is the velocity taken in the direction of y. Both velocities are therefore zero, unless the sum of the masses is zero, in which case there is no center of gravity at all. Therefore the center of gravity of the vortex filaments remains unchanged during their motion, and since this proposition holds good for every distribution of the vortex filaments, therefore we may also apply it to the individual filaments of infinitely small cross section.

Hence result the following consequences:

(1) If we have but one individual rectilinear vortex filament of infinitely small cross-section within a liquid mass of infinite extent in all directions perpendicular to the filament, then the movement of the particles of water at a finite distance from the filament depends only on the product $\xi\, da\, db = m$, or the velocity of rotation multiplied by the area of the cross-section, and not on the form of the cross-section. The liquid particles rotate about the filament with the tangential velocity $\frac{m}{\pi r}$, where r denotes the distance from the center of gravity of the vortex filament. The location of the center of gravity, the velocity of rotation, the area of the cross section, and therefore also the quantity m remains unchanged although the form of the infinitely small cross-section may change.

(2) If we have two rectilinear vortex filaments of infinitely small cross-sections and an indefinitely large liquid mass, each will drive the other in a direction that is perpendicular to the line joining them together. The length of this connecting line will not be changed thereby; therefore both will revolve about their common center of gravity, remaining at equal constant distances therefrom. If the rotatory velocity is in the same direction in the two filaments and therefore has the same sign, then their center of gravity must lie between them. If the rotations are mutually opposed to each other and therefore of opposite signs, then their center of gravity lies in the prolongation of the line connecting the filaments. If the products of the rotatory velocity by the cross section are numerically equal for the two but of opposite signs, thereby causing the center of gravity to be at an infinite distance, then both filaments advance with equal velocity and in the same direction perpendicular to their connecting line.

The case where a vortex filament of infinitely small section lies close to an infinitely extended plane surface parallel to it can be reduced to this last case. The boundary condition for the movement of the liquid along a plane (*i. e.*, that the motion must be parallel to this plane) is satisfied when we imagine a second vortex filament, which is as the reflected image of the first, introduced on the other side of the plane. Hence it follows that the vortex filament within the liquid mass ad-

vances parallel to the plane in the direction in which the liquid particles, between it and the plane, themselves move, and with one-fourth of the velocity possessed by the particles that are at the foot of the perpendicular drawn from the filament to the plane.

The assumption of the infinitely small cross-section leads to no inadmissible results, because each individual filament exerts no force upon itself affecting its own progression, but is driven forwards only by the influence of the other filaments that may be present [or by the action at the boundary]. But it is otherwise in the case of curved filaments.

VI. CIRCULAR VORTEX FILAMENTS.

In a liquid mass of indefinite extent let there be present only circular filaments whose planes are perpendicular to the axis of z, and whose centers lie in this axis, so that all are symmetrical about this axis. Transform the coördinates by putting

$$\begin{aligned} x &= \chi \cos \varepsilon, & a &= g \cos e, \\ y &= \chi \sin \varepsilon, & b &= g \sin e, \\ z &= z, & c &= c. \end{aligned}$$

Agreeably to the assumption just made, the velocity of rotation σ is only a function of χ and z, or of g and c, and the axis of rotation is everywhere perpendicular to χ (or g) and to the axis of z. Therefore the rectangular components of the rotation at this point whose coördinates are g, e, and c become

$$\xi = -\sigma \sin e,\ \eta = \sigma \cos e,\ \zeta = 0.$$

In the equation ($5a$) we now have,

$$r^2 = (z-c)^2 + \chi^2 + g^2 - 2\chi g \cos (\varepsilon - e)$$

$$L = \frac{1}{2\pi} \int\int\int \frac{\sigma \sin e}{r}\, g\, dg\, de\, dc$$

$$M = -\frac{1}{2\pi} \int\int\int \frac{\sigma \cos e}{r}\, g\, dg\, de\, dc$$

$$N = 0$$

From the equations for L and M by multiplying by $\cos \varepsilon$ and $\sin \varepsilon$ and adding we obtain

$$L \sin \varepsilon - M \cos \varepsilon = -\frac{1}{2\pi} \int\int\int \frac{\sigma \cos (\varepsilon - e)}{r}\, g\, dg\, d(\varepsilon - e)\, dc.$$

$$L \cos \varepsilon + M \sin \varepsilon = \frac{1}{2\pi} \int\int\int \frac{\sigma \sin (\varepsilon - e)}{r}\, g\, dg\, d(\varepsilon - e)\, dc,$$

In both these integrals the angles e and ε occur only in the connection $(\varepsilon - e)$ and this quantity can therefore be considered as the variable under the sign of integration. In the second integral the terms that contain $(\varepsilon - e) = \mathfrak{c}$ balance those that contain $(\varepsilon - e) = 2\pi - \mathfrak{c}$; therefore this integral is equal to zero.

Therefore if we put

$$\psi = \frac{1}{2\pi} \int\int\int \frac{\sigma \cos e\, g\, dg\, de\, dc}{\sqrt{(z-c)^2+\chi^2+g^2-2g\chi \cos e}} \tag{7}$$

then will

$$M \cos \varepsilon - L \sin \varepsilon = \psi$$
$$M \sin \varepsilon + L \cos \varepsilon = 0,$$

or

$$L = - \psi \sin \varepsilon, \quad M = \psi \cos \varepsilon. \tag{7a}$$

Let τ denote the velocity in the direction of the radius χ, and consider the fact that on account of the symmetrical position of the vortex ring in reference to the axis z the velocity must be zero in the direction of the circumference of the circle, we must have

$$u = \tau \cos \varepsilon, \quad v = \tau \sin \varepsilon$$

and according to equations (4)

$$u = \frac{\partial M}{\partial z}, \quad v = \frac{\partial L}{\partial z}, \quad w = \frac{\partial L}{\partial x} - \frac{\partial M}{\partial y}.$$

Hence it follows that

$$\tau = -\frac{\partial \psi}{\partial z}, \quad w = \frac{\partial \psi}{\partial \chi} + \frac{\psi}{\chi}$$

or

$$\tau \chi = - \frac{\partial(\psi \chi)}{\partial z}, \quad w\chi = \frac{\partial (\psi \chi)}{\partial \chi}. \tag{7b}$$

Therefore the equation of the stream line is

$$\psi \chi = \text{const.}$$

When we execute the integrations indicated in the value of ψ, first for a vortex filament of infinitely small cross-section, putting therein $m_1 = \sigma\, dg\, dc$ and indicating by ψ_{m_1} the part of ψ depending thereon, we have

$$\psi_{m_1} = \frac{m_1}{\pi} \sqrt{\frac{g}{\chi}} \left\{ \frac{2}{\varkappa} (F - E) - \varkappa F \right\} .$$

$$\varkappa^2 = \frac{4 g \chi}{(g+\chi)^2+(z-c)^2},$$

wherein F and E indicate the complete elliptic integrals of the first and second order respectively for the modulus $\varkappa$.

For brevity we put

$$U = \frac{2}{\varkappa} (F-E) - \varkappa F,$$

where U is therefore a function of $\varkappa$, then is

$$\tau \chi = \frac{m_1}{\pi} \sqrt{g\chi} \cdot \frac{\partial U}{\partial \varkappa} . \varkappa \cdot \frac{z-c}{(g-\chi)^2+(z-c)^2}.$$

If now a second vortex filament m exist at the point determined by χ and z, and if we let τ_1 be the velocity in the direction of g that m communicates to the filament m_1, we then obtain the value of this ve-

locity if in the expression for τ we substitute τ_1, g, χ, c, z, m, in place of τ, χ, g, z, c, m_1.

In this process $\varkappa$ and U remain unchanged and we obtain,

$$m\tau\chi+m_1\tau_1 g=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If now we determine the value of the velocity w parallel to the axis, caused by the vortex filament m_1 whose coördinates are g and c, we find:

$$w\chi=\tfrac{1}{2}\frac{m_1}{\pi}\sqrt{\frac{g}{\chi}}\,U+\frac{m_1}{\pi}\sqrt{g\chi}\frac{\partial U}{\partial\varkappa}\cdot\frac{\varkappa}{2\chi}\cdot\frac{(z-c)^2+g^2-\chi^2}{(g+\chi)^2+(z-c)^2}.$$

If now we call w_1 the velocity at the locality of m_1 parallel to the axis of z, which is caused by the vortex ring m whose coördinates are z and χ, then in order to determine this, we only need to execute the interchange of appropriate coördinates and masses as above shown. Thus we find:

$$2mw\chi^2-2\,m_1\,w_1\,g^2-m\tau\chi z-m_1\tau_1 gc=\frac{2mm_1}{\pi}\sqrt{g\chi}\,U. \quad . \quad . \quad (8a)$$

Sums similar to (8) and (8a) can be found for any number of vortex rings. For the nth of these rings I designate the product $\sigma\,dg\,dc$ by m_n; the components of the velocity that is communicated to this ring by all the other rings are τ_n and w_n, in which however I provisionally omit the velocities that every vortex ring can communicate to itself. Further I call the radius of this ring ρ_n and its distance from a surface perpendicular to the axis λ, which two latter quantities agree with χ and z as to direction, but, as belonging to this particular ring, they are functions of the time and not independent variables as are χ and z. Finally let the value of ψ, in so far as it depends on the other vortex rings, be ψ_n. By forming and adding the equations (8) and (8a) corresponding to each pair of vortex rings, there results

$$\Sigma\,[m_n\,\rho_n\,\tau_n]=0.$$

$$\Sigma\,[2\,m_n\,w_n\,\rho^2_n-m_n\,\tau_n\,\rho_n\,\lambda_n]=\Sigma\,[m_n\,\rho_n\,\psi_n].$$

So long as we have in these sums only a finite number of separate and infinitely slender vortex rings, we must understand by w, τ, and ψ only those parts of these quantities that are due to the presence of the other rings. But when we imagine an infinite number of such rings keeping the space continuously filled, then ψ becomes the potential function of a continuous mass, w and τ become partial differential coefficients of this potential function, and it is known* that both for such functions and for their differential coefficients, the portions of the function that depend upon the presence of matter within an infinitely small space surrounding a point for which the function is determined are infinitely small with respect to those portions that depend on finite masses at finite distances.

* See Gauss, *Allgemeine Theorie des Erdmagnetismus* in the *Resultate des magnetischen Vereins im Jahre*, 1839, page 7, or the translation in Taylor's Scientific Memoirs, vol. II.

Therefore if we change the sums into integrals we can understand by w, τ, and ψ the total values of these quantities that exist at the point in question, and can put

$$w=\frac{d\lambda}{dt}, \qquad \tau=\frac{d\rho}{dt}.$$

To this end we replace the quantity m by the product $\sigma d\rho d\lambda$, and the summations thus become converted into the following integrals:

$$\int\int\sigma\rho\frac{d\rho}{dt}d\rho\, d\lambda=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$2\int\int\sigma\rho^2\frac{d\lambda}{dt}d\rho\, d\lambda-\int\int\sigma\rho\lambda\frac{d\rho}{dt}d\rho\, d\lambda=\int\int\sigma\rho\psi\, d\rho\, d\lambda \quad . \quad . \quad (9a)$$

Since, in accordance with Sect. II, the product $\sigma\, d\rho\, d\lambda$ does not vary with the time, therefore, the equation (9) can be integrated with respect to t, and we obtain

$$\tfrac{1}{2}\int\int\sigma\rho^2\, d\rho\, d\lambda = \text{Const.}$$

Imagine the space divided by a plane that passes through the axis of z, and therefore intersects all the vortex rings that are present; then consider σ as the density of one layer of the mass, and let $\mathfrak{M}$ be the total mass in this layer adjoining this dividing plane; therefore,

$$\mathfrak{M}=\int\int\sigma\, d\rho\, d\lambda,$$

and let R^2 be the mean value of ρ^2 for all the elementary masses, then

$$\int\int\sigma\rho.\,\rho d\rho\, d\lambda=\mathfrak{M}\, R^2,$$

and, since this integral and the value of $\mathfrak{M}$ do not vary with the time, it follows that R also remains unchanged during the motion of translation.

Therefore if there exists in the unlimited mass of liquid only one circular vortex filament of infinitely small sectional area, then its radius remains unchanged.

According to equation (6c), the total living force in our case is

$$\begin{aligned}K&=-h\int\int\int(L\xi+M\eta)da\, db\, dc.\\ &=-h\int\int\int\psi\sigma\cdot\rho\, d\rho\, d\lambda\, d\varepsilon.\\ &=-2\pi h\int\int\psi\sigma\cdot\rho\, d\rho\, d\lambda.\end{aligned}$$

This also does not change with time.

Furthermore, because $\sigma\, d\rho\, d\lambda$ does not vary with time, therefore,

$$\frac{d}{dt}\int\int\sigma\rho^2\lambda\, d\rho\, d\lambda=2\int\int\sigma\rho\lambda\frac{d\rho}{dt}d\rho\, d\lambda+\int\int\sigma\rho^2\frac{d\lambda}{dt}d\lambda\, d\rho:$$

therefore if we indicate by l the value of λ for the center of gravity of the vortex filament treated of in equation (9a), and multiply (9) by this l, and add the result to (9a), and substitute therein the equation last given, we obtain

$$2\frac{d}{dt}\int\int\sigma\rho^2\lambda\, d\rho\, d\lambda+5\int\int\sigma\rho(l-\lambda)\frac{d\rho}{dt}d\rho\, d\lambda=-\frac{K}{2\pi h} \quad . \quad . \quad (9b)$$

When the section of the vortex thread is infinitely small and ε is an infinitely small quantity of the same order as $(l-\lambda)$ and the remaining linear dimensions of the section, but $\sigma\, d\rho\, d\chi$ is finite, then ψ and also K are of the same order of infinitely large quantities as $\log \varepsilon$. For very small values of the distance v from the vortex ring we have

$$v=\sqrt{(g-\chi)^2+(z-c)^2},$$

$$\kappa^2=1-\frac{v^2}{4\,g^2},$$

$$\psi\, m_1=\frac{m_1}{\pi}\log\left(\sqrt{\frac{1-\kappa^2}{4}}\right)=\frac{m_1}{\pi}\log\frac{v}{8g}.$$

In the value of K, ψ is multiplied by ρ or g. If g is finite, and v of the same order as ε, then K is of the same order as $\log \varepsilon$. Only when g is infinitely large of the order $\frac{1}{\varepsilon}$ will K be infinitely large of the order $\frac{1}{(\varepsilon)}\log \varepsilon$. But in this case the circle becomes a straight line. On the other hand, if $\frac{d\rho}{dt}$, which is equal to $\frac{\partial\psi}{\partial z}$, is of the order $\frac{1}{\varepsilon}$, then the second integral will be finite, and for a finite value of ρ will be infinitely small with respect to K. In this case we can, in the first integral, substitute the constant l in place of λ and obtain

$$2\frac{d(\mathfrak{M}R^2l)}{dt}=-\frac{K}{2\pi h}$$

or

$$2\mathfrak{M}R^2l=C-\frac{K}{2\pi h}t$$

Since $\mathfrak{M}$ and R are constant, l can only vary proportionally to the time. When $\mathfrak{M}$ is positive the motion of the liquid particles on the outer side of the ring is directed toward the side of positive z, but on the inner side of the ring toward the negative z. K, h, and R are by their nature always positive.

Hence it follows that for a circular vortex filament of very small cross-section in an infinitely extended mass of liquid the center of gravity of a cross-section has a motion parallel to the axis of the vortex ring, which is of approximately constant and very large velocity, and which is directed toward the same side as that toward which the liquid flows through the ring. Infinitely slender vortex filaments of a finite radius will have infinitely large velocities of propagation. But if the radius of the vortex ring is infinitely large of the order $\frac{1}{(\varepsilon)}$, then will R^2 be infinitely large with respect to K, and l will be constant. The vortex filament which has thus transformed itself into a straight line will be stationary, as we had already previously found for rectilinear vortex filaments.

We can now in general see how two circular vortex threads having a common axis will behave with respect to each other, since each one independent of its own translatory motion also follows the movement of the liquid particles caused by the other filament. If they have the same direction of rotation, then they both advance in the same direction, and at first the preceding one enlarges, then it advances more slowly while the following one diminishes and advances more rapidly; finally, if the progressive velocities are not too different, the second catches up with the first and passes through it. Then the same performance is repeated by the one that is now in the rear so that the rings alternately pass through each other.

If the vortex filaments have the same radii, but equal and opposite rotatory velocities, then they will approach each other and simultaneously enlarge, so that finally when they have come very close together their movement towards each other grows continually feebler, while on the other hand the enlargement goes on with increasing rapidity. If the two vortex threads are perfectly symmetrical, then midway between them the velocity of the liquid particles in the direction parallel to the axis is equal to zero. Therefore one can imagine a rigid wall located here without disturbing the motion and thus obtain the case of a vortex ring that encounters a rigid wall.

I remark further that we can easily study these movements of circular vortex rings in nature if we draw a half-immersed circular disk or the approximately semicircular end of a spoon rapidly for a short distance along the surface of a liquid and then quickly draw it out. There then remain in the liquid semi-vortex rings whose axes lie in the free upper surface of the liquid. The free upper surface thus forms, for the liquid mass, a boundary plane that passes through the axis whereby no important change is made in the motions. The vortex rings advance, broaden when they encounter a screen, and are enlarged or diminished by the action of other vortex rings precisely as we have deduced from the theory.

III.

ON DISCONTINUOUS MOTIONS IN LIQUIDS.*

By Prof. H. von Helmholtz.

It is well known that the hydro-dynamic equations give precisely the same partial differential equations for the interior of an incompressible fluid that is not subject to friction and whose particles have no motion of rotation, as obtain for stationary currents of electricity or heat in conductors of uniform conductivity. One might therefore expect that for the same external form of the space traversed by the current and for the same boundary conditions the form of the current (except for differences depending on small incidental conditions), would be the same for liquids, for electricity, and for heat. In reality however in many cases there exist easily recognizable and very fundamental differences between the currents in a liquid and the above mentioned imponderables.

Such differences are especially notable when the currents flowing through an opening with sharp edges enter into a wider space. In such cases the stream lines of electricity radiate from the opening outwards immediately towards all directions, while a flowing fluid, water as well as air, moves from the opening at first forward in a compact stream which at a less or greater distance then ordinarily resolves itself into a whirl. The portions of the fluid in the larger receiving vessel lying near the opening but outside the stream can, on the other hand, remain almost at perfect rest. Every one is familiar with this mode of motion, especially as a current of air impregnated with smoke shows it very plainly. In fact the compressibility of the air does not come much into consideration in these processes, and with slight variations air shows the same forms of motion as does water.

On account of the great differences between the facts as observed and the results of theoretical analysis as hitherto achieved the hydrodynamic equations must necessarily appear to the physicist as a prac-

* From the *Monatsberichte* of the Royal Academy of Science, Berlin. 1868, April 23, pp. 215–228. Helmholtz *Wissenschaftliche Abhandlungen*, vol. I, pp. 146–157. Berlin, 1882.

tically very imperfect approximation to the reality. The cause of this might be suspected to lie in the internal friction or viscosity of the fluid, although all forms of infreqent and sudden irregularities (with which certainly everyone has to contend who has instituted observations on the movements of fluids) can evidently never be explained as the effect of the steadily and uniformly acting friction.

The investigation of cases where periodical movements are excited by a continuous current of air, as, for example, in organ pipes, showed me that such an effect could only be produced by a discontinuous motion of the air, or at least by a kind of motion coming very near to it, and this has lead me to the discovery of a condition that must be taken into consideration in the integration of the hydro-dynamic equations, and that, so far as I know, has been overlooked hitherto, whose consideration on the other hand, in those cases where the computation can be carried out, really gives, in fact, forms of motions such as those that are actually observed. This condition is due to the following circumstance:

In the hydro-dynamic equations the velocity and the pressure of the flowing particles are treated as continuous functions of the coördinates. On the other hand, there is no reason in the nature of a liquid, if we consider it as perfectly fluid, therefore not subject to viscosity, why two contiguous layers of liquid should not glide past each other with definite velocities. At least those properties of fluids that are considered in the hydro-dynamic equations, namely, the constancy of the mass in each element of space and the uniformity of pressure in all directions, evidently furnish no reasons why tangential velocities of finite difference in magnitude should not exist on both sides of a surface located in the interior. On the other hand, the components of velocity and of pressure perpendicular to the surface must of course be equal on both sides of such a surface. I have already in my memoir on vortex motions called attention to the fact that such a case must occur when two moving masses of liquid previously separate and having different motions come to have their surfaces in contact. In that memoir I was led to the idea of such a surface of separation,* or vortex surface as I there called it† through the fact that I imagined a system of parallel vortex filaments arranged continuously over the surface whose mass was indefinitely small without losing their moment of rotation.

Now, in a liquid at first quiet or in continuous motion a definite difference in the movement of immediately adjoining particles of liquid can only be brought about through moving forces acting discontinuously. Among the external forces the only one that can here come into consideration is impact.

But in the interior of liquids there is also a cause present that can

[* Ordinarily called *surface of discontinuity* or "a discontinuous surface" by English writers.]

[† That is, an infinitely thin layer of parallel vortex filaments, the "*vortex sheet*" of English writers.]

bring about discontinuity of motion—namely, the pressure, which can assume any positive value whatever while the density of the liquid will continuously vary therewith; but as soon as the pressure passes the *zero* value and becomes negative, a discontinuous variation of the density occurs; the liquid is torn asunder.

Now, the magnitude of the pressure (at any point) in a moving fluid depends on the velocity (at that point), and in incompressible fluids the diminution of pressure under otherwise similar circumstances is directly proportional to the living force of the moving particles of liquid. Therefore if the latter exceed a certain limit the pressure must, in fact, become negative, and the liquid tears asunder. At such a place the accelerating force, which is proportional to the differential quotient of the pressure, is evidently discontinuous, and thus the condition is fulfilled which is necessary in order to bring about a discontinuous motion of the liquid. The movement of the liquid past any such place can now take place only by the formation from that point onward of a surface of discontinuity.

The velocity that will cause the tearing asunder of the liquid is that which the liquid would assume when it flows into empty space under the pressure that the liquid would have at rest at the point in question. This is indeed a relatively considerable velocity; but it is to be remarked that if liquids flow continuously like electricity the velocity at every sharp edge around which the current bends must be infinitely great.* Thence it follows that *at every geometrically perfect sharp edge past which liquids flow, even for the most moderate velocity of the rest of the liquid, it must be torn asunder and form a surface of discontinuity.* On the other hand, for imperfectly somewhat rounded edges such phenomena first occur for certain larger velocities. Pointed protuberances on the surface of a canal through which a current flows will have similar effects.

As concerns gases, the same circumstance occurs as with liquids, only with this difference,—that the living force of the motion of a particle is not directly proportional to the diminution of the pressure (p); but taking into consideration the cooling of the air by its expansion the living force is proportional to the diminution of p^m, where $m = 1 - \frac{1}{\gamma}$ and γ is the ratio of the specific heat at constant pressure to that for constant volume. For atmospheric air the exponent m has the value 0.291. Since this is positive and real, therefore p^m, like p, for high values of the velocity can only diminish to zero and not become negative. It would be otherwise if gases simply followed the law of Mariotte and experienced no change of temperature. Then instead of p^m the quantity $\log p$ would occur, which can become negative and infinite without

*At the very small distance ρ from a sharp edge whose surfaces meet each other at the angle α the velocities will be infinite, or as ρ^{-m}, where $m = \frac{\pi - \alpha}{2\pi - \alpha}$.

p being negative. Under this condition the tearing asunder of the mass of air would not be necessary.

It is possible to convince one's self of the actual existence of such discontinuities when we allow a stream of air impregnated with smoke to issue from a round opening or a cylindrical tube with moderate velocity so that no hissing occurs. Under favorable circumstances one obtains thin rays or jets of this kind of a few lines diameter and a length of many feet. Within the cylindrical surface the air is in motion with constant velocity, but outside it, on the other hand, in the immediate neighborhood of the jet it moves not at all or very slightly. One sees this very sharp separation clearly when we conduct a steadily flowing cylindrical jet of air through the point of a flame, out of which it cuts a sharply defined piece, while the rest of the flame remains entirely undisturbed, and at most a very thin stratum of flame, which corresponds to the boundary layer of the jet influenced by friction, is carried along a little way.

As concerns the mathematical theory of this motion I have already given the boundary conditions for the existence of an interior surface of separation within the liquid. They consist in this that the pressures on both sides the surface must be equal and equally so the components of the velocity normal to the discontinuous surface. Since now the movement throughout the entire interior of a liquid whose particles have no motion of rotation is wholly determined when the motion of its entire exterior surface and its interior discontinuities are given, therefore in general for a liquid whose exterior boundary is fixed, it is only necessary to know the movement of the surfaces of separation and the variations of the discontinuity.

Now such a discontinuous surface can be treated mathematically precisely as if it were a *vortex sheet*, that is to say, as if it were continuously enveloped by vortex filaments of indefinitely small mass but finite moments of rotation. For each element of such a vortex sheet there is a direction for which the components of the tangential velocities are equal. This gives at once the direction of the vortex filaments at the corresponding place. The moment of this filament is to be put proportional to the difference existing between the components, taken perpendicular to it, of the tangential velocity on both sides of the surface.

The existence of such vortex filaments in an ideal frictionless liquid is a mathematical fiction that facilitates the integration. In a real liquid subject to friction, this fiction becomes at once a reality inasmuch as by the friction the boundary particles are set in rotation, and thus vortex filaments originate there having finite gradually increasing masses, while the discontinuity of the motion is thereby at the same time compensated.

The motion of a vortex sheet and the vortex filaments lying in it is to be determined by the rules established in my *Memoir on Vortex*

Motions. The mathematical difficulties of this problem however can be overcome only in a few of the simpler cases. In many other cases, however, one can from the above given method of consideration of this matter at least draw conclusions as to the general nature of the variations that occur.

Especially is it to be mentioned that in accordance with the laws established for vortex motions, the vortex filaments and with them the vortex sheets in the interior of a frictionless liquid can neither originate nor disappear, but rather each vortex filament must retain permanently the same constant moment of rotation; furthermore that the vortex filaments themselves advance along the vortex sheet with a velocity that is the mean of the two velocities existing on the two sides of the discontinuous surface. Thence it follows that *a surface of discontinuity can only elongate in the direction towards which the stronger* of the two currents that meet in it is directed.

I have first sought to find examples of permanent discontinuous surfaces in steady currents, for which the integration can be executed, in order thereby to prove whether the theory gives forms of currents that correspond to experience better than when we disregard the discontinuity of motion. If a surface of discontinuity that separates quiet and moving water from each other is to remain stationary, then along this surface the pressure within the moving layer must be the same as in the quiet layer, whence it follows that the tangential velocity of the particles of liquid must be constant throughout the whole extent of the surface; equally so must the density of the fictitious vortex filament be constant. The beginning and end of such a surface can only lie on the boundary of the inclosure or at infinity. Where the former alternative is the case they must be tangent to the wall of the inclosure, assuming that the latter is continuously curved, because the component-velocity normal to the wall of the inclosure must be zero.

Moreover the stationary forms of the discontinuous surface are distinguished, as experiment and theory agree in showing, by a remarkably high degree of variability under the slightest perturbations, so that to a certain extent they behave similarly to bodies in unstable equilibrium. The astonishing sensitiveness to sound waves of a cylindrical jet of air impregnated with smoke has already been described by Tyndall; I have confirmed this observation. This is evidently a peculiarity of surfaces of discontinuity that is of the greatest importance in operating sonorous pipes.

Theory allows us to recognize that in general wherever an irregularity is formed on the surface of an otherwise stationary jet, this must lead to a progressive spiral unrolling of the corresponding portion of the surface, which portion, moreover, slides along the jet. This tendency towards spiral unrolling at every disturbance is moreover easy to see in the observed jets. According to the theory a prismatic or cylindrical jet can be indefinitely long. In fact however such an one can not be

formed, because in an element so easily moved as is the air small disturbances can never be entirely avoided.

It is easy to see that such an endless cylindrical jet, issuing from a tube of corresponding section into a quiet exterior fluid and everywhere containing fluid that is moving with uniform velocity parallel to its axis, corresponds to the requirements of the "steady condition."

I will here further sketch only the mathematical treatment of a case of the opposite kind, where the current from a wide space flows into a narrow canal, in order thereby also at the same time to give an example of a method by which some problems in the theory of potential functions can be solved that hitherto have been attended by difficulties.

I confine myself to the case where the motion is steady and dependent only upon two rectangular coördinates, x and y; where moreover no rotating particles are present in the frictionless fluid at the beginning, and where none such can be subsequently formed. If we indicate by u the component parallel to x of the velocity of the fluid particle at the point (xy) and by v the velocity parallel to y, then, as is well known, two functions of x and y can be found such that

$$\left.\begin{aligned} u &= \frac{\partial \varphi}{\partial x} = \frac{\partial \psi}{\partial y} \\ v &= \frac{\partial \varphi}{\partial y} = -\frac{\partial \psi}{\partial x} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

By these equations the conditions are also directly fulfilled that in the interior of the fluid the mass shall remain constant in each element of space, viz:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

For a constant density, h, and when the potential of the external forces is indicated by V, the pressure in the interior is given by the equation—

$$V - \frac{p}{h} + c = \tfrac{1}{2}\left[\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2\right] = \tfrac{1}{2}\left[\left(\frac{\partial \psi}{\partial x}\right)^2 + \left(\frac{\partial \psi}{\partial y}\right)^2\right] \quad . \quad . \quad . \quad . \quad (1b)$$

The curves

$$\psi = \text{constant}$$

are the stream lines of the fluid, and the curves

$$\varphi = \text{constant}$$

are orthogonal to them. The latter are the equi-potential curves when electricity, or the equal temperature curves when heat, flows in steady currents in conductors of uniform conductivity.

From the equation (1) it follows as an integral equation that the quantity $\varphi + \psi i$ is a function of $x + yi$, where $i = \sqrt{-1}$. The solutions hitherto found generally express φ and ψ as the sums of terms that are

themselves functions of x and y. But inversely we can consider and develop $x + yi$ as a function of $\varphi + \psi i$. In problems relative to currents between two stationary walls, ψ is constant along the boundaries, and therefore if φ and ψ are presented as rectangular coördinates in a plane, then in a strip of this plane bounded by two parallel straight lines, $\psi = c_0$ and $\psi = c_1$, the function $x + yi$ is to be so taken that on the edge it corresponds to the equation of the wall, but in the interior it assumes a given variability.

A case of this kind occurs when we put

$$x + yi = A \{ \varphi + \psi + e^{\phi + \psi i} \} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or

$$x = A\,\varphi + Ae^{\phi} \cos \psi$$
$$y = A\,\psi + Ae^{\varphi} \sin \psi$$

For the value $\psi = \pm\,\pi$ we have y constant and $x = A\,\varphi - Ae^{\phi}$.

When φ varies from $-\infty$ to $+\infty$ the value of x changes at the same time from $-\infty$ to $-A$, and then again back to $-\infty$.

The stream lines $x = \pm\,\pi$ correspond thus to a current along two straight walls, for which $y = \pm\,A\,\pi$ and x varies between $-\infty$ and $-A$.

Therefore when we consider ψ as the expression of the stream curve the equation (2) corresponds to the flow out into endless space from a canal bounded by two parallel planes. On the border of the canal however where $x = -A$ and $y = \pm\,A\,\pi$ and where further, $\varphi = 0$ and $\psi = \pm\,\pi$, we have

$$\left(\frac{\partial x}{\partial \varphi}\right)^2 + \left(\frac{\partial y}{\partial \varphi}\right)^2 = 0.$$

therefore

$$\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2 = \infty$$

Electricity and heat flow in this manner, but liquids must tear asunder.

If from the border of the canal there extend stationary dividing discontinuous lines that are of course prolongations of the stream lines $\psi = \pm\,\pi$ that follow along the wall and if outside of these discontinuous lines that limit the flowing fluid there is perfect quiet, then must the pressure be the same on both sides of these dividing lines. That is to say, along that portion of the line $\psi = \pm\,\pi$ which corresponds to the free dividing line, in accordance with the equation (1b), we must have

$$\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2 = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In order now, in the solution of this modified problem, to retain the fundamental idea of the motion expressed in equation (2), we will add

to the above expression of $x+y\,i$ still another term $\sigma+\tau\,i$, which is also always a function of $\varphi+\psi\,i$, we have then

$$\left.\begin{aligned} x &= A\,\varphi + A\,e^{\phi}\cos\psi + \sigma \\ y &= A\,\psi + A\,e^{\phi}\sin\psi + \tau \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (3a)$$

and must determine $\sigma+\tau i$ so that along the free portion of the discontinuous surface where $\psi=\pm\pi$ we shall have

$$\left(A - Ae^{\phi} + \frac{\partial\sigma}{\partial\varphi}\right)^2 + \left(\frac{\partial\tau}{\partial\varphi}\right)^2 = \text{constant.}$$

This condition is fulfilled if we make

$$\frac{\partial\sigma}{\partial\varphi} = 0 \text{ or } \sigma = \text{Constant} \quad \ldots\ldots\ldots \quad (3b)$$

and

$$\frac{\partial\tau}{\partial\varphi} = \pm A\sqrt{2e^{\phi} - e^{2\phi}} \quad \ldots\ldots\ldots \quad (3c)$$

Since ψ is constant along the wall we can integrate the last equation with respect to φ, and change the integral into a function of $\varphi+\psi\,i$ by substituting everywhere instead of φ the expression $\varphi+i\,(\psi+\pi)$. Thus by an appropriate determination of the constants of integration we obtain

$$\sigma+\tau\,i = A\,i\left\{\sqrt{-2e^{(\phi+\psi i)} - e^{(2\phi+2\psi i)}} + 2\,\text{arc sin}\left[\frac{i}{\sqrt{2}}\cdot e^{\frac{1}{2}(\phi+\psi i)}\right]\right\} \quad \ldots \quad (3d)$$

The cusp points of this expression lie where

$$e^{(\phi+\psi\,i)} = -2;$$

that is to say where

$$\psi = \pm\,(2\,a+1)\,\pi \text{ [}a\text{ being any whole number],}$$

and

$$\varphi = \log 2.$$

Thus neither one lies between the limits from $\psi=+\pi$ to $\psi=-\pi$.

The function $\sigma+\tau i$ is here continuous.

Along the wall we have

$$\sigma+\tau i = \pm A\,i\left\{\sqrt{2e^{\phi}-e^{2\phi}} - 2\text{ arc sin}\left[\frac{1}{\sqrt{2}}\,e^{\frac{\phi}{2}}\right]\right\}$$

If $\varphi > \log 2$, then all these values become purely imaginary, therefore $\sigma=0$, while $\frac{d\tau}{d\varphi}$ has the value given above in equation (3c). This portion of the lines $\psi=\pm\pi$ therefore corresponds to the free portion of the jet.

If $\varphi < \log 2$ the whole expression is real up to the additive quantity $\pm\,A\,i\,\pi$, which latter is to be added to the value of $\tau\,i$ and $y\,i$ respectively.

The equations (3a) and (3d) correspond therefore to the outflow from an unlimited basin into a canal bounded by two planes, whose breadth is $4\,A\,\pi$ and whose walls extend from $x=-\infty$ to $x=-A\,(2-\log 2)$. The free discontinuous line of the flowing fluid curves from the nearest edge of the opening at first a little towards the side of the positive x, where for $\varphi=0$, $x=-A$ and reaches its greatest x value when $y=\pm A\left(\frac{3}{2}\pi+1\right)$; then it turns inward towards the inside of the canal and at last asymptotically approaches the two lines $y=\pm A\,\pi$, so that finally the breadth of the outflowing jet is equal only to the half breadth of the canal.

The velocity along the discontinuous surface and at the extreme end of the outflowing jet is $\frac{1}{A}$, so that this form of motion is possible for every velocity of efflux.

I present this example especially as it shows that the form of the liquid stream in a tube can for a very long distance be determined by the form of the initial portion.

ADDITION, BEARING ON ELECTRICAL DISTRIBUTION.

When in equation (2) we consider the quantity ψ as the electric potential it gives the distribution of electricity in the neighborhood of the edges of two plane disks quite near together, assuming that their distance is indefinitely small with respect to the radius of curvature of their curved edge. This is a very simple solution of the problem that has been considered by Clausius.* It gives moreover the same distribution of electricity as he found for it; at least so far as it is independent of the curvature of the edges.

I will further add that the same method also suffices to find the distribution of electricity on two parallel, infinitely long, plane strips, whose four edges in cross section form the corners of a rectangle, that is, the cross section of the strips gives two lines which are opposite and parallel to each other. The potential function ψ in this case is given by an equation of the form

$$x+y\,i=A\,(\varphi+\psi\,i)+B\frac{1}{H\,(\varphi+\psi\,i)} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $H\,(u)$ represents the function designated by Jacobi in the *Fundamenta Nova*, p. 172, as the numerator of the function developed in terms of sin *am u*. The overlying strips correspond, according to Jacobi's notation, to the values $\varphi=\pm 2\,K$ where $x=\pm 2\,KA$ gives the half distance of the strips, while the width of the strip depends on the ratio of the constants A and B.

The form of the equations (2) and (4) allows us to recognize that φ and ψ can be expressed as function of x and y only by means of most complicated serial developments.

* Poggendorff's *Annalen*, Bd. LXXXVI.

IV.

ON A THEOREM RELATIVE TO MOVEMENTS THAT ARE GEOMETRICALLY SIMILAR IN FLUID BODIES, TOGETHER WITH AN APPLICATION TO THE PROBLEM OF STEERING BALLOONS.*

By Prof. H. von Helmholtz.

The laws of motion of cohesive and non-cohesive fluids [namely, liquids and gases] are sufficiently well known in the form of differential equations, that take into consideration not only the influence of exterior forces acting from a distance, as well as the influence of the pressure of the fluid, but also the influence of the friction [namely, both internal and external frictions, or both viscosity and resistance]. When in the application of these equations one remembers that under certain circumstances [namely, wherever a continuous motion would give a negative pressure] there must form surfaces of separation with discontinuous motion on the two sides, as I have sought to prove in a previous communication to this academy,† then will disappear the contradictions that by neglect of this consideration have hitherto been made to appear to exist between many apparent consequences of the hydro-dynamic equations on the one hand and the observed reality on the other. In fact, so far as I see, there is at present no ground for considering the hydro-dynamic equations as not being the exact expression of the laws controlling the motions of fluids.

Unfortunately it is only for relatively few and specially simple experimental cases that we are able to deduce from these differential equations the corresponding integrals appropriate to the conditions of the given special cases, especially if the nature of the problem is such that the internal friction [viscosity] and the formation of surfaces of discontinuity can not be neglected. The discontinuous surfaces are extremely variable, since they possess a sort of unstable equlibrium, and with every disturbance in the whirl they strive to unroll themselves; this circumstance makes their theoretical treatment very difficult. Thus it happens

* From the *Monatsberichte* of the Royal Academy of Berlin, June 26, 1873, pp. 501 to 514. *Wissenschaftliche Abhandlungen*, vol. II, pp. 158–171, Berlin, 1882.

† Berlin *Monatsberichte*, April 23, 1868. See also No. III of this collection of Translations.

that where we have to do practically with the motions of fluids we are thrown almost entirely back upon experimental trials, and can often, from theory, predict but very little, and that only in an uncertain manner, as to the result of new modifications of our hydraulic machines, aqueducts, or propelling apparatus.

In this state of affairs I desire to call attention to an application of the hydro-dynamic equations that allows one to transfer the results of observations made upon any fluid and with an apparatus of given dimensions and velocity over to a geometrically similar mass of another fluid and to apparatus of other magnitudes and to other velocities of motion.

To this end I designate by u v w the components of the velocity of the first fluid in the directions of the rectangular coördinate axes x y z; by t the time, by p the pressure, by ε the density, by k its coefficient of friction (viscosity). The equations of motion in the Eulerian form introducing the frictional forces, as is done by Stokes, in case no exterior forces act upon the fluid, will now have the following form:

$$-\frac{\partial \varepsilon}{\partial t}=\frac{\partial(u.\varepsilon)}{\partial x}+\frac{\partial(v.\varepsilon)}{\partial y}+\frac{\partial(w.\varepsilon)}{\partial z} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$-\frac{1}{\varepsilon}\cdot\frac{\partial p}{\partial x}=\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-k\left\{\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right\}$$

$$-\frac{k}{3}\frac{\partial}{\partial x}\left\{\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

To these are still to be added the two equations that are deduced from the latter equation ($1a$) by interchanging x and u with y and v or with z and w.

When now for another fluid the velocities are designated by U, V, W, the pressure by P, the coördinates by X, Y, Z, the time by T, the density by E, the viscosity constant by K, and if we introduce three constants q, r, and n, and put

$$K=qk \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$E=r\varepsilon \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

$$U=nu \qquad X=\frac{q}{n}x$$

$$V=nv \qquad Y=\frac{q}{n}y$$

$$W=nw \qquad Z=\frac{q}{n}z$$

$$P=n^2rp+\text{constant.} \qquad T=\frac{q}{n^2}t$$

then the quantities designated by these capital letters will also fulfill the above differential equations. If we substitute these in those equations, the result, E, is as if all the terms of equation (1) were multiplied by

the factor $\frac{rn^2}{q}$ and all the terms of equation (1a) by the factor $\frac{n^3}{q}$. Of the constants q, r, n, two are determined through the equations (2) and (2a) by the nature of the fluid, but the third, n, is arbitrary so far as the conditions hitherto considered come into consideration.

If the fluid is incompressible, then ε is to be considered as a constant and $\frac{\partial \varepsilon}{\partial t}=0$, and the above equations then suffice to determine the motion in the interior.

If the fluid is compressible, we can put

$$p=a^2\varepsilon-c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$
$$P=A^2E-C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

where c and C indicate constants to be added to the pressure and which have no influence on the equation 1a.

For gases c and C are to be put equal to zero if the motion occurs under such circumstances that the temperature remains constant. For rapid variations of density in gases without equalization of temperature (namely non-adiabatic motions), the equations (3) and (3a) would only apply for the case of slight variations in density.

The equation (3a) is only satisfied by the above-given values for P and E when

$$A^2=a^2n^2.$$

By this condition therefore the third constant, n, is determined. The quantities a and A in this latter equation are the velocities of sound in the respective fluids. These quantities must change in the same ratio as the other velocities.

If the boundaries of the fluid are in part infinitely distant and in part given by moving or quiet, perfectly wetted, rigid bodies, and the coördinates and component velocities of these limiting rigid bodies are transferred from one case to the other in the same manner as has just been done for the particles of fluid, then will the boundary conditions for U, V, W be fulfilled when they are fulfilled for u, v, w. In this I assume that on completely wetted bodies the superficial layer of fluid is held perfectly adherent; that therefore the component velocities of the surfaces of the rigid bodies and those of the adherent fluid are equal.

For imperfectly wetted solids it is as a rule assumed that there is a relative motion of the superficial fluid layers with respect to the solid. In this case the application of our principles would require that a certain ratio be assumed between the coefficients of sliding superficial friction of the fluid on the respective rigid bodies, and the internal friction (or viscosity) of the fluid.

Similarly the boundary conditions at the free surfaces of a liquid over which the surface pressure is constant, would be satisfied in case no

outside forces like gravity have an influence. But since this case occurs only in liquids [*i. e.*, fluids that form drops] that can be regarded as incompressible, therefore (for these) it is not necessary to satisfy equations (3) and (3*a*). Therefore (for these) the constant n remains arbitrary, and when for this case this latter constant is so determined that $\frac{n^3}{q}=1$, then in equation (1*a*) the intensity of gravity (*i. e.*, the acceleration, $-g$) can be added to the left-hand member.

The boundary condition for a discontinuous surface is that the pressure shall be equal on both sides of such a surface, which condition will be satisfied for P when it is so for p.

As regards the re-action of the fluid against a solid body moving in it, the pressure against the unit of area of surface increases as $n^2 r$. In the same ratio, the frictional forces increase that are proportional to the product of $k\,\varepsilon$, with the differential quotients such as $\frac{\partial u}{\partial x}$, and other similar ones. But for corresponding similar portions of the surfaces of the bounding bodies of the forces of pressure and of friction increase as

$$\frac{q^2}{n^2} \cdot n^2 . r = q^2 r.$$

The work needed to be done by the immersed bodies to overcome these resistances will therefore for equal intervals of time increase as nq^2r.

In general therefore for compressible fluids [gases] and for heavy cohesive fluids [liquids under gravitation] with free surfaces, if the movement is to be completely and accurately transferred from the first fluid to the other, the three constants n, q, r are completely determined by the nature of the two fluids. Only in the case of incompressible fluids without free surfaces does one constant remain indeterminate.

Now there is a large series of cases where the compressibility not only for cohesive, but also for gaseous fluids, has only an inappreciably small influence. To such cases the following considerations apply: If the constant n becomes smaller while r and q remain unchanged, this indicates that in the second fluid the velocity of sound diminishes proportionally with n, and similarly for the velocities of the moving material portions, whereas the linear dimensions increase proportional to the reciprocal of n. For a constant value of r, that is to say, a constant density of the second fluid, a diminution of the velocity of sound corresponds to an increased compressibility of the fluid. Therefore with an increased compressibility, the movements remain similar. Hence it follows that when we diminish n, while leaving the compressibility of the fluid unchanged, the movements of the fluid themselves change and become similar to those that a more incompressible fluid would execute in a narrower space. Therefore for smaller velocities,

even in extensive spaces, the compressibility loses its influence. Under such circumstances gases move like cohesive incompressible fluids [viz, liquids], as is well known practically from many examples.

If the velocities of the material parts are in general very small, as in the case of exceedingly small oscillations, so that the course of the movement remains sensibly unchanged for a uniform increase in these velocities, then it will only be the velocity of sound that changes, and our proposition will take the following form: The sonorous vibrations of a compressible fluid can, in larger spaces, behave mechanically the same as more rapid oscillations of a less compressible fluid in smaller spaces. An example of the utilization of the similarity here spoken of is found in my investigations on the acoustic movement at the ends of open organ pipes.* In that study the possibility of replacing the analytical conditions of the motion of the air by the simpler ones of the motion of water depended on the principle that the dimensions of the given spaces must be very small in comparison to the wave lengths of the existing acoustic vibrations.

On the other hand the viscosity also shows itself less influential in the movements of fluids in large spaces. If we let n remain unchanged while q increases we obtain the same ratio between the frictional forces and the pressure forces. That is to say, if we increase the dimensions and the friction constants in the same ratio, then the movements in the enlarged system remain similar so long as the velocities do not change. Hence it follows that in such an enlarged model, when the friction constant is not increased in the same ratio, but remains unchanged, the friction loses in influence for the same velocity. That which holds good for greater dimensions with unchanged velocities also obtains for increased velocities with unchanged dimensions. For one can also simultaneously let n increase proportional to q.

In fact, in most practical experiments in extended fluid masses, the resistance that arises from the accelerations of the fluid,† and especially in consequence of the formation of surfaces of discontinuity is by far the most important. Its magnitude increases proportionally to the square of the velocity, whereas the resistance depending upon the friction proper (internal friction or viscosity and surface-hesion), which increases simply in proportion to the velocity, becomes appreciable only in experiments in very narrow tubes and vessels.

Neglecting the friction, that is to say, if in the above equations we put the constants

$$k=K=0$$

then will the constant q also become arbitrary, and we can change the dimensions and velocities in any ratio whatever.

If however the force of gravity comes into consideration as in the

* Borchardt's *Journal für Mathematik*, 1859, vol. LVII, pp. 1-72.

† [These resistances are those that I have called "convective" in my Treatise on Meteorological Methods and Apparatus.—*C. A.*]

case of waves on the free surface of water, then, according to the remarks already made the ratio $\frac{n^3}{q}$ must remain unchanged, therefore q must be put $=n^3$. Then will

$$X=n^2x \qquad Y=n^2y \qquad T=nt. \qquad Z=n^2z$$

Therefore when the wave lengths increase in the ratio n^2 the duration of the oscillations will increase only in the ratio n, which corresponds to the well-known law of the velocity of propagation for the surface waves of water, which velocity increases as the square root of the wave length. Thus this result is attained very simply and for all wave forms, without the necessity of knowing a single integral of wave motion.

The same principle is applicable to the relative resistances that ships having n^2 times the dimensions and n times the velocity, experience by reason of the waves that they excite on the surface of the water. The total resistance in this case increases as q^2r, and since for the same fluid $r=1$ therefore the resistance increases as n^6 and the work needed to overcome it as n^7, therefore in a rather larger ratio than the volume of the ship, while the supply of fuel and the size of the boiler that must do the work can increase only in the same ratio as the volume of the ship, namely as n^6. Therefore so long as lighter machinery can not be applied (including the supply of coal) the velocity of such an enlarged ship can increase above a certain limit only by a ratio that is smaller than that of the square root of the increase of the linear dimensions.

A similar computation holds good for the model of the bird in the air. When we increase the linear dimensions of a bird and would take into consideration the viscosity, we must put q and r equal to unity because the medium, namely the air, remains unchanged. Let n be a vulgar fraction, then will the velocity be reduced in the same proportion as the volume of the bird increases and the pressure (of the air) against the total surface of the larger bird will only attain the same value as for the smaller bird, therefore will not be able to bear up the weight of the larger bird.

If we allow ourselves to neglect the friction, which according to the above remarks we can do so much the more readily the more we increase the dimensions, or for the same dimensions increase the velocities, then q is arbitrary and the change of dimensions and velocities must be so made that the total pressure against the surfaces shall increase as the weight of the body or we must have $q^2=\frac{q^3}{n^3}$ or $q=n^3$. In order to execute the corresponding motions, the work that will be necessary will be

$$q^2n=n^7=\left(\frac{q}{n}\right)^{\frac{7}{2}};$$

but the volume of the body and of the muscles that do the work increases only in the ratio $\left(\frac{q}{n}\right)^3$.

Hence it follows that the size of a bird has a limit, unless the muscles can be further developed in such a manner that for the same mass as now they shall perform more work. Now it is precisely among the larger birds, that are capable of the greater performances in flying, that we find those that eat only flesh and fish; they are animals that consume concentrated food and need no extensive system of digestive organs. Among the smaller birds many grain eaters like doves and the smaller singing birds are also good flyers. It therefore appears probable that in the model of the great vulture, nature has already reached the limit that can be attained with the muscles as working organs, and under the most favorable conditions of subsistence, for the magnitude of a creature that shall raise itself by its wings* and remain a long time in the air.

Under these circumstances it is scarcely to be considered as probable that man even by means of the most ingenious wing-like mechanism that must be moved by his own muscles will ever possess the strength needed to raise his own weight in the air and continue there.

Concerning the question as to the possibility of driving balloons forward relative to the surrounding air, our propositions allow us to compare this problem with the other one that is practically executed in many ways, namely, to drive a ship forward in water by means of oar-like or screw-like organs of motion. In studying this we must not consider movement on the surface, but rather imagine to ourselves a ship driven along under the surface. But such a balloon which presents a surface above and below that is congruent with the submerged surface of an ordinary ship scarcely differs in its powers of motion from an ordinary ship.

If now we let the small letters of the two above given systems of hydro-dynamic equations refer to water and the large letters to the air, then for 0° temperature and 760 mm. of the barometer, we have

$$\frac{1}{r}=773$$

According to the determination of O. E. Meyer and Clerk Maxwell,

$$q=0.8082;$$

the velocity of sound gives for n the value

$$n=0.2314$$

Hence the increase of linear dimensions is

$$\frac{q}{n}=3.4928$$

*[That is, by the work done by its wings; this of course does not cover the case of soaring where the muscles do no lifting work but simply keep the wings in the best position for the wind to act on them.—*C. A.*]

and the increase of volume is

$$\left(\frac{q}{n}\right)^3=42.61$$

The work in this case is very slight, namely,

$$q^2nr=\frac{1}{5114.3}$$

The ship, including the crew and the load, must weigh as much as the volume of water displaced by it. The balloon, filled with hydrogen, in order to carry an equal weight with the ship, must have a volume 837 times as great. If it is filled with illuminating gas of a specific gravity 0.65 relative to that of the air, it must have a volume 2,208.5 times as great as the ship. Thus, the weight that the balloon must have for the given dimension is now determined. The weight for the hydrogen balloon would be $\frac{42.6}{837}=\frac{1}{19.6}$ that of the ship; that of the illuminating gas balloon would be $\frac{42.6}{2208.5}=\frac{1}{51.8}$ that of the ship.

The work that is necessary under such circumstances to propel the balloon, as the above number for the value of q^2nr shows, would, however, for the adopted small velocity, be reduced in much greater proportion than that of the weight of the balloon to the weight of the ship, so that the work here required for the given weight is easy to accomplish in the balloon. For even when we so choose the ship that its load in excess of that of the driving machine (or in excess of the men who act as the machine) is negligible, then the weight of the illuminating-gas balloon need be only $\frac{1}{52}$ part of the weight of this driving machine, but the machine thus carried by it would also have to do only the one $\frac{1}{5114}$ of the work of the ship's machine, it would, therefore, need to have a less weight in about this latter ratio. Especially would this latter be the case when we utilize men as the driving machine, whose work and weight both increase proportionally to the number.

So far we can therefore apply the transference from ship to balloon with complete consideration of the peculiarities of air and water. As a maximum velocity for fast ships (large naval steamers), "The Engineer's Pocket Book," published by the society "*Die Hütte*," gives 18 feet per second, or 2.7 German miles, or 21 kilometers per hour. Similarly built balloons, with relatively very feeble or small propelling machinery, can attain about one-fourth of this velocity.

Ships of the above-given dimensions find the limit of their efficiency bounded by the limits of the power of the machinery (including the fuel) that they can carry. However, the practical experience thus far attained allows us to neglect the influence of viscosity for large, swift

ships, and therefore to arbitrarily assume the constant q, as also n (when we can neglect the movements at the surface). If we assume that q increases proportionally to n, then the dimensions remain unchanged, the velocities increase as n, the resistance as n^2, the work done as n^3. If therefore we were able to build a marine engine of the same weight as the present ones, but of greater efficiency, we would then be able also to attain greater velocities.

We must compare the balloon with such a ship, although the latter has not yet been constructed, in order to attain complete utilization of the propelling machine that goes up with it. But for this case also and for unchanged dimensions, when the velocity increases as n the work must increase as n^3.

Now the ratio between weight and work done by the men who are carried by a balloon can only, for balloons of very large dimensions, be perhaps more favorable than for a war ship and its machinery. For the latter I compute from the technical data that to attain a velocity of 18 feet requires an expenditure of one horse-power to 4636.1 kilograms weight.* On the other hand, a man weighing 200 pounds, who under favorable circumstances can do 75 foot-pounds of work per second during eight hours daily, gives on the average for the day one horse-power per 1,920 kilograms. When therefore the balloon weighs one and a half times as much as the laboring men whom it carries, then the ratio is the same as for the ship. Dupuy de Lôme has carried out his experiments under somewhat less favorable circumstances; in his balloon were a crew of 14 men whose weight was one-fourth of the whole, and of whom only eight worked. Under these circumstances it is a relatively very favorable assumption when for the balloon we assume the ratio between the weight and the work to be the same as for a war steamer. We can therefore for the illuminating gas balloon increase the ratio $\frac{51.831}{5114n^3}$ between work and weight by increasing n so that the ratio shall equal unity; that is to say, equal to the value for ships. In this case we must have

$$n = 4.6208.$$

Since now the velocity U of the balloon which we have before computed under the assumption of a perfect geometrical similarity in the

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*The special data on which the computation is based are as follows:

L = length of the ship over all = 230 Prussian feet.

B = breadth of the ship over all = 54 " "

H = total height of the ship = 24 feet.

T = depth under water = $H - \frac{1}{8} B$

V = volume of water displacement = $0.46\ L.\ B.\ T.$

Weight of one cubic foot of sea water = 63.343 lbs.

A the area of the immersed principal section = 1000 sq. feet.

The total work = ζ A V^3

Where $\zeta = 0.46$.

movements has only 0. 2314 that of the velocity u of the ship, therefore there results:

$$U=0.2314.\,n.\,u=1.06925u.$$

For the hydrogen balloon under the same assumptions the velocity will be somewhat larger, since in this case we have to assume

$$\frac{19.6}{5114}n^3=1.$$

Hence,

$$n=6.390$$
$$U=0.2314\,.n\,.u=1.4786u.$$

which is nearly one and a half times the velocity hitherto attained in naval steamers. This last velocity for a hydrogen balloon would suffice to go slowly forwards against a fresh breeze.

But it is to be remarked that these computations relate to colossal balloons whose linear dimensions are three and a half times larger than those of the immersed portion of a large man-of-war, and that the inflammable gas balloon would weigh 60220 kilograms, while that of Dupuy de Lôme only weighed 3799 kilograms. In order to return to dimensions that are attainable in actual practice, one must so diminish q and n as that the ratio of the work to the weight shall remain unchanged, therefore, so that

$$q^2\,n : \left(\frac{q}{n}\right)^3=1,$$

whence $$q=n^4.$$

In this way the velocity n will diminish as the cube root of the linear dimensions or as the ninth root of the volume or the weight. This reduction is relatively unimportant. If we pass, for example, from our ideal balloon down to one of the weight of that of Dupuy, there results a reduction of the velocity in the ratio of 1.36 to 1; this would give a velocity of 14.15 feet per second, or 16.5 kilometres per hour. The linear dimensions of the balloon would therefore exceed in the ratio 1.4 to 1 the dimensions of the ship that is compared with it.

The ratio between work and load in Dupuy's experiments correspond to the above assumptions very nearly. The eight men that worked for him are, according to our previous estimate, to be put down at 800 kilograms, which is rather more than one-fifth of the total weight. Since however the experiment only lasted a short time, therefore these men could work the whole time through with their whole energy, whereas in our computation only the average value of eight hours of work is assumed for the whole day. Therefore these eight men are equal to twenty-four steady workers, whereby the difference is more than made up. Dupuy gives, as having been attained independent of the wind,

on the average 8 kilometers per hour for the whole duration of the experiment, and $10\frac{1}{4}$ kilometers attained by intense work. He is therefore not very far behind the limit that my computations show attainaable with a balloon of such dimensions.

In the preceding computation we have however only taken account of the ratio between the effective force and the weight, and have assumed that the form of such a balloon and of its motor can be attained with the materials at our disposal. But here seems to me to lie one of the principal difficulties of the practical execution. For the parts of a machine made of rigid bodies do not by a geometrically similar increase in their linear dimensions retain the necessary stiffness; they must be made thicker, and therefore heavier. If on the other hand with small motors one would attain the same effect, by means of greater velocity, then work is dissipated. The pressure against the whole surface of a motor (a ship's propeller, or oars or paddles) increases as $q^2 r$. If this pressure, which determines the propelling force, is to remain unchanged, we can only diminish the dimensions in so far as we increase n, and therefore also the velocities; but then the work increases also as $q^2 n r$, and therefore proportionally to n. Therefore one can work economically only with relatively slow-moving motors of large surface. And to realize this in the necessary dimensions without too great a load for the balloon will be one of the greatest practical difficulties.

V.

ON ATMOSPHERIC MOTIONS.*

(*FIRST PAPER.*)

By Prof. H. VON HELMHOLTZ.

I. INFLUENCE OF VISCOSITY ON THE GENERAL CIRCULATION OF THE ATMOSPHERE.

The influence of fluid friction in the interior of very extended regions that are filled with fluid and contain no vortex motion is always relatively very small. This can be proved from considerations that are based upon the principle of mechanical similarity. If we form the Eulerian hydro-dynamic equations and in them indicate by u, v, w the components of the velocity parallel to the axes of x, y, z; by ε the density, by p the pressure, by P the potential of the forces that act upon a unit of mass of the fluid; then if we consider P, ε, p, u, v, w as functions of x, y, z, t we have, as is well known, the following partial differential equations for a fluid under the influence of friction†:

$$-\frac{\partial P}{\partial x}-\frac{1}{\varepsilon}\frac{\partial p}{\partial x}=\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-\frac{k^2}{\varepsilon}\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right] \quad . \quad . \quad . \quad . \quad (1)$$

$$-\frac{\partial \varepsilon}{\partial t}=\frac{\partial(\varepsilon\, u)}{\partial x}+\frac{\partial(\varepsilon\, v)}{\partial y}+\frac{\partial(\varepsilon\, w)}{\partial z} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

Two other equations symmetrical with regard to the other coördinates are to be added to the first of these equations. If now we have found any special integral whatever of these equations, which obtains for a definite region, then the equations will also hold good for a second case where all the linear dimensions x, y, z and also the time t and the friction constant k^2 are increased by a factor n, but where P, p, ε, u, v, w retain for every value of the new coördinates nx, ny, nz, nt, the same values as they had in the first case for the original coördinates x, y, z, t. Hence it follows that when in the movement of the magnified mass the friction constant can be also simultaneously and correspondingly increased, the

* From the *Sitzungsberichte* of the Royal Prussian Academy of Science at Berlin, 1888, May 31, pp. 647–663.

[† Namely viscosity as represented by Maxwell's kinematic coefficient ν or Helmholtz' $\frac{k^2}{\varepsilon}=\frac{0.0001878}{0.001293}=0.13417$]

movement takes place in an analogous manner, only slower. When this is not the case and when the friction retains its value unchanged then will the influence of the friction on the increased mass be very much less than upon the smaller mass. In consequence of this the greater mass will show the effects of its inertia as influenced much less by friction.

It is to be remarked that the potential P remains unchanged by the increase of the mass, but the force $\frac{\partial P}{\partial x}$ is reduced to $\frac{1}{n}$ of its value and that the whole process as already remarked requires for its completion n times the time.

Since the density and pressure are to remain unchanged therefore also any temperature differences that are present retain their magnitude and influence and do not disturb the relations implied in the mechanical similarity.

Unfortunately we can not imitate in small models the varying density of the atmosphere at different altitudes since we can not correspondingly change the force of gravity that is included in the expression $\frac{\partial P}{\partial x}$. Our mechanical comparisons are only able to imitate an atmosphere of constant density. Such an one must, as is well known, have an altitude of 8026 metres at 0° C. in order to produce the mean barometrical reading of 76 centimetres of mercury. If we desire in a model to represent the atmosphere by a layer of one metre in altitude, then we would need to reduce the day to 10.8 seconds, or the year to 65.5 minutes, and the influence of friction in movements at velocities that correspond to those of the atmosphere would in a small model be 8026 times as great as in the atmosphere. The loss of living force in the atmosphere during a year would therefore correspond to that lost in our model in $\frac{65.5}{8026}$ of a minute, which corresponds to less than a half a second.

On the other hand it is possible with the measured value of the friction constant of the air to compute for some simple cases how long a time would be required in order to reduce to one-half of its velocity any motion that is hindered only by internal friction. In this case the assumption of a constant density is for our purpose more unfavorable than the adoption of the actual variable density.

Assume that a stratum of air whose constant density is such as that of the lower stratum of the atmosphere, spreads over an unlimited plane and has a forward movement whose velocity is u in the direction of x parallel to the plane. Let z be the vertical coördinate, then the equation of motion for the interior of the mass is

$$\frac{\partial u}{\partial t} - \frac{k^2}{\varepsilon} \cdot \frac{\partial^2 u}{\partial z^2} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

Assume that the fluid adheres to the earth's surface where $z = 0$, therefore for this surface we have

$$u = 0_{z=0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

At the upper boundary surface where $z = h$ the fluid experiences no friction, therefore for that surface we have

$$\frac{\partial u}{\partial z} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2b)$$

Of the special integrals of the equation (2) that fulfill the boundary condition ($2a$), namely:

$$u = Ae^{-nt} sin\ (qx)$$
$$n = \frac{k^2}{\varepsilon} q^2$$

the one that also fulfills the condition (2_b) and is the most slowly diminishing is given by the value

$$q = \frac{\pi}{2h}$$

Hence follows

$$n = \frac{k^2}{\varepsilon} \cdot \frac{\pi^2}{4h^2}$$

The factor e^{-nt} becomes 1 at the time $t=0$: in order that this factor may be equal to one-half we must have

$$nt = \text{nat. log. } 2 = 0.69315.$$

According to Maxwell's determinations (Theory of Heat, London 1871, p. 279, where $\frac{k^2}{\varepsilon}$ is expressed by ν and k^2 by μ), we have

$$\frac{k^2}{\varepsilon} = 0.13417\ [1 + 0.00366\theta_c]\ .\ \frac{[\text{centimetre}]^2}{\text{second}}$$

where θ_c indicates the temperature centigrade. From this there results, for the temperature 0° C.,

$$t = 42747 \text{ years.}$$

If we distribute the same mass of air throughout a thicker stratum with less density so that $\varepsilon\ .\ h$, as also the k^2 which is independent of ε, retains its value unchanged, then t must increase with h. Hence it follows that in the upper thinner strata of the atmosphere the effect of viscosity propagates itself through atmospheric strata of equal mass more slowly than through the lower denser strata.

On the other hand an increase of the absolute temperature θ will cause the time t to diminish as $\frac{1}{\theta}$. The lower temperature of the upper

strata of the atmosphere also diminishes the effect of the viscosity here under consideration.

This computation also shows how extremely unimportant for the upper strata of the air are those effects of viscosity that can arise on the earth's surface in the course of a year.

Only at the fixed boundaries of the space that the atmosphere fills, or at the interior surfaces of discontinuity where currents of different velocity border on each other, do the surface forces remain the same when the scale of dimensions is increased and the coefficient of friction is not simultaneously increased, and this allows us to recognize that the annulment of living force by viscosity can take place principally only at the surface of the ground and at the discontinuous surfaces that occur in vortex motions.

A similar relation obtains with regard to those temperature changes that can be effected by the true conduction of heat in the narrower sense, namely, the diffusion of moving molecules of gas between the warmer and colder strata. The coefficient $\varkappa$ of conduction for heat, when we choose as the unit of heat that which warms a unit volume of the substance by one degree in temperature (or the thermometric coefficient of conduction), is, according to Maxwell (Theory of Heat, page 302):

$$\varkappa = \frac{5}{3.\gamma} \cdot \left(\frac{k^2}{\varepsilon}\right)$$

where γ is the ratio between the two specific heats of gases.

In order to solve the corresponding problem for the conduction of heat this $\varkappa$ is to be substituted in equation (2) instead of $\frac{k^2}{\varepsilon}$, and if we put $\gamma = 1.41$ it is seen that in the above-assumed atmosphere of uniform density under a pressure of 76 centimetres of mercury and at a temperature of 0° an interval of 36164 years would be necessary in order by conduction to reduce by one-half the final difference in temperature of the upper and lower surfaces. Therefore also in the interchange of heat only its radiation and its convection by the motion of the air need be taken into consideration, except at the boundary between it and the earth's surface and at the interior surfaces of discontinuity.

On the other hand, simple computations have frequently shown that an unrestricted circulation of the air in the trade zones can not exist even up to 30° latitude.

If we imagine a rotating ring of air whose axis coincides with that of the earth and which, by the pressure of neighboring similar rings, is pushed now northward and now southward, and in which we can neglect the friction, then, according to the well-known general mechanical principle, the moment of rotation of this ring must remain constant. We will indicate this moment as computed for the unit of mass by Ω,

and the angular velocity of the ring by ω, and its radius by ρ; then, as is well known,

$$\Omega = \omega\, \rho^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and therefor ω must vary inversely proportionally with ρ^2. If we indicate the mean radius of the earth by $R = 6379600$ metres, the geographical latitude by β, and the velocity of diurnal rotation of the earth by ω_0, then the corresponding relative velocity at the earth's surface for a ring of air that preserves a calm at the equator is

$$\rho\,(\omega - \omega_0) = \omega_0 \left[\frac{R}{\cos\,\beta} - R\cos\,\beta \right].$$

For air that is resting quietly at the equator in the zone of calms and is thence pushed up to the latitude of 10°, this expression gives the acquired wind velocity 14.18 metres per second, and similarly for air pushed up to latitude 20°, 57.63 metres, and for 30°, 133.65 metres per second.

Since 20 metres per second is the velocity of a railroad express train, therefore these numbers show without further consideration that such gales do not exist over any broad zone of the earth. We therefore ought not to make the assumption that the air which has risen at the equator reaches the earth's surface again unchecked in its motion even 20° farther northwards.

The matter is not much better if we assume the atmospheric ring resting at some intermediate latitude. In that case it would give an east wind at the equator, but a west wind at 30° latitude; but both velocities would far exceed the ordinary velocities of the observed winds.

Since now in fact observations do demonstrate a circulation of the air in the trade-wind zone, therefore the question recurs: By what means is the west-east velocity of this mass of air checked and altered? The resolution of this question is the object of the following remarks:

II. ON THE EQUILIBRIUM OF ROTATING RINGS OF AIR AT DIFFERENT TEMPERATURES.

If we introduce into equations (1) only rotatory motions about the axis, whereby ω, Ω, and ρ retain the significance just given them we then have

$$u = 0$$

$$v = -z\,\omega = -z.\frac{\Omega}{\rho^2}$$

$$w = y\,\omega = y.\frac{\Omega}{\rho^2}$$

and if we consider a steady mode of motion, in which Ω, p, P, and ε are functions of x and ρ only, then the equations (1) become

$$-\frac{\partial P}{\partial x}-\frac{1}{\varepsilon}\cdot\frac{\partial p}{\partial x}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

$$-\frac{\partial P}{\partial \rho}\cdot\frac{y}{\rho}-\frac{1}{\varepsilon}\cdot\frac{\partial p}{\partial \rho}\cdot\frac{y}{\rho}=-y\cdot\frac{\Omega^2}{\rho^4}$$

$$-\frac{\partial P}{\partial \rho}\cdot\frac{z}{\rho}-\frac{1}{\varepsilon}\cdot\frac{\partial p}{\partial \rho}\cdot\frac{z}{\rho}=-z\cdot\frac{\Omega^2}{\rho^4}.$$

The two last equations combine into the one following:

$$\frac{\partial P}{\partial \rho}+\frac{1}{\varepsilon}\cdot\frac{\partial p}{\partial \rho}=\frac{\Omega^2}{\rho^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3b)$$

Equation 1_a is satisfied by the above adopted values of u, v, w. Therefore the only equations to be satisfied are (3*a*) and (3*b*).

As concerns the value of the density ε, this depends upon the pressure p and the temperature θ. Since appreciable effective conduction of heat is excluded, therefore we must here retain the law of adiabatic variations between p and ε; therefore we have

$$\left(\frac{p}{p_0}\right)^{\frac{1}{\gamma}}=\frac{\varepsilon}{\varepsilon_0},$$

wherein γ again represents the ratio of the specific heats. If we indicate by θ the temperature that the mass of air under consideration would acquire adiabatically under the pressure p_0 (wherefore θ indicates the constant quantity of heat contained in the air while its temperature is varying with the pressure), and if we put

$$\frac{p_0}{\varepsilon_0\theta}=\Re$$

then we have

$$\frac{1}{\varepsilon}\cdot\frac{\partial p}{\partial \rho}=\left(\frac{p_0}{p}\right)^{\frac{1}{\gamma}}\cdot\frac{\theta.\Re}{p_0}\cdot\frac{\partial p}{\partial \rho};$$

or if, for further abbreviation, we put

$$\frac{\gamma}{\gamma-1}\cdot\Re\cdot p^{\frac{1-\gamma}{\gamma}}=q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3c)$$

$$p^{\frac{\gamma-1}{\gamma}}=\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3d)$$

we shall have

$$\frac{1}{\varepsilon}\cdot\frac{\partial p}{\partial \rho}=q\cdot\theta\cdot\frac{\partial \pi}{\partial \rho},$$

wherein q indicates a constant peculiar to the gas and independent of θ and p. Similarly we also have

$$\frac{1}{\varepsilon} \cdot \frac{\partial p}{\partial x} = q\theta \frac{\partial \pi}{\partial x}$$

and therefore within a stratum of air having a constant θ and Ω we have, according to equations ($3a$) and ($3b$),

$$P + q \cdot \theta \cdot \pi = -\frac{1}{2} \cdot \frac{\Omega^2}{\rho^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3e)$$

The very slight deviation of the earth from a spherical form allows us to simplify the computation on the one hand by regarding the earth's surface as a sphere, but on the other hand by giving the potential P an addition, the effect of which is that for the normal velocity of rotation ω_0 of the earth, its spherical surface becomes a level surface. To this end we put

$$P = -\frac{G}{r} + \frac{1}{2}\omega_0^2 \rho^2,$$

[Where G=normal force of gravity: r=distance from center of gravity to point or stratum in the actual atmosphere.]

This gives the component in the direction of x, of the forces acting upon the unit of mass,

$$X = -\frac{\partial P}{\partial x} = -\frac{Gx}{r^2};$$

and, for the component in the direction of ρ,

$$P = -\frac{\partial P}{\partial \rho} = -\frac{G.\rho}{r^2} - \omega^2 \rho$$

If to the latter the centrifugal force $+\omega^2.\rho$ is also added, there remains only one force on the rotating earth and which is directed normal to the spherical surface. Thus the spherical surface becomes the level surface of the combined potential force and centrifugal force, as indeed the surface of the earth really is.

Thus our equation ($3e$) becomes

$$q.\theta.\pi = -\frac{1}{2} \cdot \frac{\Omega^2}{\rho^2} + \frac{G}{r} - \frac{1}{2}\omega_0^2 \rho^2 + C \quad . \quad . \quad . \quad . \quad (3f)$$

The function π which is some power of the pressure p with positive exponent, increases and diminishes with p, and remains unchanged when p remains unchanged, so that we can determine the direction of the changes of the pressure easily by the changes of π.

Within a uniform stratum and with unchanged r, that is to say, for

a constant elevation above the earth's surface, π has a maximum value at the station and latitude where

$$\frac{\Omega^2}{\rho^3}=\omega_0{}^2\rho;$$

or, if we introduce ω instead of Ω from equation (3), the maximum occurs where

$$\omega^2=\omega_0{}^2;$$

that is to say, where the [movement of the] ring causes a calm [on the earth's surface]. Towards this locality the pressure increases both from the pole and from the equator.

III. EQULIBRIUM BETWEEN ADJACENT STRATA HAVING DIFFERENT VALUES OF θ AND Ω.

On both sides of the surfaces separating such strata, p and therefore also $q\cdot\pi$ (see equation $3d$) must have the same value. If we distinguish the quantities on either side [of the boundary surface] by the indices 1 and 2 we obtain from equation ($3f$)

$$\left(\frac{1}{\theta_1}-\frac{1}{\theta_2}\right)\cdot\frac{G}{r}=\tfrac{1}{2}\cdot\frac{1}{\rho^2}\left[\frac{\Omega_1{}^2}{\theta_1}-\frac{\Omega_2{}^2}{\theta_2}\right]+\tfrac{1}{2}\omega_0{}^2\rho^2\left[\frac{1}{\theta_1}-\frac{1}{\theta_2}\right]-\frac{C_1}{\theta_1}+\frac{C_2}{\theta_2}\quad . \quad . \quad (4).$$

This should be the equation of the boundary curve, linear with respect to r and quadratic with respect to ρ^2.

In order to find the direction of the tangent to this curve we differentiate equation (4) with respect to r and ρ, whence we get

$$\frac{G}{r^2}dr=\frac{d\rho}{\rho^3}\left[\frac{\Omega_1{}^2\theta_2-\Omega_2{}^2\theta_1}{\theta_2-\theta_1}-\omega_0^2\rho^4\right]\quad . \quad . \quad . \quad . \quad . \quad (4a)$$

or, if instead of Ω we introduce the corresponding value of ω from equation (3),

$$+\frac{G}{r^2}dr=\rho.d\rho.\quad\frac{(\omega_2{}^2-\omega_0{}^2)\theta_1-(\omega_1{}^2-\omega_0{}^2)\theta_2}{\theta_1-\theta_2}\quad . \quad . \quad . \quad . \quad (4b).$$

In order to decide how the two layers must lie with respect to the boundary surface if they are to have stable equilibrium, we reason as follows: The equation of the boundary surface (4) can, in accordance with the method of its deduction, be also written

$$\pi_1-\pi_2=\text{constant}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4c);$$

or, if we designate by ds one of its elements of length,

$$\frac{\partial}{\partial s}[\pi_1-\pi_2]=0.$$

Now π_1 and π_2 are functions that also have a meaning when continued beyond the boundary curve, and can be so extended by continuous

change [*i. e.*, without discontinuity]. The difference $(\pi_1 - \pi_2)$ will therefore in general increase on one side of the surface for increasing distance dn from this surface, but decrease, that is to say, become negative, on the other side; and thus on the side where $\frac{d(\omega_1 - \omega_2)}{dn}$ is positive we must have $\frac{\partial}{\partial h}(\pi_1 - \pi_2) > 0$ or positive for every other direction dh, in which one moves from any point of the surface towards the same side as dn.

If dh is drawn toward the other side of the surface for which $\pi_1 - \pi_2 = 0$, then will

$$\frac{\partial}{\partial h}(\pi_1 - \pi_2) < 0, \text{ or negative.}$$

If now the difference is positive on that side of the surface designated by the subscript index 1, then in case there is an infinitely small protrusion of the boundary surface toward this side, this protrusion will be pressed back by the exterior and greater π_1; similarly an infinitely small protrusion toward the negative side will also be pushed back, since there, on the other hand, π_1 diminishes more rapidly in the interior of such protrusion. Therefore in both these cases the equilibrium is stable. On the other hand, the equilibrium is unstable when the difference $(\pi_1 - \pi_2)$ on the side of π_1 is negative.

Now we need not form the differential quotients for the direction dn. It suffices to form them for dr or $d\rho$, and to merely determine whether the positive dr or $d\rho$ look toward the side whose index is 1 or that whose index is 2.

By forming these differential quotients from the equation $(3f)$ there results

$$q.\frac{\partial(\pi_1 - \pi_2)}{\partial r} = -\frac{G}{r^2}\left[\frac{1}{\theta_1} - \frac{1}{\theta_2}\right] \quad . \quad . \quad . \quad . \quad (4d).$$

The differential quotient is positive when $\theta_1 > \theta_2$. The partial differentiation with respect to r while ρ remains unchanged, indicates a progress in an ascending direction parallel to the earth's axis; that is to say, in the direction of a line pointing towards the celestial pole.

The equilibrium is stable when the strata containing the greater quantity of heat lie at higher elevations on the side towards the celestial poles.

We now form the other differential quotients

$$q.\frac{\partial}{\partial \rho}(\pi_1 - \pi_2) = \frac{1}{\rho^3}\left(\frac{\Omega_1^2}{\theta_1} - \frac{\Omega_2^2}{\theta_2}\right) - \omega_0^2\rho\left(\frac{1}{\theta_1} - \frac{1}{\theta_2}\right) \quad . \quad . \quad . \quad . \quad (4e).$$

$$= \rho\left[\frac{\omega_1^2 - \omega_0^2}{\theta_1} - \frac{\omega_2^2 - \omega_0^2}{\theta_2}\right] \quad . \quad . \quad . \quad . \quad . \quad (4f).$$

If in these equations θ_1 indicates the greater quantity of heat, then the equilibrium is stable when everywhere along the boundary surface we have

$$\rho\frac{\omega_1^2 - \omega_0^2}{\theta_1} > \rho.\frac{\omega_2^2 - \omega_0^2}{\theta_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4g).$$

Both these values are positive where the west wind prevails; both negative where the east wind prevails.

The equation (4e) can also be written

$$q.\frac{\partial}{\partial\rho}(\pi_1-\pi_2)=\frac{1}{\rho^3}.\frac{\theta_1-\theta_2}{\theta_1\ \theta_2}\left[\omega_0^2\rho^4+\frac{\Omega_1^2\theta_2-\Omega_2^2\theta_1}{\theta_1-\theta_2}\right].$$

In order that this may be positive at all latitudes, the following inequality must be satisfied

$$\Omega_1^2\theta_2>\Omega_2^2\theta_1$$

or,

$$\frac{\Omega_1^2}{\theta_1}>\frac{\Omega_2^2}{\theta_2}.$$

Ordinarily this will be the case, since in general θ increases simultaneously with ρ and from a definite value at the pole to a finite value at the equator. Similarly Ω_1^2 also increases with ρ, and from zero at the pole to $\omega_0^2\ \rho^2$ at the equator, so that $\frac{\Omega^2}{\theta}$ also increases from zero at the pole to a definite positive value at the equator. We will therefore designate this case as the normal case. Exceptions can only occur under special conditions within limited zones.

In the normal case as we progress along the same level, the warmer π_1 lies on the side of the greater ρ; that is to say, on the side towards the equator, and equally on the side of the greater r if we progress toward the celestial pole; that is to say, ρ and r increase toward the same side of the boundary surface, and this surface must be so inclined that the tangent of its meridian section intersects the celestial sphere between the pole and the point of the horizon lying immediately beneath it. Near the equator, where the pole rises very little above the horizon, this gives an inclination to the boundary surface such that it makes a very small acute angle with the horizon.

In accordance with this, equation (4a) shows us that under those circumstances $\frac{dr}{d\rho}$ is negative along the boundary surface itself.

Therefore the normal inclination of the bounding surface is in an ascending direction toward a point situated beneath the celestial pole.

If on the other hand exceptional localities should exist at which

$$\omega_0^2\ \rho^4+\frac{\Omega_1^2\ \theta_2-\Omega_2^2\theta_1}{\theta_1-\theta_2}<0 \quad . \ . \ . \ . \ . \ . \ . \quad (4h)$$

then in such cases according to equation (4a) $\frac{dr}{d\rho}$ will be positive; that is to say, the boundary line will ascend to higher levels as we depart from the earth's axis.

Since moreover equation (4d) shows that as we proceed in the direction of a line drawn to the pole, the warmer air must lie higher, there-

fore this line can not twice intersect the boundary surface between two layers, and consequently in the abnormal case this line must necessarily lie between the boundary surface and the horizontal plane located at the pole. Therefore the tangents to the meridional section of the boundary surfaces must intersect the greater arcs on the celestial sphere somewhere between the pole and the equatorial side of the horizon.

The smaller the difference of temperature is relative to the difference of the velocities of rotation so much the nearer does the tangent just referred to approach the pole.

Moreover at different points of the bounding line of the same two layers there can occur both normal and abnormal inclinations. For since in the expression (see equation $4h$) on whose positive or negative value such occurrence depends, the Ω and θ throughout the extent of each layer are constant, therefore for the same altitude above the earth this value can have a positive value near the equator but a negative value near the poles. Between these the boundary curve must attain a maximum altitude where the quantity under consideration passes from positive through zero to negative. At this place also, according to equation ($4a$), we have $\frac{dr}{d\rho} = 0$, therefore r is a limiting value and is here a maximum.

Location of the strata in the case when the velocity of rotation varies continuously with the quantity of heat contained.—The considerations hitherto set forth can also be extended to the case where Ω is a continuous function of θ, and the value of θ in the atmospheric strata is continually changing. The individual strata are in this case to be considered as indefinitely thin. Equation ($4a$) now becomes.

$$G\frac{dr}{r^2} = \frac{d\rho}{\rho_3}\left[\frac{d\left[\frac{\Omega^2}{\theta}\right]}{d\left(\frac{1}{\theta}\right)} - \omega_0^2\,\rho^4\right]$$

$$= \frac{d\rho}{\rho_3}\left[\Omega^2 - \theta\frac{d\Omega^2}{d\theta} - \omega_0^0\,\rho^4\right]$$

In order that the equilibrium may be stable the quantity of contained heat (see equation $4h$) must increase in the direction towards the celestial pole. But the layers of similar air are less inclined than the inclination of the polar axis at all places where the quantity

$$\Omega^2 - \theta.\ \frac{d\Omega^2}{d\theta} < \omega_0^2\,\rho^4\,;$$

but on the other hand their inclination is steeper where the left-hand side of this inequality is greater than the right.

IV. GRADUAL VARIATIONS OF THE EQUILIBRIUM BY FRICTION AND HEATING.

It is well known how very differently the propagation of changes of temperature in the air goes on according as heat is added or withdrawn below or above.

If the lower side of a stratum of air is warmed, as occurs at the surface of the earth, by action of the solar rays, then the heated stratum of air seeks to rise. This is effected very soon all over the surface in small tremulous and flickering streams such as we see over any plane surface strongly heated by the sun, but soon these smaller streams collect into larger ones when the locality affords opportunity, especially on the side of a hill. The propagation of heat goes on relatively rapidly through the whole thickness of the atmospheric layer, and when it has a uniform quantity of heat throughout its whole depth and is therefore in adiabatic equilibrium then also the newly added air seeks *de nova* to distribute itself through the entire depth.

The same process occurs with like rapidity when the upper side of a stratum of air is cooled.

On the other hand, when the upper side is warmed and the lower side cooled such convective movements do not occur. The conduction of heat operates very slowly in large dimensions, as I have already explained above. Radiation can only make itself felt to any considerable extent for those classes of rays that are strongly absorbed. On the other hand, experiments on the radiation from ice and observations of nocturnal frosts show that most rays of even such low temperatures can pass through thick layers of clear atmosphere without material absorption.

Therefore a cold stratum of air can lie for a long time on the earth, or equally a warm stratum remain at an altitude, without changing its temperature otherwise than very slowly.

Similar differences exist also in the case of the change of velocity by friction. For the normal inclination of an atmospheric stratum its upper end is nearer to the earth's axis than its lower end. If the stratum appears at the earth's surface as a west wind, then the moment of rotation of the lowest layer is delayed [by resistance of the earth's surface], its centrifugal force is diminished, and on the polar side of the stratum this lowest portion will slide outwards, approaching the axis in order to find its position of stable equilibrium at the upper end of the stratum. This movement will ordinarily take place in small tremulous streams similar to the ascent of warm air and must diminish the moment of rotation of the whole layer rather uniformly, but in the upper portions a little later than in the lower. Since, however, this latter effect distributes itself throughout the whole mass of air, it will become much less apparent on the lower side of the stratum than if it were confined to the lower stratum.

For the east wind matters are reversed. Its moment of rotation is increased by the friction on the earth's surface. The accelerated mass of air [the ground layer] already finds itself in that position of equilibrium which it has to occupy within its stratum, and can only press forward equatorially along the earth's surface into the stratum lying in front of it. If it is also simultaneously heated then the resulting ascent takes place more slowly than would occur in a stratum of air that is at rest at the bottom.

Hence it is to be concluded that in the east wind, the change due to friction is confined to the lower layer of air, and furthermore that it is relatively more effective here than in the case of a west wind of equal velocity. In general, the retarded layer of air will press forward toward the equator, in the Northern Hemisphere as northeast wind. In this motion it will continue to appear as an easterly wind since it is continually arriving at more rapidly rotating zones on the earth. The air of the stratum lying above the retarded layer will, where the region is free from obstruction, as at the outer border of the trade wind zone, fall behind and will appear as an east wind, retaining its moment of rotation unchanged and gradually pushing toward the equator will itself in its turn experience the above described influence of friction. I would here further remark that the water so abundantly evaporated in the tropical zone also enters into the trade wind, but with the greater velocity of rotation of the revolving earth and must diminish the retardation of the latter with respect to the earth.

The lower layers of the trade wind can press in under the equatorial calm zone itself only when any difference between their velocity of rotation and that of the earth's surface is entirely destroyed. They then blend with the zone of calms and increase its mass so that the latter broadens with its inclined boundary surface always higher above the layer of diminishing east wind beneath it.

Thus it is brought about that whereas below [nearer the earth's surface] mostly continuous changes are taking place in the temperature and the moment of rotation of the strata, on the other hand above, the boundaries of the broadening zones of calms (that have the great moment of rotation that pertains to the equatorial air and which at 10° latitude must appear as a strong west wind, and at 20° latitude as a westerly storm), occur in direct contact with the underlying stratum that has less velocity of rotation and lower temperature. Evidently the upper side of this latter [lower] stratum can scarcely be changed as to the quantity of its contained heat and of its moment of rotation, while after the loss of its lower layer it is being pushed sidewise and towards the equator.

As I have already shown in my communication to this Academy, April 23, 1868, on "Discontinuous Fluid Motions,"* such discontinuous motions can continue for a while, but the equilibrium at their boundary

* [See No. III of this collection of Translations.]

surfaces is unstable, and sooner or later they break up into whirls that lead to general mixture of the two strata. This statement is confirmed by the experiments with sensitive flames and by those in which by means of a cylindrical current of air blown from a tube we make a section in a flame and thus make visible the boundary of the moving and the quiet mass. If, as in our case, the lower stratum is the heavier it can be shown that the perturbations must at first be similar to the waves of water that are excited by the wind. The process is made evident by the striated cirrus clouds that are visible when fog is precipitated at the boundary of the two strata. The great billows of water that are raised by the wind show the same process which is different in degree only, by reason of the greater difference of the specific gravities. The severer storms even turn the aqueous billows to breakers, that is to say, they form caps of froth and throw drops of water from the upper crest high into the air. Up to a certain limit, this process can be mathematically deduced and analyzed, on which subject I propose a later communication. For slighter differences of specific gravity the result of this process must be a mixture of the two strata with a formation of whirls and under some circumstances with heavy rainfall. An observation of one such process under very favorable circumstances I once made accidentally upon the Rigi and have described.*

The mixed strata acquire a temperature and moment of inertia whose values lie between those of the component parts of the mixture, and its position of equilibrium will therefore be found nearer the equator than the position previously occupied by the colder stratum that enters into it. The mixed stratum will descend toward the equator and push back the strata lying on the polar side. Into the empty space thus created above, the strata from which this descending portion has been drawn stretch upwards, and thus their cross section must be diminished. Wherever the lower layers are pushed apart by descending masses of air, as is well known, there arise anti-cyclones; wherever cavities or gaps arise by reason of ascending masses of air, there arise cyclones. Anti-cyclones and the corresponding barometric maxima are shown, with very great regularity, by the meteorological charts † along the very irregularly varying limits of the northeast trade in the Atlantic Ocean—in the winter, under latitude 30°; in summer, under 40° latitude. On account of the inclined position of the strata, the rain that frequently forms by reason of the mixture of air (Dove's Subtropical Rain) falls somewhat farther northward because the water must fall down almost vertically. ‡

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* See Proceedings of the Physical Society in Berlin, October 22, 1886.

† Daily Synoptic Weather Charts. Published by the Danish Meteorological Institute and the German Seewarte, Copenhagen and Hamburg.

‡ [The results stated in the above paragraph were subsequently greatly modified by Helmholtz. See Section V of his second memoir, or page 98 of these Translations.—*C. A.*]

Therefore the zone of cyclones begins there, but these become more frequent farther northward. We can certainly assume that the process of mixture is not perfected immediately at the exact border of the trade-wind zone, but that a part of the rapidly-rotating warm upper stratum remains unchanged or half mixed, which will presently bring about new mixtures farther on toward the pole.

In general, in this zone of mixture, even below at the earth's surface, the west wind must retain the upper hand because the increase of the total moment of rotation which the mass of air, through friction, experiences in the east wind of the trade zone must finally rise to such a pitch that somewhere the west wind again touches the earth and experiences sufficient friction to entirely give back the increase that it had. The masses of air resting in the equilibrium of stratification can certainly have no long-continued motion of rotation that differs essentially from that of the earth beneath them. When therefore they are mixed with the stronger west wind of the air from above, they receive a movement toward the east. Moreover the falling rain that in great part comes from the upper west winds, must transmit its motion to the lower strata through which the rain falls. Eventually all zones that are pressed polewards by intermixed masses moving equatorially and descending from them will become west winds.

Another permanent source of winds is the cooling of the earth at the poles. The cold layers endeavor to flow outwards from each other at the earth's surface and form east wind (or anti-cyclones). Above these the warmer upper strata must fill the vacancy and continue as west winds (or cyclones). Thus an equilibrium would come about, as is shown in Sect. II, if it were not that the lower cold stratum acquires, through friction, a more rapid movement of rotation, and is therefore competent for further advance. In doing this, according to the above given views this lower stratum must remain on the earth's surface. That in fact it does so is shown by frequent experiences during our northeast winter winds whose low temperatures frequently enough do not extend up to even the summit of the North German Mountains. Moreover on the front border of these east winds advancing into the warmer zone, the same circumstances are effective in order to bring about a discontinuity between the movement of the upper and lower currents, as in the advancing trade-winds, and there is therefore here a new cause for the formation of vortex motions.

The advance of the polar east wind, although recognizable in its principal features, proceeds relatively very irregularly since the cold pole does not agree with the pole of rotation of the earth, and also because low mountain ranges have a large influence. In addition to this comes the consideration that in the cold zone fog causes only a moderate cooling of the thicker stratum of air, but clear air brings about a very intense cooling of the lower layer. By such irregularities, it is brought about that the anti-cyclonic movement of the lower stratum

and the great and gradually increasing cyclone of the upper stratum (that should otherwise be expected at the pole) break up into a large number of irregular, wandering cyclones and anti-cyclones, with a preponderance of the former.

From these considerations, I draw the conclusion that the principal obstacle to the circulation of our atmosphere, which prevents the development of far more violent winds than are actually experienced, is to be found not so much in the friction on the earth's surface as in the mixing of differently moving strata of air by means of whirls that originate in the unrolling of surfaces of discontinuity. In the interior of such whirls the strata of air originally separate are wound in continually more numerous, and therefore also thinner layers spirally about each other, and therefore by means of the enormously extended surfaces of contact there thus becomes possible a more rapid interchange of temperature and equalization of their movement by friction.

The present memoir is intended only to show how by means of continually effective forces, there arises in the atmosphere the formation of surfaces of discontinuity. I propose, at a future time, to present further analytical investigations as to the phenomena of such disturbances of continuity.

VI.

ON ATMOSPHERIC MOTIONS.*

(*SECOND PAPER.*)

By Prof. H. von Helmholtz.

ON THE THEORY OF WINDS AND WAVES.

In my previous communication made to the Academy on the 31st of May, 1888, I endeavored to prove that conditions must regularly recur in the atmosphere where strata of different density lie contiguous one above another. The reason for the greater density of the lower stratum is conditioned by the fact that the latter has either a smaller amount of heat or a smaller velocity of rotation, if in fact both conditions do not work together. As soon as a lighter fluid lies above a denser one with well-defined boundary, then evidently the conditions exist at this boundary for the origin and regular propagation of waves, such as we are familiar with on the surface of water. This case of waves as ordinarily observed on the boundary surfaces between water and air is only to be distinguished from the system of waves that may exist between different strata of air, in that in the former the difference of density of the two fluids is much greater than in the latter case. It appeared to me of interest to investigate what other differences result from this in the phenomena of air waves and water waves.

It appears to me not doubtful that such systems of waves occur with remarkable frequency at the bounding surfaces of strata of air of different densities, even although in most cases they remain invisible to us. Evidently we see them only when the lower stratum is so nearly saturated with aqueous vapor that the summit of the wave, within which the pressure is less, begins to form a haze. Then there appear streaky, parallel trains of clouds of very different breadths, occasionally stretching over the broad surface of the sky in regular patterns. Moreover it seems to me probable that this which we thus observe under special conditions that have rather the character of exceptional cases, is present in innumerable other cases when we do not see it.

* From the *Sitzungs-berichte* of the Royal Prussian Academy of Sciences at Berlin, July 25, 1889, pp. 761–780.

The calculations performed by me show further that for the observed velocities of the wind there may be formed in the atmosphere not only small waves, but also those whose wave-lengths are many kilometres which, when they approach the earth's surface to within an altitude of one or several kilometres, set the lower strata of air into violent motion and must bring about the so-called gusty weather. The peculiarity of of such weather (as I look at it) consists in this, that gusts of wind often accompanied by rain are repeated at the same place, many times a day, at nearly equal intervals and nearly uniform order of succession.*

I think it may be assumed that this formation of waves in the atmosphere most frequently gives occasion to the mixture of atmospheric strata and, under favorable circumstances, when the ascending masses form mist, give opportunity for disturbances of an equilibrium that had already become nearly unstable. Under conditions, such as those where we see water waves breaking and forming white caps, thorough mixtures must form between the strata of air.

In the beginning of my previous paper I have explained how insufficient are the known intensities of the internal friction and the thermal conductivity of gases in order to explain the equilibration of motions and temperatures in the atmosphere. Since now the mechanical theory of heat has taught us to consider friction in gases as the mixture of strata having different movements, but the conduction of heat as the mixture of strata having different temperatures, it is therefore intelligible that a more thorough mixture of strata in the atmosphere must bring about, to a still higher degree, the effects of friction and conduction,† but certainly not in a quiet, steady progress, but proceeding irregularly as is indeed the special character of meteorological processes.

Therefore I have considered it important to develop the theory of waves at the common boundary surface of two fluids. Hitherto in studies on waves of water, so far as known to me, the influence of the air and its motion with the water has always been neglected, but this may not be done in the present work. The problem becomes thereby much more complicated and difficult; and since even the simpler problem that takes no account of the influence of the wind has at the hands of many excellent mathematicians received only incomplete and approximate solutions, under assumptions chosen to simplify the problem, therefore I pray to be excused in that I also have at first treated the simplest case of the problem, namely, the movement of rectilinear waves which propagate themselves with unchanged forms

* This assumption of the formation of billows in the atmosphere that I recently briefly expressed in my first contribution has since then also been propounded by Jean Luvini (*La Lumière Électrique.* T. xxx, pp. 368, 617, 620).

† Perhaps this would correspond to the assumptions that form the basis of the theory submitted to this (Berlin) Academy by Oberbeck, March 15, 1888. [See Nos. XII and XIII of this collection.—*C. A.*]

and with uniform velocity on the plane boundary surface between indefinitely extended layers of two fluids of different densities and having different progressive movements. I shall call this kind of billows stationary billows, since they represent a stationary motion of two fluids when they are referred to a system of coördinates which itself advances with the waves. Since in the relative motion of the different parts of a closed material system nothing is changed when the whole receives a uniform rectilinear velocity toward any direction, therefore this rearrangement of our problem is allowable.

Moreover I propose to-day to give only the results of my mathematical investigations. The complete presentation of these I reserve for publication in another manner.

Before I advance to the theory of atmospheric billows, I will however introduce a supplement to the considerations given in my communication of May, 1888, by which the region in which we have to look for the conditions that give rise to atmospheric billows is better defined.

V. THE ASCENT OF MIXED STRATA.

In Section III of my previous communication I have shown what would be the law of equilibrium, in case such a condition should be attained, between atmospheric rings of different temperatures and different speeds of rotation, which however are all assumed as being composed of mixtures that are similar to each other. I now return to equation $4a$ (page 85). Let the location of a point in the atmosphere be given by the quantities

ρ, the distance from the earth's axis.

r, the distance from the center of the earth.

Let ω_0 be the angular velocity of the solid earth; and Ω_1 and Ω_2 be the constant moments of rotation of the unit of mass of one or the other layer of air:

Let θ_1 and θ_2 be the quantities that I have called the contained caloric of the unit of mass of air, and that certainly may be better designated by the term *potential temperatures*, so well chosen by Bezold, namely, those temperatures which the respective masses of air would assume when brought adiabatically to the normal pressure.

Let G = constant of gravitation. In accordance with equation $(4a)$ we now have at the boundary surfaces the relation

$$\frac{G}{r^2}dr = \frac{d\rho}{\rho^3}\left[\frac{\Omega_1^2\ \theta_2 - \Omega_2^2\ \theta_1}{\theta_2 - \theta_1} - \omega_0^2\ \rho^4\right] \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The ratio $\frac{d\rho}{dr}$ indicates also the ratio of the sines of the two angles which the tangent to the curve in the meridional plane makes on the one hand with the earth's axis, and on the other hand with the horizon. When, as is ordinarily the case, the warmer layer has also the greater

moment of rotation, the ratio $\frac{d\rho}{dr}$ is then negative, and the tangent to the boundary surface cuts the celestial vault below the pole. The colder, more slowly rotating mass, which we will designate by the subscript (2), lies in the acute angle between the boundary surface and that part of the terrestrial surface which is on the polar side of the given point.

When now at the boundary surface of the two strata, a mixture takes place of the component masses m_1 and m_2, then will the moment of rotation (Ω) of the mixed masses be given by the equation

$$(m_1+m_2)\Omega=m_1\Omega_1+m_2\Omega_2,$$

since the sum of the moments of rotation does not vary when no exterior rotatory forces are at work. Equally will the potential temperature θ of the mixture be given by

$$(m_1+m_2)\ \theta=m_1\ \theta_1+m_2\ \theta_2.$$

If now in equation (1), we at first substitute the mixture in place of the cooler mass (2), in order to find the direction of the boundary line between the mass (1) and the mixture, and indicate by $d\rho_1$ and dr_1 the corresponding values of $d\rho$ and dr, then our equation (1), after an easy transformation, gives

$$\rho^3\frac{G}{r^2}\left[\frac{dr_1}{d\rho_1}-\frac{dr}{d\rho}\right]=\frac{m_1\ \theta_1}{m_1+m_2}\ \frac{(\Omega_1-\Omega_2)^2}{\theta_2-\theta_1}\quad . \quad . \quad . \quad . \quad (1a)$$

Since in stable equilibrium $\theta_2<\theta_1$, therefore this equation shows that

$$\frac{dr_1}{d\rho_1}<\frac{dr}{d\rho}\ \text{or}\ \frac{d\rho_1}{dr_1}>\frac{d\rho}{dr},$$

that is to say, that the boundary surfaces between mass (1) and the mixture must ascend more steeply with reference to the horizon than the boundary surface between (1) and (2).

Similarly it follows that the ratio $\frac{dr_2}{d\rho_2}$ between the cooler mass (2) and the mixture will be given by the equation—

$$\rho^3\frac{G}{r^2}\left[\frac{dr_2}{d\rho_2}-\frac{dr}{d\rho}\right]=\frac{m_2\theta_2}{m_1+m_2}\ \frac{(\Omega_1-\Omega_2)^2}{\theta_1-\theta_2}.$$

Therefore $\frac{dr_2}{d\rho_2}>\frac{dr}{d\rho}$; that is to say, the boundary surface between the cooler mass (2) and the mixture must make a more acute angle with the horizon towards the pole than does the boundary surface between the mixture and the warmer mass (1).

It is to be noted that the ratios $\frac{d\rho}{dr}$ are positive when the tangent to the boundary line is more inclined than the line to the pole—in the other cases they are negative—and furthermore that the increase of a negative quantity means the diminution of its absolute value.

But the required directions for the two boundary lines of the mixture can only exist when this mixture passes upwards between the two masses (1) and (2). Only thus can there be a condition of equilibrium.

Hence results the important consequence that all newly formed mixtures of strata that were in equilibrium with each other must rise upwards between the two layers originally present, a process that of course goes on more energetically when precipitations are formed in the ascending masses.

While the mixed strata are ascending, those parts of the strata on the north and south that have hitherto rested quietly approach each other until they even come in contact, by which motion the difference of their velocities must necessarily increase since the strata lying on the equatorial side acquire greater moment of rotation with smaller radius, while those on the polar side acquire feebler rotation with a larger radius. If this occurs uniformly along an entire parallel of latitude we should again obtain a new surface of separation for strata of different rates of rotation whose equatorial side would show stronger west winds than the polar side, which latter might occasionally show east winds. On account of the numerous local disturbances of the great atmospheric currents there will, as a rule, be formed no continuous line of separation, but this will be broken into separate pieces which must appear as cyclones.

But as soon as the total mixed masses have found their equilibrium the surfaces of separation will again begin to form below, and new wave formations will initiate a repetition of the same processes.*

From these considerations it follows that the locality for the formation of billows between the strata of air is to be sought especially in the lower parts of the atmosphere, while in the upper parts an almost continuous variation through the different values of rotation and temperature is to be expected. The boundary surfaces of different strata of air, along which the waves travel, have one edge at the earth's surface and there the strata becomes superficial. Experience also teaches, as does the theory, that water-waves that run against a shallow shore break upon it, and even waves which originally run parallel to the shore propagate themselves more slowly in shallow water. Therefore waves that are originally rectilinear and run parallel to the banks will

* In the last section of my previous paper [see *ante* p. 91] I located the origin of the discontinuity principally in the upper strata of the atmosphere. But in that paper the point of departure was different from the present. In that the question considered was: If at any time the atmosphere has attained an initial stage of continuous steady motion without surfaces of separation, where will such a surface first form? To this the answer is: At the upper boundary of the tropical belt of calms.

At present the question is, Where in consequence of processes of mixture will the surfaces of separation necessarily be renewed? But I must take back the proposition on page 91 that treats of the descent of mixed strata, now that I have found the law expressed in this paragraph.

in consequence of the delay become curved, whereby the convexity of their arcs is turned toward the shore; in consequence of this they run upon the shore and break to pieces there.

In the next paragraphs I will show in what respects the movements and forms of water-waves must be changed in order to be applicable to the air. These relations are indeed not to be rigidly transferred from water-waves that break upon the shore to the air, and even the simpler theory hitherto developed, which neglects the influence of the air, gives no complete explanation on this point.

But the conditions are not very different from those cases in which we can make a strict application, and I therefore believe there is no reason to doubt that waves of air which in the ideal atmospheric circulation symmetrical to the axis could only progress in a west east direction, must, when once they are initiated in the real atmosphere, turn down toward the earth's surface and break up by running along this in a northwesterly direction (in the northern hemisphere).

Another process that can cause the foaming of the waves at their summits is the general increase in velocity of the wind. My analysis also demonstrates this: it shows that waves of given wave-length can only co-exist with winds of definite strength. An increase in the differential velocities within the atmosphere indeed often happens, but one can not yet give the conditions generally effective for such a process.

I will here also mention another point that may give rise to considerations against my explanation. Water-waves forced up to a great height always have narrow, strongly curved ridges and broad, flat, curved troughs. Analysis shows that this feature is independent of the nature of the medium. Atmospheric waves have, on the other hand, rounded heads when they become visible to us as bands of cirri. But we must remember that according to the proposition first formulated by Reye, air that has formed cloud or mist is lighter than it was before. Therefore what we see as mist rises up and increases the size of the summit of the wave more than would be the case in transparent air.

VI. CONSEQUENCES DEDUCED FROM THE PRINCIPLE OF MECHANICAL SIMILARITY.

If we confine ourselves to the search for such rectilinear waves as advance with uniform velocity without change of form, we may, as before remarked, represent such a movement as a stationary one, by attributing to both the media a uniform rectilinear velocity equal and opposite to that of the wave. It is well known no change is thereby introduced into the relative motions of the different parts of the masses. In this way the bounding surface of the two media appears as a surface fixed in space; above it the upper medium flows in one direction; below it the other medium in the opposite direction. At a great distance from the bounding surface both movements become rectilinear

currents of uniform velocity, but in the neighborhood of the wavy boundary surface the motion must follow its direction.

Designate by u and v the components of the velocities of the fluid particles at the point corresponding to the rectangular coördinates x and y; these velocities are by assumption, independent of the time, and (for an incompressible fluid whose current is free from vortices) can be presented in the form

$$u = -\frac{\partial\psi}{\partial y}$$

$$v = \frac{\partial\psi}{\partial x}.$$

where ψ is such a function of the coördinate as satisfies the differential equation

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The equations

$$\psi=\text{const.}$$

are in this case, as is well known, the stream-lines of the fluid. The boundary line of both fluids must be such a stream-line, and we will give it for both sides the value

$$\overline{\psi}_1=0 \text{ and } \overline{\psi}_2=0.$$

The above overscored letters will, in what follows, always indicate values on the boundary surface.

The first boundary condition that we have to satisfy is therefore that, when we express ψ_1 and ψ_2 as functions of x and y. then the two equations

$$\psi_1=0=\psi_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

shall admit of an accordant solution.

The second boundary condition is that the pressure at the bounding surface shall be the same on both sides, or

$$\overline{p}_1=\overline{p}_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2b)$$

Now, under the adopted assumptions and when s is the density of the fluid and C is a constant, we have

$$p=C-s\,g\,x-\tfrac{1}{2}s\left[\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2\right].$$

Therefore the equation ($2b$) can be written:

$$\text{Const.}=(s_1-s_2)\,g\,\overline{x}+\tfrac{1}{2}s_1\left(\frac{\partial\psi_1}{\partial N_1}\right)^2-\tfrac{1}{2}s_2\left(\frac{\partial\psi_2}{\partial N_2}\right)^2 \quad . \quad . \quad . \quad . \quad (3)$$

The equations (2) and ($2a$) remain true when we increase either the values of the two coördinates x and y or those of ψ_1 or ψ_2 in any given ratio. Since the densities s_1 and s_2 do not occur in these two equations, therefore also these can change to any amount. But equation (3) requires that the quantities

$$\frac{s_1}{s_2-s_1}\left(\frac{\partial\psi_1}{\partial N_1}\right)^2\frac{1}{x} \text{ and } \frac{s_2}{s_2-s_1}\left(\frac{\partial\psi_2}{\partial N_2}\right)^2\frac{1}{x}$$

shall remain unchanged. When therefore s_1 and s_2 vary and we put their ratio

$$\frac{s_1}{s_2}=\sigma$$

and when further the coördinates increase by the factor n, but ψ_1 by the factor a_1 and ψ_2 by the factor a_2, then the quantities

$$\frac{\sigma}{1-\sigma}\cdot\frac{a_1^2}{n^3} \text{ and } \frac{1}{1-\sigma}\cdot\frac{a_2^2}{n^3}$$

must both remain unchanged.

Or when we, in the expressions for these quantities, put

$$b_1=\frac{a_1}{n} \text{ and } b_2=\frac{a_2}{n}$$

as the ratios by which the velocities are altered, then the above proposition becomes equivalent to saying that the geometrically similar waveforms can occur when

$$\frac{\sigma}{1-\sigma}\cdot\frac{b_1^2}{n} \text{ and } \frac{1}{1-\sigma}\cdot\frac{b_2^2}{n}$$

remain unchanged.

(1) *If the ratios of the densities are not changed then in geometrically similar waves, the linear dimensions increase as the squares of the velocities of the two media; the velocities therefore will increase in equal ratios.*

Therefore for a doubled velocity of the wind we shall have waves of four times the linear dimensions.

This proposition is not limited to stationary movements, but is quite general.* The following propositions however will hold good only for stationary waves.

(2) *When the ratio of the density σ is varied, the quantities*

$$\sigma\frac{b_1^2}{b_2^2}=\frac{s_1}{s_2}\frac{b_1^2}{b_2^2}=\text{const.}$$

*See my paper "On a Theorem relative to geometrically similar movements of Fluid Bodies," in the *Monats b. der Akad. Berlin*, 1873, pages 501 to 514; [or see No. IV of this collection of Translations.]

must remain constant; that is to say, *the ratio of the living forces of the corresponding units of volume must remain unchanged.* As corresponding units of volume, those must be used that hold good in the region of rectilinear flow far from that of the wave surface; but also for such units of volume as have centers that are corresponding images for each other the same proposition holds good.

(3) If for a varied density the geometrically similar waves are to have the same wave-length, namely, $n=1$, then

$$b_1 \text{ must increase as } \sqrt{\frac{1}{\sigma}-1}=\sqrt{\frac{s_2-s_1}{s_1}}$$
$$b_2 \text{ must increase as } \sqrt{1-\sigma}=\sqrt{\frac{s_2-s_1}{s_2}}.$$

For air and water at a temperature of 0° C. we have the ratio

$$\sigma=\frac{1}{773.4}$$

For two strata of air whose temperatures are 0° and 10° the ratio becomes

$$\sigma=\frac{273}{283}$$

If both boundary surfaces are to show congruent waves and therefore also equal wave-lengths, and if I designate by β_1 and β_2 the values of the quantities b_1 and b_2 in this last case, then we have

$$b_1=145.21\beta_1$$
$$b_2=5.316\beta_2$$

therefore both the velocities, especially that of the wind relative to the waves of water, must be considerably diminished for the case of ærial billows.

The value of the quantity

$$\mathfrak{p}=\frac{s_2\,.\,b_2^2}{s_1\,.\,b_1^2}$$

which is invariable for any change in the material for a given form of wave whose store of energy is equal to that of the rectilinear flow along a plane boundary surface is given at least approximately according to my computations, as

$$\mathfrak{p}=0.43103.$$

If by a wind-force w we understand the difference of the movement of the two media

$$w=b_1+b_2$$

then will for air and water

$$\frac{b_2}{w}=0.069469$$

and if $w=10\ \dfrac{\text{metres}}{\text{second}}$

$$\lambda=0.208965 \text{ metre};$$

on the other hand for the two strata of air

$$\frac{\beta_2}{\beta_1+\beta_2}=0.67135$$

and for $w=10\ \dfrac{\text{metres}}{\text{second}}$ we have

$$\lambda=549^{\text{m}}.65$$

Hence it results that when we would obtain for this form of atmospheric wave the same wind velocity as for geometrically similar water-waves we must increase the wave-length of the air wave in the ratio of 1 to 2630.3.

This ratio becomes somewhat smaller when we execute the computation for the lowest waves for which

$$\mathfrak{p}=0.15692$$

This gives for air and water

$$\frac{b_2}{w}=0.090776$$

and for a wind velocity of 10 metres per second,

$$\lambda=0.^{\text{m}}83222$$

The necessary magnification of the wave-length for equal strength of wind would be 1:2039.6 which gives a wave-length of more than 900 metres for a wind of 10 metres per second.

Since the moderate winds that occur on the surface of the earth, often cause water-waves of a metre in length, therefore the same winds acting upon strata of air of 10° difference in temperature, maintain waves of from 2 to 5 kilometres in length. Larger ocean-waves from 5 to 10*m* long would correspond to atmospheric-waves of from 15 to 30 kilometres, such as would cover the whole sky of the observer and would have the ground at a depth below them less than that of one wave-length, therefore comparable with the waves in shallow water, such as set the water in motion to its very bottom.

The principle of mechanical similarity, on which the propositions of this paragraph are founded, holds good for all waves that progress with an unchanged form and constant velocity of progress. Therefore these propositions can be applied to waves in shallow water, of uniform

depth, provided that the depth of the lower stratum in the image varies in the same ratio as the remaining linear dimensions of the waves.

The velocity of propagation of such waves in shallow water depends on the depth of the water. For water waves of slight height and without wind it can be computed by well-known formulæ. When we indicate the depth of the water by h and put $n=\frac{2\pi}{\lambda}$, then is

$$b^2=\frac{g}{n}\cdot\frac{e^{nh}-e^{-nh}}{e^{nh}+e^{-nh}}$$

which for $h=\infty$ becomes

$$b^2=\frac{g}{n}=\frac{g\lambda}{2\pi}$$

and for small values of h becomes

$$b^2=gh$$

When however the depth of the water is not small relatively to the wave length, then the retardation is unimportant, thus for

$\frac{h}{\lambda}=\frac{1}{2}$ the speed of propagation diminishes as 1:0.95768

$=\frac{1}{4}$ the speed of propagation diminishes as 1:0.80978

$=\frac{1}{10}$ the speed of propagation diminishes as 1:0.39427

When it is calm at the earth's surface the wind beneath the trough of the aerial billow is opposed to the direction of propagation, but under the summit of the billow it has the same direction as that. Since the amplitudes at the earth's surface are diminished in the proportion $e^{-nh}:1$ with respect to the amplitudes at the upper surface, therefore these latter variations can only make themselves felt below when the depth is notably smaller than the wave-length. Variations of barometric pressure are only to be expected when decided changes in the wind are noticed during the transit of the wave.

VII. FUNDAMENTAL FORMULÆ FOR THE COMPUTATION.

I will here give the theory of the calculation only so far as is necessary, so that any investigator familiar with analytical methods can verify my results. I introduce two new variables, η and θ, which are so connected with rectangular coördinates x and y that

$$e^{n(x+yi)}=a[\cos(\theta+\eta i)-\cos\varepsilon] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

wherein n, a, and ε are constants. The boundary line between the two fluids corresponds to a constant positive value of η, namely:

$$\eta=h$$

Hence for this boundary line result the equations

$$\left.\begin{aligned} e^{n\bar{x}} \cos (n\bar{y}) &= a(\cos ih \cos \theta - \cos \varepsilon) \\ e^{n\bar{x}} \sin (n\bar{y}) &= -\frac{a}{i} \sin (ih) \sin \theta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

By the elimination of θ this gives an equation between $\bar{x}$ and $\bar{y}$ as the equation of the boundary line. Beside the constant a which determines the initial point of the x coördinate and the n which determines the wave-length this equation contains two arbitary parameters h and ε that determine the form of the curve.

We take x vertical, increasing upwards, and then for the space occupied by the upper fluid, for which we use the subscript $_1$ put

$$\psi_1 + \varphi_1 i = b_1(\eta - h - i\theta)$$

by which $\psi_1 + \varphi_1 i$ becomes simultaneously a function of $(x+yi)$. When $h=\eta$, then $\psi_1=0$, so the boundary line on the lower side coincides with the stream line. When $\eta=+\infty$ then

$$n(x+yi) = \eta - i\theta = \frac{1}{b_1}[\psi_1 + \varphi_1 i] + h$$

or

$$\psi_1 = nb_1 x,$$
$$\varphi_1 = nb_1 y,$$

so that at great altitudes the motion is a rectilinear flow with the velocity nb_1.

For the lower space where $\eta < h$ and x has generally a negative value, I put

$$\frac{1}{b_2}[\psi_2 + \varphi_2 i] =$$

$$-nx - nyi + \log\left(\frac{a}{2}\right) + h - 2\sum_{\mathfrak{a}=1}^{\mathfrak{a}=\infty}\left[\frac{1}{\mathfrak{a}} \cdot e^{-\mathfrak{a}h} \cdot \frac{\cos(\varepsilon\mathfrak{a}) \, . \, \cos \mathfrak{a}\,(\theta + \eta i)}{\cos(\mathfrak{a}hi)}\right].$$

Hence for $\eta = h$ there results

$$\frac{1}{b_2}\psi_2 = -n\bar{x} + \log\left(\frac{a}{2}\right) + h - 2\sum_{1}^{\infty}\left[\frac{1}{\mathfrak{a}} \cdot e^{-\mathfrak{a}h} \cdot \cos(\varepsilon\mathfrak{a}) \cos \mathfrak{a}\theta\right].$$

When we determine the value of $\bar{x}$ from the equation (1) it is seen that for $\eta=h$ there results $\psi_2=0$, therefore it is seen that the boundary line for the second medium is also a stream-line.

According to equation (1) for $x=-\infty$ we have

$$\cos\theta \, . \, \cos \eta i = \cos\varepsilon$$

$$\sin\theta \, . \, \sin \eta i = 0$$

The values corresponding to these are

$$\sin \eta i = 0$$

$$\cos \theta = \cos \varepsilon$$

In consequence of this the equation above given becomes

$$\frac{1}{b_2} \cdot \psi_2 = -nx + \log\left(\frac{a}{2}\right) + h - 2 \sum_1^\infty \left[\frac{e^{-\mathfrak{a}h} \cdot \cos^2(\varepsilon \mathfrak{a})}{\mathfrak{a} \cdot \cos(\mathfrak{a}hi)}\right]$$

$$(x = -\infty)$$

The first term of the right-hand member is infinite, but all the others finite when h is a positive quantity. Therefore at great depths the value of ψ_2 reduces to

$$\psi_2 = -nb_2 x$$

that is to say that even there also the motion is a rectilinear flow with the velocity $-nb_2$.

The second boundary condition which has respect to the equality of pressure on both sides of the boundary surface can, however, by reason of the assumptions already made, be satisfied only approximately for waves of small altitude. The convergence of the series under consideration in this case depends upon the factor $e^{-\mathfrak{a}h}$. When the quantity h is positive and not too small the series converges relatively rapidly and we obtain for this case sufficient approximation to the true value, in that in the value of the pressure as deduced from equation (3) we equate to zero the terms multiplied by the first to the third power of e^{-h}, or of $\frac{1}{\cos hi}$. The terms that do not contain these factors serve only to determine the value of the constant of integration which forms the left-hand side of the equation. These terms just mentioned are linear functions of cos θ, cos 2θ, cos 3θ, and by equating to zero the coefficients of these three quantities we satisfy equation (3) to terms that contain the fourth or higher power of $\frac{1}{\cos hi}$. But this assumption corresponds only to a single possible form of wave, not to the most general form. It has been chosen as an example on account of the simplicity of computation. The three equations that we obtain in this manner are those given below. For brevity we have put

$$\mathfrak{P} = \frac{s_1 \cdot b_1^2 \cdot \pi}{g \cdot \lambda \cdot (s_2 - s_1)}$$

$$\mathfrak{Q} = \frac{s_2 \cdot b_2^2 \cdot \pi}{g \cdot \lambda (s_2 - s_1)}$$

$$\zeta = \frac{1}{\cos hi}$$

$$z = \zeta \cos \varepsilon$$

The quantity z determines the altitude of the wave, which according to equation ($1a$) is—

$$H = \frac{\lambda}{2\pi} \cdot \log \text{nat} \cdot \left(\frac{1+z}{1-z} \right).$$

The three equations referred to may now be written:

$$\text{I.} \quad z \left\{ \mathfrak{Q} \left[2 - 2z^2 + \tfrac{1}{2}\zeta^2 \right] + \mathfrak{P} \left[2 + \tfrac{3}{2}\zeta^2 \right] - (1 - \zeta^2) \right\} = 0.$$

$$\text{II.} \quad \mathfrak{Q} \left[2z^2 - \zeta^2 \right] - \mathfrak{P} \cdot \zeta^2 - \tfrac{1}{2}z^2 + \tfrac{1}{4}\zeta^2 = 0.$$

$$\text{III.} \quad z \left\{ \mathfrak{Q} \left[2z^2 - \tfrac{3}{2}\zeta^2 \right] + \mathfrak{P} \cdot \frac{\zeta^2}{2} - \tfrac{1}{3}z^2 + \tfrac{1}{4}\zeta^2 \right\} = 0.$$

Of the four quantities that occur herein any three may therefore in general be determined by the fourth. Only one system of values, namely,

$$z = 0 \text{ and } \mathfrak{Q} + \mathfrak{P} = \tfrac{1}{4},$$

leaves ζ undetermined. This solution holds good for the entire lower wave, for which z is to be neglected as compared with ζ.

Since in general one of the four quantities in the equations I to III remains undetermined, therefore for given properties of the medium and for a given strength of the wind, there remains always one variable parameter of the stationary wave; and in fact the further investigation shows that this variable is connected with the quantity of energy that is accumulated in the wave.

The simplest method of computation is to express the remaining quantities as functions of $\cos \varepsilon$.

$$\mathfrak{Q} = \frac{7}{36} \cdot \frac{\cos^2 \varepsilon - \frac{9}{14}}{\cos^2 \varepsilon - \frac{2}{3}}$$

$$\mathfrak{P} = -\tfrac{1}{9} \cos^2 \varepsilon + \tfrac{1}{3} \mathfrak{Q} = -\tfrac{1}{9} \frac{(\cos^2 \varepsilon - \frac{1}{2}) \cdot (\cos^2 \varepsilon - \frac{3}{4})}{\cos^2 \varepsilon - \frac{2}{3}}$$

$$\zeta^2 \left[\mathfrak{Q} (\cos^2 \varepsilon - \tfrac{1}{4}) - \tfrac{3}{4} \mathfrak{P} + \tfrac{1}{2} \right] = \mathfrak{Q} + \mathfrak{P} - \tfrac{1}{2}$$

Since $\mathfrak{Q}$ and $\mathfrak{P}$ must necessarily be positive, it follows from the first of these equations that

$$\cos^2 \varepsilon > \tfrac{2}{3} = 0.666667;$$

or

$$\cos^2 \varepsilon < \tfrac{9}{14} = 0.642857.$$

The equation for $\mathfrak{P}$ would also allow $\cos^2 \varepsilon > \frac{2}{3}$, but also

$$0.5 < \cos^2 \varepsilon < 0.642857.$$

Finally the equation for ζ^2 can be written

$$\zeta^2 = 0.4 \times \frac{(0.68615 - \cos^2 \varepsilon)\ (\cos^2 \varepsilon + 2.18615)}{(\cos^2 \varepsilon - 0.66537)\ (\cos^2 \varepsilon + 1.46537)}.$$

Since ζ^2 must be positive it follows that

$$0.66537 < \cos^2 \varepsilon < 0.68615\,;$$

so that values of $\cos^2 \varepsilon$ that are smaller than 0.643 are thereby excluded. But when we consider that for values of ζ that are larger than 1, the above-given series for the coördinates of the boundary surface are no longer convergent, there results a lower limit that is still higher than the preceding, which corresponds to the value

$$\cos^2 \varepsilon > 0.67264 = -\tfrac{1}{2} + \sqrt{\tfrac{11}{8}}.$$

For this value the altitude of the wave will still be finite, namely:

$$H = \frac{\lambda}{2\pi} \times 2.5112 = \lambda\, 0.39967.$$

But the fact that the value of the coördinates can no longer be developed in converging series, according to the powers of cos $(\alpha\theta)$ and sine $(\alpha\theta)$, shows that a discontinuity or an ambiguity of the coördinates must have come into existence. In fact the equations (1a) also show that for small values of h

$$\text{tang}\,(n\,y) = -\frac{h \sin\theta}{\cos\theta - \cos\varepsilon}$$

$$e^{2nx} = a^2(\cos\theta - \cos\varepsilon)^2.$$

From the first of these it follows that wherever tan $(n\,y)$ has a finite value then $\cos\theta$ must be nearly equal to $\cos\varepsilon$, and only at the points where tan $(n\,y)$ is very small and passes through zero can θ increase and rapidly pass through the interval to the next point, where $\cos\theta$ approaches again the value, $\cos\varepsilon$.

Now for such values of h the diminution of the terms in the series expressing the value of the pressure will not be rapid enough, in order to express the value of the function sufficiently well by using only the first three terms of the series, and the true form of the wave curve for such values of h can only be obtained by further approximations. However, these relations show that waves which rise too high lose the continuity of their surfaces. But sharp ridges can not occur on the surfaces of the waves except when they are at rest relatively to the medium into which they protrude. For when the medium flows around the edge there would occur infinite velocity and infinite pressure at the place in question, which must violently draw up the other liquid, as in fact is occasionally observed in high and foaming waves.

In the case of waves that advance with the same velocity as the wind the summits can in fact have a ridge of 120° before they break into foam.

The above given formulæ show that when cos ε diminishes from its upper to its lower value, then both $\mathfrak{Q}$ and $\mathfrak{P}$ and ζ^2 must continually increase. For waves whose lengths remain constant the increase of $\mathfrak{P}$ and $\mathfrak{Q}$ means an increase of the two velocities b_1 and b_2 as well as their sum, *i. e.*, the wind velocity $w=b_1+b_2$. If the latter remains constant, then the wave length must necessarily diminish with increasing cos ε.

It follows from this, that within certain limits the same wind can excite this form of waves of greater and smaller wave lengths. The longer waves will at the same time have a relatively greater altitude. This relation depends upon the store of energy that is accumulated in the wave.

VIII. THE ENERGY OF THE WAVES.

When we investigate the energy of the waves of water raised by the influence of the wind, and compare it with that which would be appropriate to the two fluids uniformly flowing with the same velocity when the boundary surface is a plane, we find that a large number of the possible forms of stationary wave motion demand a smaller storage of energy than the corresponding current with a plane boundary. Hence the current with a plane boundary surface plays the part of a condition of unstable equilibrium to the above-described wave motion. Besides these, there are other forms of stationary wave motion where the store of energy for both the masses that are in undulating motion is the same, as in the case of currents of equal strength with plane bounding surfaces; and finally, there are those in which the energy of the wave is the greater.

The reason for this is to be found in the following circumstances: In the undulating masses of water two forms of energy occur, namely:

First, potential energy, represented by the water raised from the wave valley to the wave summit. This quantity of work increases with the increasing height of the wave, and must always be positive; it is only absent for perfectly smooth surfaces.

Second, living force is common to the two forms of motion under comparison, and according to the original assumption there is an equal quantity of it in the portions of the fluid masses distant from the boundary surface. The difference of the two modes of motion is not affected by the participation of the more distant strata of fluid, the difference between the two motions depends only on the strata that lie near the boundary surface. The wave surface which we again imagine to ourselves fixed in space affords to the two fluids streaming along it an alternately broad and narrow channel; where the bed is broader the fluid moves more slowly, the upper fluid above the wave valley, the lower fluid under the wave summit. Thereby the living force of the portion flowing through a broadening of the channel will be alternately smaller, while that flowing through a narrowing of the channel will be greater than the living force in the corresponding part of the uniform

stream with the plane bounding surface. But the volumetric extension of the part with diminished living force, that fills the broader channel, is greater than the volume of increased velocity in the narrow channel. Therefore in the sum total the living force of the diminished portion prevails.

Nevertheless only the terms of the fourth degree in ζ which first occurred in the computation by considering the terms with ζ^3 in the values of x and y, give a basis for the computation of the difference of energy. This difference, as computed for one wave-length according to my calculation in the class of waves discussed in Section VII, is as follows:

$$E \frac{1}{2\,\pi g\,(s-s_0)} = \mathfrak{Q}.\,\frac{z^2}{4}\left[5\zeta^2-2z^2\right]+\frac{1}{48}\left[z^4-15z^2\zeta^2-\frac{3}{4}\zeta^4\right]$$

or

$$E\frac{1}{2\pi g(s_2-s_1)}=$$

$$\frac{7\cos^2\varepsilon}{144}.\,\zeta^4\,\frac{[5-2\cos^2\varepsilon]\,[\cos^2\varepsilon-\frac{9}{14}]}{\cos^2\varepsilon-\frac{2}{3}}-\frac{1}{48}\zeta^4[15.0845-\cos^2\varepsilon].[\cos^2\varepsilon+0.0845]$$

In this the $\mathfrak{Q}$ is the only factor that changes rapidly for small changes of $\cos\varepsilon$, a circumstance that very materially lightens the numerical computation. For $E=0$, we find the value $\cos^2\varepsilon=0.675148$, which is not very far from the limit of convergency or $\cos\varepsilon^2 = 0.67264$.

Corresponding to $E=0$ we find

$$\begin{aligned}\mathfrak{Q} &= 0.740333\\ \mathfrak{P} &= 0.1717613\\ \zeta &= 0.6899\\ z &= 0.56686\\ H &= 0.20464\times\lambda\\ e^h &= 2.52006\end{aligned}$$

Since these are the waves that can be immediately produced by a constant wind, therefore these are the values that lie at the foundation of the computation quoted in Article 6, whereas the values for the lowest waves are found when we assume for $\cos^2\varepsilon$ the upper limit of its values, namely, 0.68615.

Theory shows, moreover, as also the above numerical example, that the waves of this form for large values of $\cos\varepsilon$ and for the same material and same strength of wind have greater wave-lengths; that, however, their altitudes form a smaller fraction of the wave-length, and that their energy when $\cos^2\varepsilon > 0.675148$ is smaller than that of the rectilinear flow of both media with the same velocities. The difference of energy is zero for very low waves; it is negative when we pass to relatively high waves; it reaches a maximum, then diminishes, and is again zero for the given boundary value.

It is sufficient to have proven that for one form of wave billows due to wind are possible, which billows have a less store of energy than the same wind would have over a plane boundary surface. Hence it follows that the condition of rectilinear flow with plane boundary surface appears at first as a *condition of indifferent or neutral equilibrium*, when we consider only the lower powers of small quantities. But if we consider the terms of higher degree, then this condition is one *of unstable equilibrium*, in view of certain disturbances that correspond to stationary waves between definite limits as to wave-length; but on the other hand is a condition of *stable equilibrium* when we consider shorter waves.

This result is evidently of great importance for the origin of waves. It follows from this, as we everywhere see confirmed in nature, that even the most uniform wind can not blow over a plane surface of water without on the slightest disturbance causing waves of a certain length, which for a given height acquire regular form and speed of propagation. If the wind increases then the heights of all these waves increase, the shorter ones among them break foaming, so that new longer ones of less height can be formed.

The greater energy that is necessary in this case in order to push the shorter waves up higher becomes possible in that the previous feebler wind had already given a part of its energy to the mass of water, and the new stronger wind finds this part already present there.

Breaking, foaming atmospheric billows cause mixture of strata in the mass of air. Since the elevations of the air-waves in the atmosphere can amount to many hundred metres, therefore precipitation can often occur in them which then itself causes more rapid and higher ascent. Waves of smaller and smallest wave-length are theoretically possible. But it is to be considered that perfectly sharp limits between atmospheric strata having different motions certainly seldom occur, and therefore in by far the greater number of cases only those waves will develop whose wave-length is very long compared with the thickness of the layer of transition.

The circumstance that the same wind can excite waves of different lengths and velocities, will cause interferences to occur between the waves, and also higher and lower wave summits to follow each other interchangeably. This is a process observed often enough on the shore of the ocean. But where two wave summits of different groups of waves reënforce each other a height will easily be attained at which they break into foam, and thereby, as in the analogous case of the production of sonorous combination tones, longer waves can be formed which, when they are favored by the strength of the wind, can also grow larger. This is one of the processes by which waves of great length can arise.

VII.

THE ENERGY OF THE BILLOWS AND THE WIND.*

By Prof. H. Von Helmholtz.

In my communication to the Academy on July 25, 1889, I called attention to the fact that a plane surface of water above which a steady wind is blowing is in a state of unstable equilibrium and that the origin of large waves or billows of water is essentially due to this circumstance. I have there also shown that the same process must be repeated at the boundary of two strata of air of different densities gliding over each other, but that in this case it can assume much larger dimensions and without doubt has an important meaning as a cause of nonperiodic meteorological phenomena.

The importance of these processes has induced me to investigate still more thoroughly the relations of the energy and its distribution between the air and the water; at first, however, as before, with the limitation to stationary waves in which the motions of the particles of water only take place parallel to a vertical plane in which the coördinates are respectively (x) vertical and (y) horizontal. Since however we can only solve even this special problem by the development into a converging series whose higher terms rapidly diminish in magnitude but offer comparatively complex forms therefore the conclusions that we may have drawn from a knowledge of the first largest term of such a series are necessarily always limited to waves of slight altitude and cause the correctness of many more important generalizations to appear doubtful.

Many of these difficulties have been surmounted in that I have been able to reduce the law of stationary rectilinear waves to a problem of minima, in which the variable quantities are the potential and actual energies of the moving fluids. From this problem in variations many general conclusions can be deduced as to the decrease and increase of the energy, and the difference between stable and unstable equilibrium of the surface of water.

Theoretically considered, there arises here a rather new problem in so far as we have to do, not with the difference between stable and un-

* From the *Sitzungsberichte* of the Royal Prussian Academy of Sciences at Berlin, 1890, vol. VII, pp. 853–872. Wiedemann, *Annalen*, 1890, XLI, pp. 641–662.

stable equilibrium of masses at rest, but with moving masses that are in steady motion.

Some examples of such differences have indeed been already treated, as in the rotation of a solid body about the axis of its greatest or least moment of inertia, and in the rotation of a fluid ellipsoid subject to gravity. But a general principle such as is given for bodies at rest, in the proposition that stable equilibrium requires a minimum of potential energy, has never yet been established for a moving system of bodies.

The following investigations lead to such propositions, which moreover can also be considered as generalizations of the propositions that I have deduced from the general equations of motion given by Lagrange in their application to the motion of "poly-cyclic" systems.*

I. THE THEOREM OF MINIMUM ENERGY APPLIED TO STATIONARY WAVES HAVING A CONSTANT QUANTITY OF FLOW.

As in my paper of last year,† I indicate by u and v the component velocities of the particles of water during any motion that is free from vortices by the equations:

$$\left.\begin{aligned} u &= -\frac{\partial \psi}{\partial y} = \frac{\partial \varphi}{\partial x}, \\ v &= \frac{\partial \psi}{\partial x} = \frac{\partial \varphi}{\partial y}. \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (1)$$

I again assume, whenever the opposite is not expressly stated, that the coördinate system for $x\ y$ is at rest with reference to the wave, x being vertical, positive upward, y horizontal. Therefore the wave surface is at rest with reference to these coördinates while the two fluids flow steadily along it. The wave curve will be considered as periodical with the wave length λ. On the other hand, the flowing fluid will be considered as bounded by two horizontal planes whose equations are

$$x = H_1 \text{ and } x = -H_2 \quad \ldots\ldots\ldots \quad (1a)$$

Corresponding to this, I indicate the remaining quantities that refer to the fluid which is on the positive side of x by the subscript 1; those that are on the negative side of x by the subscript 2.

The wave-lines and these two horizontal boundary lines must be stream lines—that is to say, ψ must have a constant value throughout their whole length. Since each of the functions ψ can contain an arbitrary additive constant, therefore we can assume arbitrarily both of the values of ψ for one of the stream lines. I assume that for the wave line for which

$$x = \bar{x}$$

we have the value

$$\bar{\psi} = 0 \quad \ldots\ldots\ldots\ldots \quad (1b)$$

* *Kronecker und Weyerstrass, Journ. für Mathemat.*, 1884, vol. XCVII, p. 118.

† [See the previous paper, No. VI, in this collection of Translations.]

On the other hand, for the boundary line, for which

$$\left.\begin{array}{l} x = H_1 \\ \psi_1 = \mathfrak{p}_1 \end{array}\right\} \qquad (1c)$$

we have

and for the other boundary line, whose equation is

$$\left.\begin{array}{l} x = -H_2 \\ \psi_2 = \mathfrak{p}_2 \end{array}\right\} \qquad (1d)$$

we have

The quantities $\mathfrak{p}_1$ and $\mathfrak{p}_2$, as is well known, give respectively the volumes of the fluid that flow in the unit of time through every section between the wave surfaces for which $\psi_1 = \psi_2 = 0$, and through the upper or lower boundary surface.

These are the quantities which I have above designated as *quantities of flow.* In taking the variations of these quantities, I shall, in this paragraph, consider $\mathfrak{p}_1$ and $\mathfrak{p}_2$ as invariable.

That altitude will be adopted as the initial point for x, at which the boundary surface of the two quantities of fluid under consideration would be at rest, which is expressed by the equation

$$\int_{y_0}^{y_0+\lambda} x\, dy = 0 \qquad (1e)$$

that is to say, $x = 0$ is a plane such that as much water is raised above it as sinks below it.

Finally the space within which lie the quantities that are subject to variation is also bounded by two vertical planes that are separated from each other by one wave length. Since the movements are to be periodical and consistent with the wave length λ, the velocities at the right vertical surface and at the left vertical surface must be equal or

$$\frac{\partial \psi_r}{\partial x} = \frac{\partial \psi_l}{\partial x}$$

therefore for the same values of x

$$\psi_r = \psi_l \qquad (1f)$$

and

$$\frac{\partial \psi_r}{\partial y} = \frac{\partial \psi_l}{\partial y} \qquad (1g)$$

According to Eq. (1) this last equation can also be written

$$\frac{\partial \varphi_r}{\partial x} = \frac{\partial \varphi_l}{\partial x}$$

or

$$\varphi_r - \varphi_l = \text{constant} \qquad (1h)$$

Now it is known that equations (1) are resolvable when $(\psi + \varphi i)$ can be represented as a function of $(x + yi)$, which function must show no discontinuity and no infinite values within the region filled by the fluid in question.

When the form of the wave-line is given, the values of the two functions, ψ, as is well known, are completely determined by the above given boundary conditions ($1b$) to ($1g$) and in that case the two integrals, which multiplied by one-half of the density of the respective fluids, give the living forces, namely:

$$\frac{2L_1}{s_1}=\int\int\left[\left(\frac{\partial\psi_1}{\partial x}\right)^2+\left(\frac{\partial\psi_1}{\partial y}\right)^2\right]d\,S_1 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

$$\frac{2L_2}{s_2}=\int\int\left[\left(\frac{\partial\psi_2}{\partial x}\right)^2+\left(\frac{\partial\psi_2}{\partial y}\right)^2\right]d\,S_2 \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

become absolute minima for such variations of the functions ψ_1, as are possible under the given circumstances, when at the same time the values $\mathfrak{p}_1$ and $\mathfrak{p}_2$ are considered as invariable.

On the other hand the form of the wave-line is not yet determined by the conditions hitherto given, except in so far that it must be periodical with the period λ. We can however determine the form of this boundary line corresponding to the physical condition that the pressure shall be the same on either side of it, in that we require that the variation of the difference between the potential energy Φ and the living force $L=L_1+L_2$ shall disappear, or

$$\delta[\Phi-L]=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2b)$$

The potential energy depends upon the unequal elevation of the different parts of the surface of heavier fluid above the level surface $x=0$. Its amount is easily seen to be given by the equation

$$\Phi=\tfrac{1}{2}g(s_2-s_1)\int x^2 dy. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2c)$$

If s_2 is the denser fluid, then the positive x, as already remarked, must be assumed as ascending perpendicularly and y must be taken as a positive quantity.

When the linear element $d\,s$ of the boundary-line of the two fluids is displaced upwards normal to its own direction by the infinitely small quantity $\delta\,N$, then the variation becomes

$$\delta\,\Phi=g(s_2-s_1)\int x\,\delta\,N.\,d\,s. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2d)$$

The variation of L can be executed in two steps. In the *first* of these we imagine the boundary-line displaced in the above-given manner and first allow the two functions ψ_1 and ψ_2 in each point of space to remain unchanged, but in doing so, on that side where space is gained by the displacement $d\,s$, imagine this strip so gained to be filled with the continuous prolongation of the ψ that pertains to this side, and so that the equation $\Delta\,\psi=0$ continues to be satisfied in that region [and so that the prolongation of ψ just mentioned enters here instead of the value of the other function of ψ previously existing here]. This prolongation of the function ψ into the strip just described is, as well known, only possible in one manner without forming discontinuities. Only when a cusp of

the appropriate function ψ exists in the original boundary, therefore, especially when the boundary-line forms a sharp corner, is a continuous prolongation of the function excluded. The special physical significance of such a case we shall have to consider later on.

By this first step in the variation of L we obtain

$$\delta' L = \tfrac{1}{2}\int\left[s_2\left(\frac{\partial\psi_2}{\partial N_2}\right)^2 - s_1\left(\frac{\partial\psi_1}{\partial N_1}\right)^2\right] ds\,\delta N.$$

But now the values of ψ_1 and ψ_2 are no longer zero at the new boundary, but we have there, approximately

$$\psi_1 = \frac{\partial\psi_1}{\partial N_1}\,\delta N$$

$$\psi_2 = -\frac{\partial\psi_2}{\partial N_2}\,\delta N$$

and in order again to make these equal to zero we must execute a *second step* in the variation, such that the function ψ shall so vary that these now again become zero at the new boundaries. Since according to the general laws of potential functions we have

$$\delta'' L = -s_1\int\frac{\partial\psi_1}{\partial N_1}\,\delta\psi_1\,ds - s_2\int\frac{\partial\psi_2}{\partial N_2}\,\delta\psi_2\,ds$$

therefore when we (as is necessary in our case) put

$$\delta\psi_1 = -\frac{\partial\psi_1}{\partial N_1}\,\delta N$$

$$\delta\psi_2 = +\frac{\partial\psi_2}{\partial N_2}\,\delta N$$

we obtain the final value:

$$\delta L = \delta' L + \delta'' L = -\tfrac{1}{2}\int\left[s_2\left(\frac{\partial\psi_2}{\partial N_2}\right)^2 - s_1\left(\frac{\partial\psi_1}{\partial N_1}\right)^2\right] ds\,\delta N. \quad . \quad (2e)$$

Since finally the volume of each of the two liquids must remain unchanged during the variation, therefore it is necessary that

$$\int \delta N\,ds = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2f)$$

Hence results the variation,

$$\delta\left\{\Phi - L\right\} = -\int ds\,\delta N\left\{g\,(s_1 - s_2)\,\bar{x} + \frac{s_1}{2}\left(\frac{\partial\psi_1}{\partial N_1}\right)^2 - \frac{s_2}{2}\left(\frac{\partial\psi_2}{\partial N_2}\right)^2 + c\right\}$$

$$= -\int ds\,\delta N\,[p_2 - p_1.] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2g)$$

Here p_2 and p_1 designate the fluid pressure on the upper and lower sides, respectively, of the boundary surface as they result from Euler's hydrostatic equations. Since p_2 and p_1 contain arbitrary additive constants c can be omitted.

When, therefore, the equation (2b) is to be satisfied, that is to say, when we must have

$$\delta \left\{ \Phi - L \right\} = 0$$

then must $p_2 = p_1$ throughout the boundary surface, which is the condition of a stationary surface.

The stability of the steady motion.—For any form of surface that nearly corresponds to a stationary form, and which therefore still shows differences of pressure, it follows from the preceding that such a surface when it changes with the differences of the pressures experiences therefore a positive displacement δN where $p_2 > p_1$, therefore the quantity $(\Phi - L)$ diminishes and consequently approximates to a neighboring minimum of $(\Phi - L)$, and must therefore depart from the neighboring maximum of the same quantity.

The hydro-dynamic equations show in fact that the equality of pressure in such cases can only be brought about by accelerations which act in the direction from the stronger to the feebler pressure and must disturb the steady motion.

Therefore the *stable equilibrium* of a stationary wave-form must (among all possible variations of such a form) correspond to a minimum of the quantity $(\Phi - L)$, just as in the polycyclic systems for a constant velocity of their cyclic motions. When on the other hand this same quantity $(\Phi - L)$ attains a maximum value or a cusp value for some other form of curve, then the condition of equality of pressure on both sides of the boundary surface is at least temporarily fulfilled; but individual or the very smallest disturbances of the form of equilibrium must continue to increase: the equilibrium will thus become *unstable* as is actually recognized in natural water-waves by the foaming and breaking of the crests of the waves.

On the other hand it is to be remarked that these propositions hold good only when the functions L_1 and L_2 are determined as minima in accordance with the boundary conditions of the spaces within which they hold good, and for every variation in the form of the boundary line the functions experience a change in accordance with this condition that they shall be minima.

Under the assumptions already made, the function Φ is certainly positive and finite, since only a finite quantity of liquid is present which can be raised up only through the finite altitude H_1. L is also necessarily positive but can become $+\infty$, since the summit of the wave can approximate to the upper but the trough of the wave to the lower boundary surface and the total constant quantity of moving fluid must then be pressed with infinite velocity through infinitely narrow crevices.

The quantity $(\Phi - L)$ must therefore have a positive value for plane boundary surfaces where $\Phi = 0$, and it can become $-\infty$ for increasing wave altitudes. Whether a minimum occurs between these limits, and for what value of $\mathfrak{p}$ this could occur, can only be decided by investigation

of the individual forms of the waves. At least one cusp value occurs for a plane surface.

Only this much can be at once seen, that when an absolute minimum exists there must be a transition leading from this to the infinite negative value of $(\Phi - L)$, which transition at first begins with an ascending value and then again diminishes. There must then be a lowest value on the transition curve between the ascending and the descending values that corresponds to a maximo-minimum (absolute minimum) of the quantity $(\Phi - L)$, therefore also to a stationary form of wave, but such an one as corresponds to an unstable equilibrium, and which is on the point of becoming a breaker.

If such a minimum exists, then for it any variation in the form of the wave that makes Φ increase will make L increase by the same amount. The same is true of the cusp value when we consider such waves as form trough-lines. But if we increase the values of $\mathfrak{p}_1$ and $\mathfrak{p}_2$, that is to say, if we increase the velocity of the wind and the rate of propagation of the waves through water, then the partial differential coefficient of L will be greater at both places and the two limiting values must approach each other and finally coincide, whereby the absolute minimum ceases to exist and the equilibrium becomes unstable. Hence it is to be concluded that with increasing rate of flow, stationary waves of a given wave-length will finally become impossible.

Necessary formation of breakers when the velocity is excessive.—That, for a constant definite value of the wave-length, minima of the function $(\Phi - L)$ are no longer possible for large values of $\mathfrak{p}_1$ and $\mathfrak{p}_2$ exceeding a certain definite amount, can easily be shown as follows: We compute the values of L_1 and L_2 under the assumption that $\mathfrak{p}_1 = \mathfrak{p}_2 = 1$, for any arbitrarily chosen form of wave and then for an arbitrarily chosen value of $\delta\Phi$ seek the two variations of the curve which respectively make δL_1 and δL_2 to become maxima.

Among the possible variations of the form of the wave that give positive values of $\delta\Phi$ are those that give higher summits and lower troughs for the wave. Since the upper fluid has the greatest [least ?—*C. A.*] section above the summits of the waves, but the smallest [greatest ?—*C. A.*] section above their valleys therefore above the summits a greater velocity of flow must prevail than above the valleys, that is to say the value of $\frac{\partial \psi_1}{\partial N_1}$ must be greater absolutely on the summits than in the troughs. Hence follows from equation (2*e*) that when we raise the summits and depress the valleys we obtain not only positive values of $\delta\Phi$ but also positive values of δL_1 and δL_2. Consequently the desired maximum values of the two quantities δL_1 and δL_2, that belong to the prescribed positive values of $\delta\Phi$ are necessarily positive, and for a finite altitude of the wave the ratio $\frac{\delta\Phi}{\delta L_1}$ as also $\frac{\delta\Phi}{\delta L_2}$ must necessarily be finite.

We now indicate by α a proper fraction and imagine that we have executed a variation of L_1 to the amount expressed by α, such as would correspond to the variation $\alpha. \delta\Phi$. On the other hand we perform the variation δL_2, to the amount $(1-\alpha)$. Then the total variation for Φ is

$$\delta\Phi = [\alpha + (1-\alpha)]\ \delta\Phi,$$

$$\delta L = \alpha.\ \delta L_1 + (1-\alpha).\ \delta L_2.$$

If now $\delta L_1 > \delta L_2$ we obtain the maximum variation of δL when we make $\alpha = 1$; but for the opposite case we should have to make $\alpha = 0$. Thus δL attains the greatest value that it can have for the given value of $\delta\Phi$ and the adopted form of wave.

When the greatest positive value of δL is smaller than $\delta\Phi$ then a value for $\mathfrak{p}_1^2$ can be found that in any case will make

$$\mathfrak{p}_1^2 \delta L > \delta\Phi$$

and therefore, for at least one method of change of form, which need not necessarily be a minimal form, will make the variation $\delta\ (\Phi - L)$ negative.

Since Φ always remains finite one can always execute finite variations in its magnitude that shall be of the same order of magnitude as the displacement δN of the elementary line ds, and which latter give always finite variations of L_1 and L_2, at least for finite velocities of flow along the surface.

Infinite velocities can only occur at the projecting cusps of the wave-lines and, when there is a current there, give infinite negative pressures, that is to say, the phenomena of breaking or frothing. Only when there exists no relative motion of the wave with respect to the medium into which the sharp edges of the waves project, namely, when the wind has precisely the same speed as that of the wave, can such cusp points long endure.

Except these latter cases, that lie on the boundary of breaking and frothing, we shall therefore for all continuously curved forms of waves have for every $\delta\Phi$ a maximum of δL of the same order of magnitude. And when we seek for the smallest value of the ratio $\frac{\delta L}{\delta \varphi}$ and seek for a value of $\mathfrak{p}^2$ which shall be greater than the greatest of the values of $\frac{1}{\frac{\delta L}{\delta\Phi}}$ thus obtained, then for the corresponding strength of current the possibility of stationary wave-formation for the prescribed wave-length λ is entirely excluded.

Therefore stationary waves of a prescribed wave-length are only possible for such values of the velocities of flow $\mathfrak{p}_1^2$ *and* $\mathfrak{p}_2^2$ *as are less than certain definite extreme limits.*

On the other hand, these same considerations further show that the

diminution of the values $\mathfrak{p}_1^2$ and $\mathfrak{p}_2^2$ will necessarily make the larger values of δL_1 and δL_2 with respect to $\delta\Phi$ disappear. Then variations of $\delta\Phi$ can not be counterbalanced by opposite and equal values of L and then can at the most only one limiting value exist, *i. e.*, that which corresponds to the plane surface. The limit for the smallest allowable values of $\mathfrak{p}_1$ and $\mathfrak{p}_2$ results from the preceding investigation as follows:*

$$\frac{2\pi s_1\mathfrak{p}_1^2}{H_1^2}+\frac{2\pi s_2\mathfrak{p}_2^2}{H_2^2}=g\cdot\lambda(s_1-s_2).$$

Hence the range of values of $(\mathfrak{p}_1)^2$ *and* $(\mathfrak{p}_2)^2$ *that permit stationary waves of the wave-length* λ *is limited on its lower side.*

It is to be noticed that the quantity $\mathfrak{p}_2$ determines the progressive velocity of the wave with respect to the water; $\mathfrak{p}_1$, on the other hand, determines the velocity of the wind relative to the wave. Either of these can be small if the other is sufficiently large.

II. THE THEOREM OF MINIMUM ENERGY APPLIED TO STATIONARY BILLOWS WITH A CONSTANT VALUE OF THE VELOCITY POTENTIAL.

The value of the living force, as given in equation (2), can by partial integration be written

$$L_1=\frac{s_2}{2}\int\mathfrak{p}_1.\frac{\partial\psi_1}{\partial x}.\,dy,$$

in which the integral relates only to the upper horizontal boundary line. The portions of the integral for the other limit of the space S_1 all disappear. Since now according to equation (1)

$$\frac{\partial\psi}{\partial x}=\frac{\partial\varphi}{\partial y}$$

there results

$$L_1=\frac{s_1}{2}\mathfrak{p}_1\int\frac{\partial\varphi_1}{\partial y}.\,dy.$$

Of, if we put

$$\mathfrak{f}=\varphi_{y+\lambda}-\varphi_y,$$

which difference is independent of x, we obtain

$$L_1=\frac{s_1}{2}\mathfrak{p}_1.\,\mathfrak{f}_1 \quad \ldots \quad (3)$$

and similarly

$$L_2=-\frac{s_2}{2}\mathfrak{p}_2\mathfrak{f}_2 \quad \ldots \quad (3a)$$

* My attention has been called to the fact that Sir William Thomson has already given this equation as the first approximation, taking into consideration the strength of the wind, *Philos. Mag.*, 1871 (4), vol. XL, p. 362, where, moreover, the influence of capillarity is also considered.

The quantities $\mathfrak{p}$ and $\mathfrak{f}$ are dependent on each other as soon as the form of the space is given for whose boundary they hold good; so that we can put

$$\mathfrak{p}=\mathfrak{f}.\ \mathfrak{R}$$

where $\mathfrak{R}$ indicates a value that depends only on the size and form of this space. Hence there results

$$L=\frac{s}{2}\mathfrak{p}.\ \mathfrak{f}=\frac{s}{2}\mathfrak{f}^2\ \mathfrak{R}=\frac{s}{2}\frac{\mathfrak{p}^2}{\mathfrak{R}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3b)$$

When therefore $\mathfrak{R}$ experiences a change $\delta\mathfrak{R}$, then if $\mathfrak{f}$ remains unchanged we have

$$\delta L=\frac{s}{2}.\ \mathfrak{f}^2.\ \delta\mathfrak{R}$$

$$\delta\mathfrak{f}=0;$$

on the other hand, when $\mathfrak{p}$ remains unchanged we have

$$\delta L=\frac{s}{2}\frac{\mathfrak{p}^2\ .\ \delta\mathfrak{R}}{\mathfrak{R}^2}=-\frac{s}{2}\mathfrak{f}^2.\ \delta\mathfrak{R}$$
$$\delta\mathfrak{p}=0.$$

Both variations therefore have the same values with opposite signs. We can therefore, instead of

$$\delta\Phi-\delta L=0$$
$$\delta\mathfrak{p}_1=\delta\mathfrak{p}_2=0.$$

which is the form of variation for the stationary condition where the variation of δL is deduced from the variation of the form of the region, also write

$$\delta\Phi+\delta L=0$$
$$\delta\mathfrak{f}_1=\delta\mathfrak{f}_2=0.$$

The quantities $\mathfrak{f}$ according to their definition have the value:

$$\mathfrak{f}=\int_{x,y.}^{x,\ y+\lambda}(u.\ dx+v.\ dy)$$

the integral being taken for any value that leads from the point (x, y) to the point $(x, y+\lambda)$. When we choose the stream-line $\psi=$ constant for this path between these points then the integral also indicates a path along which a series of material liquid particles would flow. The value of the integral $\mathfrak{f}_1$, as computed for such a series of material flowing particles as is well known remains unchanged. whatever motions may otherwise be going on in the liquid, provided there are no differences in the sum total of the pressures and potentials of the exterior forces between the beginning and the end of the series, and provided there is no friction. This is the same sum that also remains unchanged in the vortex motion in every closed ring of material particles. We can therefore in fluid motions consider $s_1\ \mathfrak{f}_1$ and $s_2\ \mathfrak{f}_2$ as the moments of

motion, which remains invariable except for the influence of direct accelerating forces, while the quantities of flow $\mathfrak{p}_1$ and $\mathfrak{p}_2$ thereby receive the significance of velocities. Thus the two problems in variations, here solved, are completely analogous to the propositions developed by me in the theory of polycyclic systems, that

$$\underset{\delta q_a = 0}{\delta(\Phi - L) = -\Sigma\,[P_a \delta\, p_a]} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3e)$$

when the velocities q_a of the cyclic motions are maintained constant. In this equation p_a, are the variable coördinates, and P_a are the forces tending to increase these coördinates. Stable equilibrium, as is easy to see, corresponds to a minimum of the $(\Phi - L)$.

On the other hand, when we assume the moment of motion $\frac{\partial L}{\partial q_a}$ to be constant we have

$$\delta(\Phi + L) = -\Sigma\,[P_a\, d\, p_a]$$

$$\delta\left(\frac{\partial L}{\partial q_a}\right) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3f.)$$

Here, also, stable equilibrium demands that the quantity $(\Phi + L)$, that is to say, the total energy of the body be a minimum.

The equation $(2g)$ corresponds throughout to the above equation $(3e)$ for polycyclic systems, only that in the former the number of variable coördinates δN of the surface elements ds is infinitely large and the force which in it corresponds to P_a, namely, the fluid pressure, is a continuous function of y; hence the integral is used instead of the sign of summation.

That stable equilibrium, even in the theory of waves, also corresponds to the minimum of energy for a constant value of $\mathfrak{f}$ is evident when we consider the influence of friction which can restore a disturbed stable equilibrium but not a disturbed unstable equilibrium. Friction always diminishes the store of energy that may be present. It can, therefore, restore a disturbed minimum of energy but not a departure from a maximum.

III. THE THEOREM OF MINIMUM ENERGY APPLIED TO LAYERS OF INFINITE THICKNESS.

In the following we shall consider the two layers of fluid on whose boundary surface the waves form, as very deep in the vertical direction, therefore the values H_1 and H_2 as very large and as respectively increasing beyond all limits to infinity, in order to free the theory of waves from those complications which are brought about by the influence of the upper and lower horizontal boundary surfaces.

Under these circumstances the motion on these two far distant horizontal boundary surfaces does not differ sensibly from rectilinear uni-

form velocity. For the surface H_1 we put a_1 for this velocity, for the surface H_2 we take $(-a_2)$ since we give the latter a motion in the opposite direction to that which would be given to it in the normal cases where the wind outruns the wave.

We have at once

$$+\mathfrak{f}_1=a_1.\ \lambda$$
$$-\mathfrak{f}_2=a_2.\ \lambda$$

and in the higher layers of the fluid

$$\psi_1+\varphi_1\, i=+a_1\ (x+yi)+h_1$$

where h_1 is a constant to be determined by the equation (1e).

Similarly

$$\psi_2+\varphi_2\, i=-a_2\ (x+yi)+h_2$$

For plane boundary surfaces when for these as above assumed $\psi_1=\psi_2=0$, and also $x=0$, we should also have h_1 and h_2 both equal to zero, and the living force in this case becomes

$$L_1{}^1=\frac{s_1\ \mathfrak{p}_1}{2}.\ \mathfrak{f}_1=\frac{s_1}{2}\ a_1{}^2.\ H_1\ \lambda$$

$$L_2{}^1=-\frac{s_2}{2}\mathfrak{p}_2.\ \mathfrak{f}_2=\frac{s_2}{2}\ a_2{}^2.\ H_2\,\lambda$$

When on the other hand billows have arisen, L_1 is smaller for a constant value of a_1 and therefore also of $\mathfrak{f}_1$, since, as we have seen then a negative value of δL_1 results from an increase in the altitude of the wave. We can therefore under these circumstances put

$$L_1=\frac{s_1}{2}a_1{}^2\ (H_1-r_1).\ \lambda$$

wherein r_1 has a positive value that depends on the form and height of the wave, but not on H_1. If we imagine H_1 increased by the quantity $D\,H_1$ and the quantity L_1 correspondingly increased by $D\,L_1$ then in the strip thus added to the field the velocity is uniformly equal to a_1 and therefore

$$D\,L_1=\frac{s_1}{2}a_1^2.\ D\,H_1$$

$$L_1+D\,L_1=\frac{s_1}{2}.a_1^2\left[(H_1+D\,H_1)-r_1\right].\lambda.$$

Therefore the same value of r_1 also holds good for the greater altitude independent of the value of $D\,H_1$.

The formula (4) gives directly

$$\mathfrak{p}_1=-\mathfrak{f}_1\,(H_1-r_1)\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ (4a)$$

Compared with galvanic conditions, $\mathfrak{p}_1$ measures the total flow or the intensity of the current; $\mathfrak{f}_1$ is the difference of potential between the boundary surfaces. Hence (H_1-r_1) is the conductivity which is pro-

portional to the sectional area. Therefore r_1 corresponds to that constant diminution of the sectional area which causes the current to diminish just as the irregular obstruction by the waves does.

For a constant value of a_1 and a_2, respectively, since λ, H_1, and H_2 remain unchanged, the condition that a minimum of $(\Phi + L)$ should exist gives

$$\delta(\Phi+L)=\delta\Phi-\frac{s_1}{2}a_1^2\,\delta r_1-\frac{s_2}{2}a_2^2\,\delta r_2=0 \quad . \quad . \quad . \quad . \quad . \quad (4b)$$

The other minimum condition in which a_1, a_2 are to be replaced by

$$a_1=\frac{\mathfrak{p}_1}{H_1-r_1} \quad \text{and} \quad a_2=\frac{\mathfrak{p}_2}{H_2-r_2}\,.$$

is

$$\delta(\Phi+L)=\delta\Phi-\frac{s_1}{2}\mathfrak{p}_1^2\frac{\delta r_1}{(H_1-r_1)^2}-\frac{s_2}{2}\mathfrak{p}_2^2\frac{\delta r_2}{(H_2-r_2)^2}$$

which agrees perfectly with that first found.

The quantities r_1 and r_2 depend only on the form of the wave, and are generally found by simple computations as soon as we have found the form of the functions ψ_1 and ψ_2.

Horizontal transportation of the superficial layer.—The quantity of flow $\mathfrak{p}_1$ and $\mathfrak{p}_2$ of the two fluids is no longer the same as it would be over plane surfaces of water for equal values of the velocities a_1 and a_2, but it is smaller than before in the upper medium by the quantity $r_1\,a_1$ and in the lower medium by the quantity $r_2\,a_2$.

Imagine now the velocity $(-a_2)$ added to both sides so that the lower medium comes to rest, but the waves progress with the velocity $(-a_2)$. Then beneath plane boundary surfaces all motion disappears, but beneath billowy surfaces a general current is set up of the magnitude $-a_2\,r_2$, and thus the wind in the upper region travels not with a uniform velocity (a_1+a_2), but just above the billowy surface there occurs a diminution of the flow of air to the amount of $a_1\,r_1$.

These two currents cause the mass of air and water taken together to have a different moment of motion in a horizontal direction than if they flowed with the same velocities a_1 and a_2 over plane boundary surfaces, and this difference of moment of motion, reckoned as positive in the direction of the wind, is

$$M = s_2\,a_2\,r_2 - s_1\,a_1\,r_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5).$$

This can only be equal to zero when

$$s_2\,a_2\,r_2 = s_1\,a_1\,r_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5a),$$

or, if we introduce w, the velocity of the wind,

$$w = a_1 + a_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5b),$$

then equation (5*a*) becomes

$$\frac{s_2 r_2 w}{s_1 r_1 + s_2 r_2} = a_1$$

$$\frac{s_1 r_1 w}{s_1 r_1 + s_2 r_2} = a_2.$$

Since now r_1 and r_2 have values that differ but little for the ordinary waves (as the subsequent computations will show), and since for air and water

$$\frac{s_1}{s_2} = \frac{1}{773.4},$$

therefore this condition gives the rate of propagation of the wave against the water as approximately

$$a_2 = \frac{w}{774.4}.$$

For waves of low altitude equation I, Section VII of my paper of the previous year,* neglecting the small quantities z and ζ, becomes

$$s_1 a_1^2 + s_2 a_2^2 = \frac{g.\lambda\ (s_2 - s_1)}{2\pi}$$

If we put w=10 metres which corresponds to a rather strong wind, then for low waves of a constant moment of motion, we have

$$a_1 = 9^{\text{m}}.98709$$
$$a_2 = 0^{\text{m}}.01291$$
$$\lambda = 0^{\text{m}}.082782$$

These waves of only 8 centimeters in length evidently can correspond only to the first crumpling of the surface, such as a strong wind striking upon it immediately excites. Only when the same wind blows for a long time over these initial waves, and gives them a part of the moment of motion of a long stretch of air, can waves be thereby produced with greater velocities of propagation.

Hence in accordance with experience it follows that wind of a uniform strength striking a quiet surface of water can only produce more rapidly running waves, namely, those that are longer and higher, when it has acted for a long time on the waves that first arose, and has accompanied these for a long distance over the surface of the water.

At the same time it also becomes clear that for a uniform wind the waves can only increase in size when the wind advances faster in the same direction than the waves themselves.

Energy of progressive waves on quiet water.—As in the case of the moment of motion, so also with the storage of energy in the wave. Our previous comparisons of the energy of different waves among themselves has reference to the energy of relative motion of the fluid with reference to the stationary wave.

* [See page 107 of this collection of Translations.]

The well known proposition that *the living force of any complex mechanical system is equal to the living force of the motions relative to its center of gravity plus the living force of the motion of the center of gravity at which we imagine the whole mass of the system to be concentrated*, can, with only a small change in the method of expression, be applied to our case. For since the total mass of the system multiplied by the velocity $\mathfrak{v}$ of the center of gravity, gives the amount of the total momentum of the system in the direction of this velocity, therefore we can also put the living force $\mathfrak{L}$ of the center of gravity

$$\mathfrak{L} = \tfrac{1}{2}\, M\, \mathfrak{v} = \tfrac{1}{2}\, \mathfrak{M}\, \mathfrak{v}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

where M is the momentum of the whole system in the direction of $\mathfrak{v}$ and $\mathfrak{M}$ is the mass of the system. If we now compare with each other two different conditions of motion and configuration of the system in which L_1 and L_2 are the living forces of the motions relative to the center of gravity, Φ_1 and Φ_2 are the potential energies, $\mathfrak{v}_1$ and $\mathfrak{v}_2$ are the parallel velocities of the center of gravity, then the difference in the total energy of the system in the two conditions is

$$E_1 - E_2 = \Phi_1 - \Phi_2 + L_1 - L_2 + \tfrac{1}{2}\,\mathfrak{M}.\,\mathfrak{v}_1^2 - \tfrac{1}{2}\,\mathfrak{M}.\,\mathfrak{v}_2^2.$$

If now, without changing the relative motions, I in both cases add the quantity c to the velocity of the center of gravity, then the above difference of energies changes into

$$E_1' - E_2' = E_1 - E_2 + c\,(M_1 - M_2).$$

If $M_1 - M_2 = 0$, then the value of the difference in energy is not changed by the addition of the velocity c. This must be true even when H_1 and H_2, and therefore the masses of the moving fluids, increase to infinity, since for our undulating fluids the differences $(E_1 - E_2)$ and $(M_1 - M_2)$ are finite for each wave length.

Therefore the difference of the energy for stationary waves and for stationary deep water will be equally great only for waves that satisfy the condition ($5a$). According to the propositions above deduced, stationary waves of this kind must have less energy than smooth water, which is therefore also true in this case for this kind of waves above quiet water.

For waves that have larger values of a_2, the addition of a common velocity $(-a_2)$, which brings the deep water into rest, changes the difference of energy between the two states, that of a smooth surface and that of a wave formation, by the quantity

$$E_1' - E_2' = E_1 - E_2 + a_2\,[s_2 a_2 r_2 - s_1 a_1 r_1].$$

The index 1 refers to the billowy surface, the index 2 to the plane surface, the accented E' refers to quiet deep water, the non-accented E refers to stationary waves.

Hence it results that when waves of considerable progressive velocity trench upon quiet deep water the generally very small differences (E_1-E_2) lose their negative and assume a positive value.

Here also the energy that is given to the previously quiet water in the form of an elevation of its surface and the living force of its motion must be abstracted from the atmosphere. In order to obtain a sufficient amount for the formation of large waves, it will on this account be necessary that long layers of air shall blow over and shall give up a part of their living force.

In the first moment when a new gust strikes the surface of the water stationary waves only can be formed for which $M=0$ and $E_1-E_2=0$ and a_2 has the value given in equation (5a). The last condition shows that these waves will be near the point of spirting, as we in fact often see in the case of small ripples suddenly excited on the surface of the water. Moreover in these small ripples, as Sir William Thomson has shown, the capillary tension of the liquid comes into consideration, which somewhat increases the store of energy of the billowy surface.

In general therefore, stationary waves are not formed immediately at the beginning, since the waves of constant momentum would leave behind an excess of energy. But when from the very beginning waves that have partly a positive and partly a negative difference of momentum and of energy are successively produced on the quiet water, then the sum of these differences can become zero. These systems of waves, having different wave-lengths and progressive velocities, cause manifold interferences as they progress, and, according to the principle given by me for combination-tones (which in its application to the tidal wave has already received a very beautiful confirmation by Sir William Thomson's analysis of the tidal observations collected by the British Association), waves of greater wave-length can gradually be formed.

So long as the wind outruns the waves it steadily increases the store of energy and the momentum of the waves, and furthermore, so long as the energies computed for stationary waves diminish and can form a still lower minimum, the inclination to attain the form of least energy under the coöperation of all the small perturbations which the other concurrent waves bring about, in the case of nature, will develop still further. This will finally lead to the value corresponding to the formation of a cusp and to the foaming of the upper ridge in case this can be produced by the given wind velocity.

In April of this year [1890] I endeavored by observations that I instituted at the Cape of Antibes [near Marseilles] to arrive at some confirmation of these consequences drawn from theory. With a small portable anemometer I measured the strength of the wind directly at the edge of the steep cliff of the narrow tongue of land which projects rather far into the sea. However, the observations showed that many times a stronger wind must have prevailed out on the sea than I had been able to observe on shore. I also counted the number of approaching billows.

With water-waves the same as with sound-waves it is to be assumed that, through all deviations, delays, and diminutions that they experience, the time of vibration remains unchanged. This time may therefore be determined near the shore even though the progressive velocity in shallow water is changed and the form and the length of the waves change. The number, N, of the waves in a minute is expressed by

$$N = \frac{60.a_2}{\lambda}.$$

When a_2 increases to na_2 then λ increases to $n^2\lambda$, as shown in my paper of a year ago, and therefore

$$N_n = \frac{N}{n}.$$

A velocity $a_2=10$ metres would give 9.4 waves per minute; on the other hand a velocity $a_2=5$ would give 18.8.

The counting of the waves without registering instruments is now not to be executed with great accuracy, since on the sea, so far as I have seen it, there are always numerous adjacent waves of rather different periodic times which interfere and give phenomena corresponding to the acoustic beats. During the minimum of motion one can easily make errors in the counting; by repeated countings at the same place we obtain therefore variations of about one-tenth or even more of the desired number.

The strength of the wind that I observed on the shore did not exceed 6.1 metres per second. This was on the evening of my arrival in Antibes, April 1, 1890; the wind was from east southeast; I counted between 8.5 and 10 waves per minute. On the next morning, April 2, there were still 10 to 10.5 waves per minute, although the wind had almost entirely gone down. This number of waves would be explicable only when a wind about 10 metres per second had blown steadily over the open sea. On the 2d of April the wind rose in the course of the day to a velocity of only 4 metres per second. Yet on the 3d of April also the number of waves was still 9.5 with a very feeble wind; on the 4th of April for the first time an increase was perceptible up to 12.3 waves per minute.

During a series of quiet days the number of steadily diminishing waves gradually increased to 17 or 18. Finally on the 7th of April the wind began again to increase. In the morning I found a velocity of 3.3 metres per second, which in the course of the day increased to 5.5 and brought the number of waves down to 11.5. This time, however, the location of the increased wind was demonstrable. In Marseilles during the previous night a severe whirlwind had prevailed and the larger waves excited by it stretched as a sharply defined dark-gray band from the sea horizon hitherward and reached Cape Antibes about midday, long before the stronger wind that had given rise to them and which had morever at the latter place by no means the same force as in Marseilles.

These few observations therefore show a connection between the number of waves per minute and the strength of the wind and even an agreement, at least in the order of magnitude. But the numbers of waves are all somewhat smaller than they should be as computed from the strength of the wind on shore and leave us to conclude that a stronger wind must have prevailed in the open sea. They show however also that the re-action of a strong wind may last many days.

For a progressive velocity of 10 metres the waves would in one day travel $7\frac{3}{4}$ degrees of longitude. Therefore, had the Mediterranean even to the Gulf of Sidra been on the 1st of April covered with waves excited by a strong breeze of 10 metres velocity, these would need two and a half days before the last ones would reach the coast of southern France.

It will of course be possible to solve the problem more thoroughly only when we have at hand continuous registers of the billows and extended observations of the velocity of the wind. These latter are unfortunately not yet collected for the month of April of this year, or at least not yet published, and could therefore not be used by me.

VIII.

THE THEORY OF FREE LIQUID JETS.*

By Prof. G. KIRCHHOFF.

Helmholtz in his communication on discontinuous motions in liquids, *Berlin, Monats-berichte*, April, 1868,† has for the first time determined the form of a free jet of liquid in a special case. The method used by him in this determination can, as will here be shown, be so generalized that it leads to the solution of the same problem for a large number of cases.

It is assumed that the fluid is incompressible, that no exterior forces act upon it, that its particles do not rotate, that the currents are steady, and finally, that the movement is everywhere parallel to a fixed plane.

Let x and y be the rectangular coördinates of any point of the space occupied by the flowing liquid reckoned parallel to the fixed plane and let φ be the velocity potential at this point, then φ is a function of x and y such that it satisfies the equation

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0$$

In this equation $\frac{\partial\varphi}{\partial x}$ and $\frac{\partial\varphi}{\partial y}$ are the velocities parallel to the axes of x and y and if p is the pressure and ρ is the density, then we have further

$$p=c-\frac{\rho}{2}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right],$$

where c indicates a constant. If the flowing liquid has a free boundary then this must correspond to a stream line and the pressure must be constant throughout it. The second of these conditions, if we adopt a proper system of units, will be expressed by the equation

$$\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2=1.$$

* From Borchardt's *Journal*, 1869, vol. LXX, or Kirchhoff *Gesammelte Abhandlungen*, Leipsic, 1882, pp. 416–427.

† [See also No. III of this present collection of Translations.]

The partial differential equation for φ is satisfied if we have

$$z=x+iy \qquad \omega=\varphi+i\psi,$$

where $i=\sqrt{-1}$, and ω can be any function of z. Therefore the equation of any curve of flow or stream line is ψ=constant, and we have

$$\frac{\partial\varphi}{\partial x}=\frac{\frac{\partial x}{\partial\varphi}}{\left(\frac{\partial x}{\partial\varphi}\right)^2+\left(\frac{\partial y}{\partial\varphi}\right)^2}$$

$$\frac{\partial\varphi}{\partial y}=\frac{\frac{\partial y}{\partial\varphi}}{\left(\frac{\partial x}{\partial\varphi}\right)^2+\left(\frac{\partial y}{\partial\varphi}\right)^2}$$

$$\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2=\frac{1}{\left(\frac{\partial x}{\partial\varphi}\right)^2+\left(\frac{\partial y}{\partial\varphi}\right)^2}$$

if we assume that x and y on the right-hand side of these equations can be represented as functions of φ and ψ. Therefore the conditions for a free boundary of the jet are that for it ψ=constant, and

$$\left(\frac{\partial x}{\partial\varphi}\right)^2+\left(\frac{\partial y}{\partial\varphi}\right)^2=1.$$

The problem is therefore to express ω as such a function of z as will satisfy these conditions.

To this end we put

$$\frac{dz}{d\omega}=f(\omega)+\sqrt{f(\omega)f(\omega)-1}$$

and select the function $f(\omega)$ so that it is real for a certain value of ψ and for a certain range of φ, and so that it lies between the limits -1 and $+1$. For this value of ψ and for this range of φ we have

$$\frac{\partial x}{\partial\varphi}=f(\omega), \qquad \frac{\partial y}{\partial\varphi}=\sqrt{1-f(\omega)f(\omega)}$$

whence

$$\left(\frac{\partial x}{\partial\varphi}\right)^2+\left(\frac{\partial y}{\partial\varphi}\right)^2=1;$$

that is to say, the stream line corresponding to the value of ψ can form a free boundary to the moving liquid in that portion which corresponds to the range of φ. If there are many values of ψ for which $f(\omega)$ has the described property then all the stream lines that correspond to these values can be free boundaries.

In general ω is defined by the equation above given for

$$\frac{dz}{d\omega}$$

as a many-valued function of z for any definite assumption as to $f(\omega)$. Let the region of z, that is to say the space filled with the moving liquid,

be so bounded that, within it, no branch of ω merges into another; such a branch, therefore, represents a possible mode of fluid motion. The desired object will be attained when the region of ω is appropriately bounded.

In reference to the boundary of the region of ω it is recognized, first, that it is a line that returns into itself and without cutting itself and that consists of parts for which ψ has a constant value and of parts for which φ has an indefinitely large positive or an indefinitely large negative value.

Within the region of ω, $f(\omega)$ is a single-valued function of ω. If we had adopted an expression for $f(\omega)$ that represented a many-valued function, then at its cusp point should start the sections for which ψ has a constant value.

Furthermore $\sqrt{f(\omega)f(\omega)-1}$ should also be made a single-valued function of ω, in that through those points for which $f(\omega)=\pm 1$, the sections pass for which ψ has a constant value. For any point of the region of ω the sign of the radical quantity is still at our disposal. If points occur for which $f(\omega)$ is infinite or infinitely great,* then for one of these points we may make

$$\sqrt{f(\omega)f(\omega)-1}=+f(\omega)$$

and assume that this equation holds good for them all.

It is further assumed that the function $f(\omega)$ is only infinite at its cusp points if it is so anywhere, and even here it is infinite only in such a way that if $f(\omega_0)$ is infinite then $(\omega-\omega_0)f(\omega)$ approximates to zero when ω has a value approximating that of ω_0.

Within the designated region of ω therefore z is a single-valued function of this variable and such that it is never infinite.

Now consider ω as a function of z. The region of z that corresponds to the adopted region of ω does not extend through infinity, and is bounded by a line that returns into itself and which is made up of the lines whose equations are $\varphi=-\infty$ and $\varphi=+\infty$ and of stream lines; a certain portion of the latter can be considered as a free boundary of the moving fluid, the other part can be considered as a fixed wall. Within this region of z, ω has no cusp point, since at no point of it does $\frac{dz}{d\omega}$ become zero. Therefore under the condition that the boundary of the region of z shall not intersect itself, ω becomes within that region a single-valued function of z.

This function of z is completely determined as soon as one has found a single value of z corresponding to a given value of ω.

(I.) An example that constitutes a generalization of the case treated of by Helmholtz is obtained if we put

$$f(\omega)=k+e^{-\omega}$$

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* By infinite, I designate the reciprocal of zero, but by infinitely great, the reciprocal of an infinitely small quantity.

where, as also in the following examples, k indicates a positive real fraction, and where the region of ω is bounded by the lines

$$\psi=0,\ \varphi=-\infty\ ;$$
$$\psi=\pi,\ \varphi=+\infty\ .$$

The expression adopted for $f(\omega)$ is single value. The multiple points of $\sqrt{f(\omega)f(\omega)-1}$ that do not lie outside the region of ω are the points

$$\varphi=-\log\,(1-k),\ \psi=0;$$
$$\varphi=-\log\,(1+k),\ \psi=\pi.$$

These lie in the boundary of this region, and, therefore, it need not be further bounded by sections.

The equations of the boundary of the region of ω are also the equations of the boundary of the region of z. If we assume that for $\varphi=-\log\,(1+k)$ and $\psi=\pi$ we have $x=0$ and $y=0$, then these equations when developed become the following

For $\psi=\pi$ and $\varphi<-\log\,(1+k)$ there results

$$y=0 \text{ and } x=\int_{-\log\,(1+k)}^{\phi}(k-e^{-\phi}-\sqrt{(k-e^{-\phi})^2-1})\,d\,\varphi$$

where the root (as also hereafter every root of a positive quantity), is taken to be positive. By these equations the positive half of the axis of x is represented; this is to be taken as a fixed wall; at the initial point of coördinates it merges into the free boundary. For this free boundary, namely, for $\psi=\pi$ and $\varphi>-\log\,(1+k)$ we have

$$x=\int_{-\log\,(1+k)}^{\phi}(k-e^{-\phi})d\,\varphi$$

$$y=-\int_{-\log\,(1+k)}^{\phi}\sqrt{1-(k-e^{-\phi})^2}\,d\varphi$$

Furthermore for $\psi=0$ and $\varphi<-\log\,(1-k)$ we have

$$x=\int_{-\log\,(1-k)}^{\phi}\left(k+e^{-\phi}+\sqrt{(k+e^{-\phi})^2-1}\right)d\varphi+a$$

$$y=b$$

and for $\psi=0$ and $\varphi>-\log\,(1-k)$

$$x=\int_{-\log\,(1-k)}^{\phi}(k+e^{-\phi})\,d\varphi+a$$

$$y=-\int_{-\log\,(1-k)}^{\phi}\sqrt{1-(k+e^{-\phi})^2}\,d\varphi+b$$

where

$$a=k\log\frac{1+k}{1-k}-2-\pi\sqrt{1-k^2}$$
$$b=-2\,\pi\,k$$

The first part of the stream line $\psi=0$ which is a straight line parallel to the axis of x and extending to the point $x=a$, $y=b$, is to be considered as a fixed wall; the second part is to be considered as the free boundary of the outflowing jet.

The approximate course of the lines $\psi=\pi$ and $\psi=0$ is shown in Fig. 4.

The completion of the boundary of the region of z is formed by the line $\varphi=-\infty$, namely,

$$x=2\,k\,\varphi-2e^{-\varphi}\cos\psi+a_1$$

$$y=2\,k\,\psi+2e^{-\phi}\sin\psi+b_1$$

and by the line, $\varphi=+\infty$, namely,

$$x=k\,\varphi+\sqrt{1-k^2}\,.\psi+a_2$$

$$y=k\,\psi-\sqrt{1-k^2}\,.\varphi+b_2$$

where a_1, b_1, a_2, b_2 are constants whose values are easily obtainable and which are partly used in the computation of a and b. The first of these two lines can be defined as a half circle that is described with an infinitely large radius about the origin of coördinates; the second is a straight line that is perpendicular to the jet at an infinitely great distance from the origin; at this distance the jet forms an angle with the positive axis of x whose cosine equals k.

Fig. 4.

If we assume that k equals 1 then a becomes infinite and the point (a, b) removes to infinity; the region of ω can in this case be bounded by the lines $\psi=\pi$ and $\psi=-\pi$ instead of by the lines $\psi=\pi$ and $\psi=0$; thus we come to the case treated of by Helmholtz and illustrated by Fig. 5.

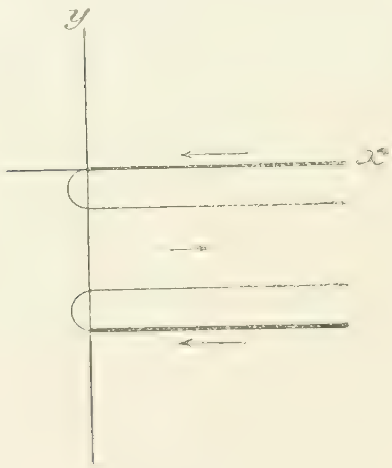

Fig. 5.

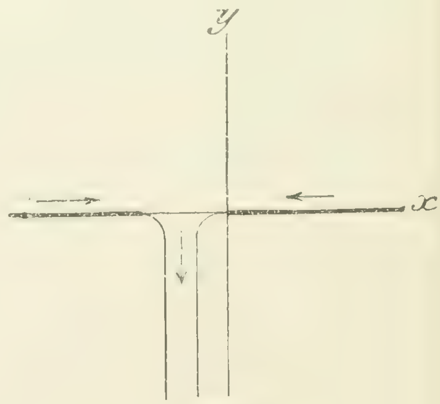

Fig. 6.

If we make k equal zero then will b equal zero; in this case the boundary of the moving fluid is represented by Fig. 6.

(II.) As a second example the case where

$$f(\omega)=k+\frac{1}{\sqrt{\omega}}$$

will be treated and the region of ω stretches indefinitely far in all directions.

In order to make $f(\omega)$ a single-valued function we draw a section from the point $\omega=0$, for which section $\psi=0$ and $\varphi>0$ and assume that for $\varphi=+0$ and $\psi=+0$ the real part of $\sqrt{\omega}$ is positive. The cusp points of the curve $\sqrt{f(\omega)f(\omega)-1}$ are the points for which $\omega=0$, $\frac{1}{\sqrt{\omega}}=1-k$, $\frac{1}{\sqrt{\omega}}=-(1+k)$; therefore they all lie on the section already drawn therefore do not require the making of a new section. As concerns the sign of $\sqrt{f(\omega)f(\omega)-1}$ it must be so determined according to the adopted rules that the real part of this radical quantity shall be positive for $\varphi=+0$, and $\psi=+0$. Finally it is assumed that ω and z disappear simultaneously.

The line for which $\psi=0$, and $\varphi>0$, is the boundary of the region of z. This line is composed of many parts which are to be distinguished from each other. For $\psi=+0$, and $0<\varphi<\frac{1}{(1-k)^2}$ we have

$$x=\int_0^\phi\left(k+\frac{1}{\sqrt{\varphi}}-\sqrt{\left(k+\frac{1}{\sqrt{\varphi}}\right)^2-1}\right)d\varphi$$

$$y=0.$$

Then again for $\psi=-0$, and $0<\varphi<\frac{1}{(1-k)^2}$

$$x=\int_0^\phi\left(k-\frac{1}{\sqrt{\varphi}}-\sqrt{\left(k-\frac{1}{\sqrt{\varphi}}\right)^2-1}\right)d\varphi$$

$$y=0.$$

These equations represent a part of the axis of x which is to be adopted as the fixed wall. If we use the relation

$$\int\sqrt{\left(k+\frac{1}{\sqrt{\varphi}}\right)^2-1}\times d\varphi=$$

$$\frac{(1-k^2)\sqrt{\varphi}-k}{1-k^2}\sqrt{(k\sqrt{\varphi}+1)^2-\varphi}+\frac{1}{(1-k^2)^{\frac{3}{2}}}\text{ arc sin}\left((1-c^2)\sqrt{\varphi}-k\right)$$

we find for the end of this part (of the axis of x) the expression

$$x=2\frac{1+k-k^2}{(1-k)(1-k^2)}+\frac{1}{(1-k^2)^{\frac{3}{2}}}\left(\frac{\pi}{2}+\text{ arc sin }k\right)$$

and

$$x=-2\frac{1-k-k^2}{(1+k)(1-k^2)}-\frac{1}{(1-k^2)^{\frac{3}{2}}}\left(\frac{\pi}{2}-\text{ arc sin }k\right)$$

where the arc whose sine is k is to be taken between zero and $\frac{\pi}{2}$.

For $\psi=+0$ and $\varphi>\frac{1}{(1-k)^2}$ we have

$$\frac{dx}{d\varphi}=k+\frac{1}{\sqrt{\varphi}};\ \frac{dy}{d\varphi}=-\sqrt{1-\left(k+\frac{1}{\sqrt{\varphi}}\right)^2}$$

and for

$$\psi=-0 \text{ and } \varphi>\frac{1}{(1+k)^2} \text{ we have}$$

$$\frac{dx}{d\varphi}=k-\frac{1}{\sqrt{\varphi}};\ \frac{dy}{d\varphi}=-\sqrt{1-\left(k-\frac{1}{\sqrt{\varphi}}\right)^2}$$

The lines that are represented by the integrals of these equations, when we determine the constants of integration so that these lines start from the previously indicated termini of the fixed walls, are the free boundaries of the moving liquid. The other boundaries of the region of z lie at infinite distances, as is seen from the fact that when $\omega=\infty$ we have

$$\frac{dz}{d\omega}=k-i\sqrt{1-k^2}$$

this equation shows at once that at an infinitely great distance from the origin of coördinates the flow takes place with the velocity 1 in a direction that forms an angle with the axis of x whose cosine is k. Figure 7 illustrates the boundary of the region of z; besides this boundary the figure also gives the stream line for which $\psi=0$, and $\varphi<0$.

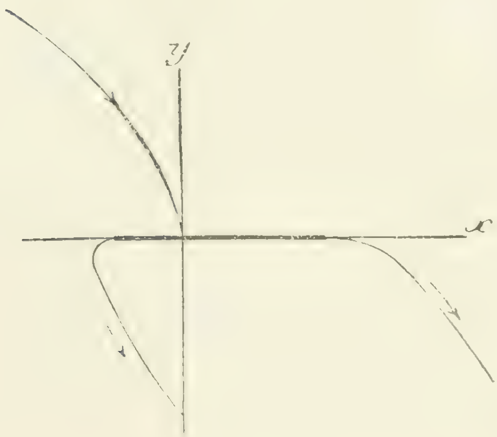

Fig. 7.

(III.) Still one more example may be introduced. Let there be

$$f(\omega)=\frac{k}{\sqrt{1-e^{-\omega}}}$$

and let ψ vary between $-\pi$ and $+\pi$, but φ between $-\infty$ and $+\infty$.

From the point $\omega=0$ draw a section for which $\psi=0$, and $\varphi>0$, and assume that for $\varphi=+0$, and $\psi=+0$, the real part of $f(\omega)$ is positive. The points of bifurcation of $\sqrt{f(\omega)f(\omega)-1}$ are the two points $\omega=0$, and $\omega=-\log(1-k^2)$ both which are found upon the section that has been drawn. The sign of the radical quantity $\sqrt{f(\omega)f(\omega)-1}$ is determined by the rule that its real part shall be positive for $\varphi=+0$, and $\psi=+0$.

Finally we assume that ω and z disappear simultaneously.

At the boundary of the region of z we have, first the line for which $\psi=0$, and $\varphi>0$. This line is composed of the following portions:

For $\psi=+0$, and $0<\varphi<-\log(1-k^2)$ we have,

$$x=\int_0^\phi\left(\frac{k}{\sqrt{1-e^{-\phi}}}+\sqrt{\frac{k^2}{1-e^{-\phi}}-1}\right)d\varphi$$

$$y=0$$

For $\psi=-0$ and $0<\varphi<-\log(1-k^2)$ we have,

$$x=-\int_0^\phi\left(\frac{k}{\sqrt{1-e^{-\phi}}}+\sqrt{\frac{k^2}{1-e^{-\phi}}-1}\right)d\varphi$$

$$y=0$$

These equations represent a portion of the axis of x that is to be assumed as the fixed partition. Adjoining this fixed partition there comes as the free boundary of the moving fluid the line for which

$$\psi=+0,\ \varphi>-\log(1-k^2),$$

therefore

$$\frac{dx}{d\varphi}=\frac{h}{\sqrt{1-e^{-\phi}}};\ \frac{dy}{d\varphi}=-\sqrt{1-\frac{k^2}{1-e^{-\phi}}}\ :$$

and also the line for which

$$\psi=-0,\quad \varphi>-\log(1-k^2),$$

whence

$$\frac{dx}{d\varphi}=-\frac{k}{\sqrt{1-e^{-\phi}}};\ \frac{dy}{d\varphi}=-\sqrt{1-\frac{k^2}{1-e^{-\phi}}}\ .$$

The remaining boundaries of the region of z are the lines

$$\psi=-\pi,\quad \psi=+\pi,\quad \varphi=-\infty,\quad \varphi=+\infty.$$

For $\psi=-\pi$ we have,

$$\frac{dx}{d\varphi}=-\frac{k}{\sqrt{1+e^{-\phi}}};\ \frac{dy}{d\varphi}=-\sqrt{1-\frac{k^2}{1+e^{-\phi}}}\ .$$

For $\psi=+\pi$ we have,

$$\frac{dx}{d\varphi}=\frac{k}{1+e^{-\phi}};\ \frac{dy}{d\varphi}=-\sqrt{1-\frac{k^2}{1+e^{-\phi}}}\ ;$$

These two stream lines are free boundaries throughout their whole extent.

For $\varphi=-\infty$, we have $\dfrac{dz}{d\omega}=-i$;

for $\varphi=+\infty$, and $\psi<0$, we have $\dfrac{dz}{d\omega}=-k-i\sqrt{1-k^2}$

and for $\varphi=+\infty$, and $\psi>0$, we have $\dfrac{dz}{d\omega}=k-i\sqrt{1-k^2}$

For $\varphi = -\infty$, we therefore have $y = +\infty$, and the stream flows with a velocity of 1 in the direction of the negative axis of y; for $\varphi = +\infty$, we have $x = \mp\infty$, and $y = -\infty$, and the stream flows with a velocity of 1 in a direction that makes an angle whose cosine is $\mp k$ with the direction of the positive axis of x.

In Fig. 8 the boundaries of the moving fluid are represented for this case.

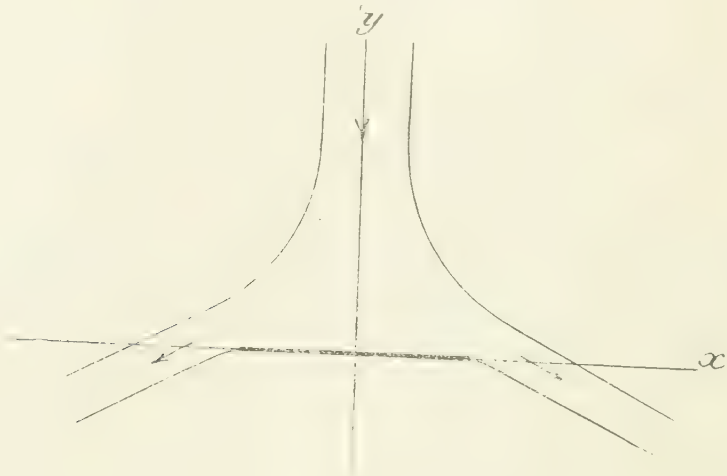

Fig. 8.

IX.

ON DISCONTINUOUS MOTIONS IN LIQUIDS.*

By Prof. A. OBERBECK.

1.

It is customary to designate by the term discontinuous fluid motions, those phenomena of movement in which the velocity is not throughout the whole space filled with the fluid a continuous function of the location. Therefore in such movements there occur surfaces within the fluid that separate from each other regions within which the velocities differ from each other by finite quantities. The fundamental principles of the theory of these motions were first given by Helmholtz.†

If we assume that a velocity potential (φ) does exist for so called steady fluid motions then the hydro-dynamic differential equations can be summarized in the one equation,

$$p=C-\tfrac{1}{2}\left\{\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2+\left(\frac{\partial\varphi}{\partial z}\right)^2\right\}$$

Now Helmholtz has shown that the pressure p and consequently the velocity can be discontinuous functions of the coordinates and that there are a great number of phenomena of motion for which the assumption of a discontinuous function is necessary. Especially has this theory been applied by Helmholtz and by Kirchhoff to fluid jets,‡ and the boundaries of free jets can be given under the following assumptions:

(a) That no accelerating force acts upon the fluid.

(b) That the movement is steady.

(c) That the movement depends only upon two variables, x and y, and is therefore everywhere parallel to a fixed plane.

If in other cases, for instance for jets that are symmetrical about an axis or that are under the influence of the accelerating force of gravity,

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* Read at the session of the Physical Society in Berlin, May 11, 1877. Translated from Wiedemann's *Annalen der Physik und Chemie*, 1877, vol. II, p. 1–16.

† See the *Berlin Monatsberichte*, 1868, p. 215 [or No. II of this series of Translations.]

‡ See Crelle's *Journal* vol. LXX, p. 289–299, [and Nos. III and VIII of this collection of Translations.]

it is not yet possible to determine the free boundaries by computation, then this is only because of the analytical difficulties. In general, however, one can judge of the nature of these boundaries from a consideration of the results already found.

The mathematical investigations just referred to hold good equally well for liquid jets that are bounded by quiet air as for those that are bounded by similar quiet liquid. In the actual production of such liquid jets it of course makes a great difference whether we allow water to flow into the air or water to flow into water. In both cases disturbing circumstances occur of which the mathematical theory takes no consideration. The jets of water projected freely into the air have been most thoroughly investigated.*

In these experiments the formation of jets occurs just as would be expected according to theory. On the other hand, however, it is known that water jets are influenced to an important extent by the capillary tension of the free surface, and that in consequence of this at certain distances from the orifice they break up into drops.

If we allow a liquid to flow into a similar quiet liquid then these capillary effects do not occur; but in place of this another disturbing cause, the viscosity, influences the phenomena. The viscosity has hitherto not been taken into consideration in the theory of the discontinuous movements of fluids. If we attempt to consider it we stumble upon a peculiar difficulty that has led the present author to experimentally investigate this class of fluid motions.

II.

It is well known the theory of viscosity of fluids can be developed from the assumption first framed by Newton,† namely, that the retarding or accelerating influence of two portions of fluid flowing past each other with different velocities is proportional to their relative velocity. Especially has O. E. Meyer from this hypothesis developed the general differential equations for the motion of fluids.‡

If we assume that all parts of the moving fluid describe parallel paths, say in the direction of the axis of y, and that the velocities v are only functions of x and that finally μ is the coefficient of viscosity, then will the influence of two neighboring parts upon each other be represented by the expression

$$\pm \mu \frac{dv}{dx}.$$

* Besides the older experiments of Bidone and Savart see especially Magnus, Poggendorff *Annalen*, vols. XCV and CVI.

† Mathematical Principles of Natural Philosophy: German translation by Wolfers, Berlin, 1872, p. 368.

‡ See Crelle, *Journal*, vol. LIX, pp. 229–303, and Poggendorff *Annalen*, vol. CXIII, pp. 68, 69.

If v is a discontinuous function of x, then at such a locality the differential quotient will be indefinitely large. Therefore two neighboring portions would exert an indefinitely great influence upon each other. If therefore one of the fluid portions is at rest while a neighboring portion that belongs to the jet flows by the first with a constant velocity communicated to it by some exterior influence, then the first or quiet particle must immediately begin to take part in the movement of the second, but the second on the other hand must begin to lose a definite fractional part of its velocity. The jet must therefore rapidly set the surrounding quiet fluid in motion with it. It would according to this appear to be doubtful whether sharply defined jets such as are demanded by the above-mentioned theory of Helmholtz could be formed in a fluid subject to viscosity.

The few experiments made hitherto upon this question appear to confirm this suspicion. Especially notable is an investigation by Magnus (Poggendorff *Annalen*, LXXX, pp. 1–40), who allowed pure water to flow from a cylindrical opening into a weak solution of salt and by means of a glass tube drawn out into a fine point, led away a small quantity of the inflowing water in the neighborhood of the opening. The liquid thus caught was examined as to its salinity. From the latter one could calculate to what extent the inflowing liquid had become mixed with that which was previously in the vessel. It resulted that pure water could not be caught at any point of the inflowing liquid; that therefore everywhere the original quiet liquid was carried along with the moving liquid.

The analogous case of jets of air and of smoke, as also that of the free jets of water in the air, demonstrates that in all these, we have to do with phenonena of very slight stability. It is well known how sensitive such jets frequently are with respect to the feeble periodic disturbances produced by waves of sound.*

It seemed to me therefore of interest to investigate more accurately the formation of water jets in water and therein to utilize a method that allows of following the course of the phenomena of motion better than was possible in the experiments of Magnus. This object is most simply attained in that we allow feebly tinted water to flow into colorless water. Fuchsin is used as coloring material. It is well known that with a very small quantity of this material an intense red color is produced with no fear lest hereby the specific gravity of the water be essentially changed. In the first experiments performed with this it resulted that the jet of colored liquid broke up at a very slight distance from the orifice into reddish clouds and drops that mixed with the quiet liquid and carried it along with them. By further investigation however, it became possible to determine conditions under which real jets of considerable length and sharp boundaries were formed. These were of great sta-

* See John Tyndall on Sound, pp. 289–292 of the German edition edited by Helmholtz and Wiedemann, Brunswick, 1869.

bility, so that small disturbances had only a rapidly diminishing influence upon their course. At the forward end of these jets there formed peculiar surfaces of flow that plainly allowed the influence of viscosity to be seen. These phenomena of motion are of remarkable beauty and delicacy, of which any one may convince himself who performs the easily repeated experiments.

Since the theoretical investigations mentioned in the introduction treat of the modifications of jets by solid bodies, and Kirchhoff especially gives a series of interesting examples bearing upon this, therefore this question has also been taken into consideration in my experiments. Very stable forms of jets are also thus formed that have more similarity with those deduced by theory than one could have expected.

III.

The experiments were made with the following simple apparatus: A cylindrical glass vessel (Fig. 9), of about 60 centimetres height and 12 centimetres diameter, was filled with water. Into this there passed a flow of water from a filter through an India-rubber tube, a glass stop-cock, and a glass tube. The filter, as also the entire tubular system, was filled with the colored liquid. After filling with water the glass cylinder (in whose place one may also use any large glass vessel), one must wait a long time until the motion of the water has been destroyed by viscosity. The experiment succeeds best when the water has stood for many hours in the cylinder, since then currents resulting from differences of temperature are no longer present. By a quick opening of the glass stop-cock one can allow a definite quantity of colored liquid to enter into the quiet liquid, or by a longer opening one can attain a steady stationary current. By elevating or depressing the filter one can easily regulate the height of the upper fluid level. The use of a small difference of pressure was found to be the principal condition for the maintenance of regular current formations.

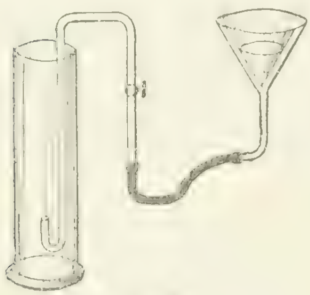

Fig. 9.

The majority of the experiments if no other problem was on hand were executed with an excess of pressure of about 20 millimeters. By means of proper arrangements solid bodies could be opposed above the jet. For exact observation it is necessary to fasten a surface of white paper behind the glass cylinder.

IV.

In order to understand the formation of jets it is advantageous first to learn the behavior of a definite quantity of liquid entering under

a small excess of pressure into the quiescent liquid. I therefore begin with a description of the experiments relative to this.

If we allow the stop-cock to be opened for only a short time, then even with the smallest differences of pressure of two or three millimetres, a sharply defined mass of liquid penetrates into the quiescent liquid. The original form of this mass is soon modified by viscosity and by the participation of the hitherto quiet liquid in its motion, in a peculiar manner, and finally it rolls itself into a ring. The colored mass of liquid goes through the series of forms presented in Figs. 10, 11, 12, and 13. Of these drawings, as of most of the following ones, it is to be noted that they represent a section of the mass of liquid by a plane that passes through the axis of symmetry of the formation. In order to find the true form therefore, one must imagine the figure revolved about this axis.

Fig. 10.

Fig. 11.

Fig. 12.

Fig. 13.

With the form of Fig. 13, the ring formation is completed. Moreover in general even for differences of pressure of 10 to 20 millimetres, the living force of the liquid was consumed so that this figure long floated motionless in the colorless liquid.

If we use somewhat larger differences of pressure we observe that the liquid within the ring continues rotating for a longer time. The original progressive movement has therefore been transformed into a vortex movement. The vortex movements have been theoretically treated by Helmholtz* and he has in the introduction to his memoir referred to the necessity of the transformation of any current or movement that has a velocity potential into a vortex movement in consequence of viscosity.

Many other consequences drawn by Helmholtz in his memoir just referred to can be easily observed by the help of the apparatus used by the present writer.

*Crelle's *Journal* LV, pp. 25–56, [and No. II of this collection of Translations.]

If by alternately opening and closing the stop-cock we allow two drops to enter into the colorless liquid in rapid succession, then there arises a ring formation for each drop and the following one always catches up with the preceding one. Different cases are then possible, according to the differences of pressure that are used; if these are slight then the second ring is not able to penetrate the first one and a formation, as shown in Fig. 14, remains for a long time visible in the fluid. With greater differences of pressure, on the other hand, ring No. 2 passes through ring No. 1, since the former contracts while the latter expands. One can then observe that afterwards ring No. 1 endeavors on its part to pass through ring No. 2. But generally the living force is by this time consumed, so that ordinarily the two rings settle into the formation shown in Fig. 15. This interchanging passage

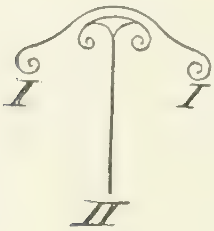

Fig. 14.

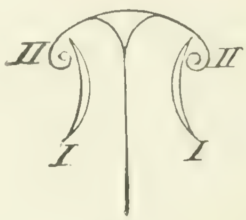

Fig. 15.

of the vortex rings through each other was predicted by Helmholtz from theory in the memoir above referred to.

Reusch has occupied himself experimentally with the formation of vortex rings.* After having described in detail the formation of smoke rings in the air, he passes to the formation of rings by the sudden entrance of a small quantity of colored liquid into colorless liquid. Although in his arrangement of the experiments the transition of the progressive into the vortex motion is very rapidly completed, still he also has frequently observed the intermediate stages shown in Figs. 11 and 12 and described them very appropriately as mushroom formations. The manner of this transition is seen directly from the examination of Figs. 10 to 13. Evidently there arise two currents in the quiescent liquid. The one current, indicated by the arrows A and B, is produced by the progressive movement of the drop, which moves forward in the liquid almost as a solid body. The other current, in the direction of the arrows C and D, is principally produced by viscosity. The formation of the spiral surface of rotation is finally the necessary consequence of these two opposite currents.

* Poggendorff's *Annalen*, vol. CX, pp. 309–316.

V.

We can now pass on to the formation of jets proper by steady currents. If we allow the stop-cock to be open for a long time there arises (at first rapidly, afterward slowly) a jet whose upper portion has great similarity with the forms hitherto described. The jet soon attains a certain altitude that depends upon the difference of pressure and above which it does not ordinarily go, or at least only with extreme slowness. Thus for a difference of pressure of 5 millimetres the altitude of the jet is about 20 millimetres; for 10 millimetres pressure the altitude is about 80 millimetres; for 20 millimetres pressure the altitude is 200 millimetres; and for 30 millimetres difference of pressure the jet attains the upper limit of the water at an altitude of about 400 millimetres in about 80 seconds. The colored liquid spreads out over the surface of the colorless water and thence diffuses very slowly downward. The above given numbers do not present any general law, but only give approximately the connection between the altitude of the jet and the difference of pressure. The former also depends somewhat on the specific gravity of the inflowing liquid, which varies a little with the quantity of added coloring material. It depends also on the size of the discharging aperture. Moreover the form of the front part of the jet is not always exactly the same; in the figs. 16 and 17 are given two of the ordinary forms of jet. In both these forms the jets proper are the same; the bell-shaped expansion, however, is formed in a somewhat different manner, perhaps conditioned by small variations of temperature in the colorless liquid.

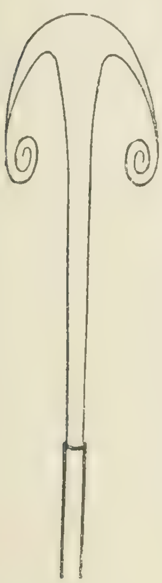

Fig. 16.

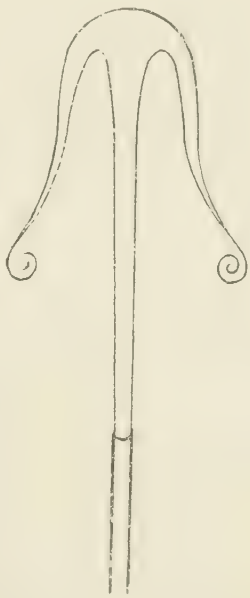

Fig. 17.

By the avoidance of all disturbances the jets here described remain many minutes entirely unchanged. Only the bell-shaped portion continues to extend slowly somewhat further downwards. Moreover, with respect to small disturbances the jets showed themselves by no means very sensitive. If by a gentle pressure on the India-rubber tube the velocity of the discharging liquid is diminished for an instant, then water presses from all sides into the jet; after the cessation of this pressure the original form of the jet is immediately resumed. Even

when the pressure on the rubber tube is periodically increased and diminished for a long time the continuity of the jet is not completely broken. Such a jet presents a very remarkable appearance, which is reproduced as well as possible in Fig. 18.

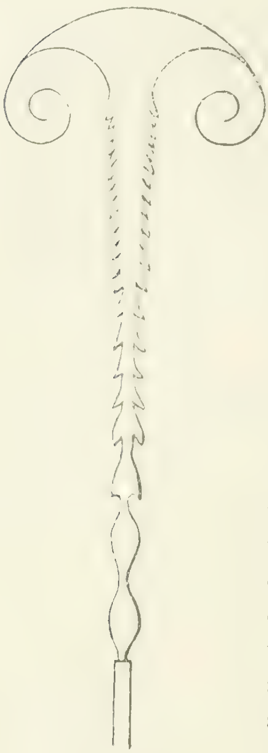

Fig. 18.

The phenomena hitherto described occur with differences of pressure of 60 millimetres at the maximum, but very different results are obtained if larger differences of pressure are used. With 80 or 90 millimetres we obtain jets of the greatest sensitiveness. By every small disturbance the continuity of the jet is broken, and it must then form for itself a new path every time. Above 100 millimetres difference of pressure there are formed only very short jets in the immediate neighborhood of the opening. These at a slight altitude break up into a cloud of individual small drops that under the rapid motion immediately mix with the colorless liquid.

When (as an experiment) colored liquids were used whose specific gravity differed considerably from that of the colorless water, no regular discontinuous currents could be obtained; thus in one experiment a solution of salt was added to the colored water, in another experiment some alcohol was added. The salt solution immediately after its discharge fell in thick, irregular, heavy drops back upon the discharge pipe, while the alcohol moved in very thin threads, frequently broken up, toward the upper free surface of the water.

From the experiments hitherto described it follows that in fact steady jets form with small differences of pressure. The viscosity therefore does not prevent discontinuous currents. Viscosity appears in general to exert so unimportant an influence upon the cylindrical portion of the jet formation that we are tempted to assume the real possibility of the sliding of moving particles of water past those at rest, as the simpler theory assumes to be the fact, without any consideration of viscosity. If, however, the transition from the finite velocity of the jet to the quiet fluid does not take place within the thickness of a mathematical cylindrical surface but gradually within a layer of a definite thickness then this thickness can only be extraordinarily small and appears not to change with the time. That on the other hand the viscosity plays an important rôle in the origin of the jets is already mentioned above. The principal proof of this consists in the invariable formation of spiral surfaces of rotation into which the jet is transformed. The origin of these assumes that the colorless liquid in the neighborhood of the jet receives a certain velocity in the direction of the jet.

The greater sensitiveness of the jets for large velocities of current, as also the impossibility of forming alcohol jets in water, is a direct consequence from the theory of discontinuous fluid motions. Since the difference of pressure in the moving and the quiet fluid is proportional to the square of the velocity, therefore for greater velocities the quiet liquid presses directly into the jet as soon as a slight disturbance occurs in its uniform course. When finally, with more rapid outflow, such disturbances occur continually, then in general a jet can not form.

VI.

As already remarked above it is of interest to know the path that a jet will describe when it meets a solid body in its path. The bodies used for this purpose by me were of different kinds, and by a simple arrangement were brought in the neighborhood of the discharging aperture before the jet was produced by opening the stopcock. It is of course understood that at the beginning of the experiment one waited a long time until the movements of the liquid caused by these operations had subsided. Equally also was the solid body first freed from the air bubbles that adhered to it.

The processes that occurred are most easily seen by considering the following experiment. If the jet strikes upon the sharp edge of a thin sheet of iron that passes parallel to the direction of the jet and through its axis, then it is cut into two portions which are deviated from the vertical direction of the current. The angle between these side currents and the original direction of the jet becomes smaller little by little. The cause of this phenomenon consists in the fact that not only the solid body but also the fluid attached thereto force the moving fluid into a departure to one side. With currents of longer duration on the other hand a part of the quiet liquid is carried along so that the two upper branches of the jet slowly change their direction of motion and more and more nearly approach the plane of the sheet-iron. Still one can always observe quiet colorless liquid between the moving colored liquid and the sheet-iron. The progress of this phenomenon depends upon the original difference of pressure or correspondingly upon the velocity of the flowing liquid. For a small velocity the current flows as shown in Fig. 19; for greater velocities, on the other hand, the two portions of the jet after a time take the position shown in Fig. 20, where the dotted part of the figure is intended to show the initial direction of the current.

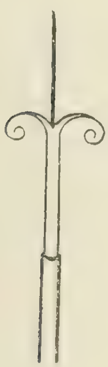
Fig. 19.

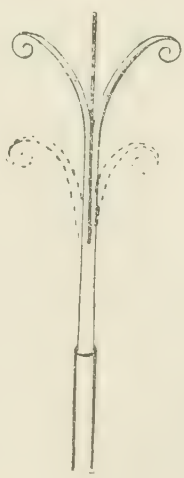
Fig. 20.

The peculiar behavior of the originally adherent quiescent liquid, which is afterward carried along, explains the slow changes in the path of the current that is also observed with other solid bodies of different shapes.

If a jet strikes upon a small brass sphere then with steady flow the stream path gradually takes the form shown in Figs. 21, 22, 23, and 24.

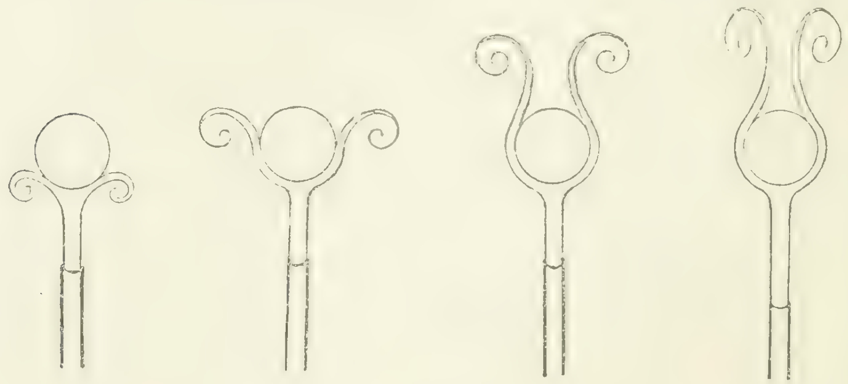

Fig. 21. Fig. 22. Fig. 23. Fig. 24.

We see how at first the sphere and the adherent liquid force the moving liquid to a deviation almost at right angles from its original course. Then gradually the quiescent liquid is carried along, the stream surface follows the surface of the sphere continually more and more closely. A consideration of the thin stream surface that finally encloses the greater part of the sphere tempts one to assume that the moving fluid glides along the surface of the sphere. At least, by means of small solid bodies occasionally occurring in the liquid, one recognizes that in the immediate neighborhood of the fixed obstacle the liquid moves with finite velocity.

The phenomena just described do not appear to depend especially on the substance of the solid body, assuming of course that it is provided with a smooth surface. Instead of the brass sphere an ivory sphere may be used. This is in the same way gradually covered over with a close-fitting stream surface. Similar to this was the process when the jet struck against the lower end of a test tube. With a steady current the lower part of the tube is slowly covered over with a thin stream surface, which at a distance of about 4 centimetres from the lower end of the glass surface bent away and ran into the spirals that here also perpetually recur.

Of further special interest is the case where the jet meets a definite thin partition perpendicular to its own direction, since this current has been theoretically treated by Kirchhoff (Crelle's *Journal*, vol. LXX, p. 298), but under the rather different conditions already mentioned. Therefore, a small circular plate was placed perpendicular to the jet. The stream lines in this case depend essentially on the ratio of the radii of the plate and the jet. If the radius of the circular plate is

materially larger than that of the jet, then the latter will be deviated at the plate through a right angle and flows in a thin layer radially along the plate, which it leaves in a horizontal direction, as in Fig. 25.

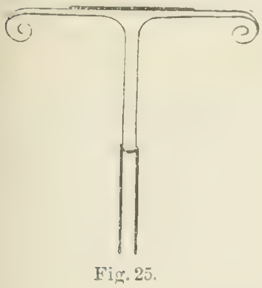
Fig. 25.

If, on the other hand, the radius of the partition is only a little larger than the radius of the jet, then will the stream lines be deviated by a smaller angle from their original direction. This process is shown in Fig. 26, which has a great similarity to the drawing given by Kirchhoff at the place just refered to.*

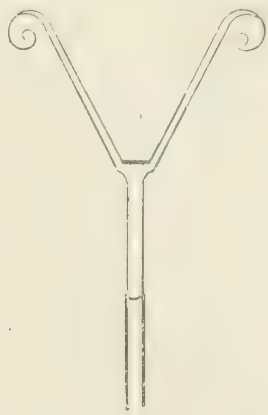
Fig. 36.

A thin sharp-edged partition that extends to about the center of the jet exerts a very similar influence to the thin circular plate. In this case, while one part of the jet spreads in a thin layer along the plate the other part is deviated through an acute angle. In this experiment also the material of the plate appears to exert no sensible influence on the course of the stream. Disks of thin glass and of glazed drawing paper were used, while the above-mentioned thin partition was replaced by a sheet of tinfoil, which was stretched over a glass frame, and one-half of which had been removed along a straight line. The stream phenomena remained exactly the same. The angle by which the jet in this last case was deviated from its initial direction depended principally upon the depth to which the thin partition penetrated into the jet.

The phenomena here described of flow against solid bodies succeeded only for small velocities of the jet such as corresponded to differences of pressure of 20 or 30 millimetres.

VII.

Since it was the main object of the author to investigate discontinuous liquid motions in their simplest form, therefore he has for the present confined himself at first to the above-described experiments. Still these shall be extended as soon as possible in different directions. As the next points for study the following especially commend themselves:

(*a*) The flow of a colored liquid into a colorless one through an opening in a thin partition. Some preliminary experiments with an imperfect apparatus showed that the jets thus formed are similar to the above described under otherwise similar circumstances.

(*b*) The discharge of a liquid into another liquid of equal specific gravity that is not miscible with the first liquid. In this experiment

* [See No. VIII of this collection of Translations.]

one could make use of the liquids employed by Plateau, namely, oil and alcohol of equal specific gravity. The question will here arise, in what manner the formation of a jet is modified by capillary action.

(*c*) A stream of air in moving air, the latter being made visible by smoke.

VIII.

The results of the present investigation can be summarized in the following theorems:

(*a*) The viscosity of fluids does not prevent the formation of steady discontinuous fluid motions. In consequence of friction these motions in the beginning suffer important modifications by reason of simultaneous spiral movements; but with long continued flow they form sharply defined fluid jets.

(*b*) The jets thus formed are very stable for small velocities, and even after small perturbations again immediately assume their original form. For greater velocities, on the other hand, they become very sensitive. If the velocity exceeds a certain limiting value, then only very short jets form in the immediate neighborhood of the opening.

(*c*) The jets are not only modified in their movement by solid bodies but also by the liquid adhering to these. The latter adherent liquid is slowly pushed aside by the jet. If then the body is bounded by a continuously curved surface the flowing liquid surrounds it in a thin layer. If, on the other hand, the solid body is bounded by a surface that at certain points has an indefinitely large curvature, such as a sharp edge, then the stream lines follow it only up to this edge and from that point on leave the solid body.

(*d*) The theory of discontinuous fluid motions, as Helmholtz and Kirchhoff have thus far developed it [for perfect fluids], also gives in general the phenomena observed in a fluid subject to friction. The only difference is the formation of vortex motions simultaneous with origination of the jets.

In conclusion we may call attention to the fact that in nature we find a whole series of processes that have a common origin with those just described. These are to be observed in the currents in rivers and canals, especially at places where the banks have sharp corners or where solid bodies, like the piers of a bridge, retard the uniform movement. The eddying motions there occurring clearly show where the quiet and the moving liquids border on each other. Since as a specially noteworthy result of the investigation here communicated has been to show that discontinuous motions arise even for very small differences of pressure, therefore it is easy to see that they must occur often enough in the last mentioned streams.

X.

THE MOVEMENTS OF THE ATMOSPHERE ON THE EARTH'S SURFACE.*

By A. Oberbeck

I. INTRODUCTION.

The investigations of Guldberg and Mohn† on the motions of the atmosphere certainly occupy a prominent place in the development of theoretical meteorology. If not the first they are at least the most extensive and successful attempt to explain the most important phenomena of the motion of the air by the principles and fundamental equations of hydrodynamics. I would especially indicate as the special service of the authors that they have brought the problem of the motions of the air into a form amenable to mathematical treatment by simple but as I believe thoroughly appropriate assumptions. They themselves have already computed a series of interesting atmospheric movements that frequently occur in nature, especially the cases where the isobaric systems consist of parallel straight lines or concentric circles.

I have attempted in the present work to go further on in the path laid out by Guldberg and Mohn, especially in that I have endeavored to apply to the atmosphere the methods developed in hydrodynamics for other problems.

In the present memoir the steady movements of the atmosphere, or, as Guldberg and Mohn call them, "invariable systems of winds," are principally treated. It is natural to refer the movements of the atmosphere back to the general modes of motion of fluids, that is to say, to motions that are characterized by a velocity potential and to vortex motions. In this way it is possible to attain solutions of great generality that can be applied to any system of isobars whatever. By this method of treatment it is further possible to overcome a difficulty that occurs in the theory of cyclones of Guldberg and Mohn, without as it would appear having been hitherto observed. These investigators distinguish correctly an inner and an outer region for each cyclone, in

* Translated from Wiedemann's *Annalen der Physik und Chemie*, 1882, vol. XVII., pp. 128–148.

† "Studies on the Motions of the Atmosphere." Christiania, Part I., 1876, Part II, 1880.

which the expressions for the velocity of the air at the earth's surface and for the pressure follow different laws. But they have not attempted so to deduce the expressions for the velocity and for the pressure in these two regions from one common principle, that these velocities and pressures merge into each other continuously at the boundary. In the computation of numerical examples they have sought to help over this difficulty by not applying their formula to the region in the neighborhood of the boundary, but have here by interpolation introduced numerical values passably good, but therefore certainly rather arbitrary. Above all however it is a serious matter that according to their theory the direction of the wind at the boundary suddenly varies through a definite angle. The want of continuity here spoken of can originate either in the assumptions adopted as a basis or in the execution of the computation. I have arrived at the conviction that the latter is the case.

I have therefore started from the same assumptions as Guldberg and Mohn; these are given in the following Section (II) and I add only thereto the following principle, about which there can be no doubt: "*The pressure of the air, as also the velocity of the air and its direction, ought to experience only continuous variations throughout the whole region under consideration.*"

By the application of this fundamental principle the theory of cyclones, even in the case of circular isobars, deviates not a little from the theory established by Guldberg and Mohn.

II. ASSUMPTIONS THAT ARE THE BASIS OF THE PRESENT TREATMENT.

The following assumptions form the foundation of my mathematical development:

(*a*) The portion of the earth's surface coming into consideration is assumed to be a plane. A constant average value will be assumed for the geographical latitude of this region.

(*b*) The air will be treated as an incompressible fluid.

(*c*) The investigation here carried out refers only to a stratum of air of moderate height above the earth's surface. The latter surface exerts a retarding influence on the movements of the air that can be considered as a force opposed to the movement and proportional to the velocity.

(*d*) The currents of air at the earth's surface are ordinarily directed toward a center or flow away from the neighborhood of such a center. Such currents can not be imagined without the existence of a vertical current in their neighborhood. If, therefore, we in general confine ourselves to the consideration of horizontal currents, still the consideration of vertical motion is not to be avoided for the neighborhood of such a center. We have, therefore, to distinguish between regions of pure horizontal motion and regions with vertical motion. As to the latter,

the following simple assumption is made:* If we adopt a system of rectangular coördinates such that the plane of $x\ y$ is the horizontal plane while the axis of z is directed vertically upward, then for the vertical component of an ascending current of air we have the expression

$$w=c.z.$$

If the boundary of the region above which this current ascends is known, while outside this boundary the movement is exclusively horizontal, then the whole system of winds (the cyclone) is thereby completely determined. The quantity c can be designated as the constant of the ascending air. For regions with descending air currents the negative sign must be given to the constant.

The region for which

$$w=c.z$$

will, for brevity, be designated as the inner region of the cyclone; that for which

$$w=0$$

will be designated as the outer region.

The vertical component is to be considered only in connection with the equation of continuity. Therefore for the outer region this equation becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

and for the inner region

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=-c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1b).$$

For both regions, moreover, the ordinary equations of hydro-dynamics for movements in one plane hold good, namely:

$$\left.\begin{aligned}\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}&=X-\frac{1}{\rho}\frac{\partial p}{\partial x}\\ \frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}&=Y-\frac{1}{\rho}\frac{\partial p}{\partial y}.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which the letters have the ordinary signification.

The accelerating forces whose components are X and Y must express the influence of the rotation of the earth and of friction.

The consideration of the earth's rotation necessitates the introduction of a force† whose components are

$$X_1=-\lambda v,\ Y_1=+\lambda u.$$

*This agrees entirely with the assumption of Guldberg and Mohn as to the vertical currents. (See *Études*, 1876, part 1, p. 28.)

†See G. Kirchhoff, *Vorlesungen über Mechanik*, Leipzig, 1876, pp. 87–95.

In these we have put

$$\lambda = 2\sigma \sin \beta,$$

wherein σ is the angular velocity of the earth (0.00007292) and β the mean geographical latitude of the region in question. Herein the system of coördinates is to be so taken that the resultant produces in the northern hemisphere a deviation of the path toward the right. Therefore the axis of X is positive toward the east and the axis of Y positive toward the south. For the resistance of friction, according to the adopted assumption (c), we put

$$X_2 = -ku, \; Y_2 = -k\, v.$$

The factor k is dependent on the nature of the earth's surface. It is smaller for the surface of the ocean than for that of the land and is of the same order of magnitude as λ.

By the introduction of these forces in the equations of motion (2) we obtain

$$\left.\begin{aligned} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -ku - \lambda v - \frac{1}{\rho}\frac{\partial p}{\partial x} \\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} &= -kv + \lambda u - \frac{1}{\rho}\frac{\partial p}{\partial y} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (3).$$

If after the addition of $\pm v\, (\partial v \,/\, \partial x)$ to the first equation and of $\pm u\, (\partial u \,/\, \partial y)$ to the second we introduce the double angular velocity ζ in reference to the axis of Z, so that

$$\zeta = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

then equations (3) can be written in the form

$$\left.\begin{aligned} \frac{\partial}{\partial x}\left\{\frac{p}{\rho} + \tfrac{1}{2}(u^2+v^2)\right\} + \frac{\partial u}{\partial t} + ku &= -(\lambda+\zeta)\, v \\ \frac{\partial}{\partial y}\left\{\frac{p}{\rho} + \tfrac{1}{2}(u^2+v^2)\right\} + \frac{\partial v}{\partial t} + kv &= \quad (\lambda+\zeta)\, u \end{aligned}\right\} \quad . \quad . \quad (5).$$

III. DEDUCTIONS FROM THE FUNDAMENTAL EQUATIONS AND THEIR TRANSFORMATION.

From equations (5) we can deduce without further special assumption a theorem that expresses a general relation between the gradient, the wind velocity and the wind direction. As is well known in meteorology, the term gradient indicates the difference in the atmospheric pressure at two localities that lie at a definite distance apart in the direction of the most rapid change of pressure. According to this we can consider the differential quotient

$$\frac{1}{\rho}\frac{dp}{dn}$$

as the analytical expression for this quantity, omitting a factor that depends upon the adopted units, and in which dn is an element of the normal to the curve whose equation is p/ρ=constant.

I put

$$\gamma=\frac{1}{\rho}\frac{dp}{dn}=\frac{1}{\rho}\sqrt{\left(\frac{\partial p}{\partial x}\right)^2+\left(\frac{\partial p}{\partial y}\right)^2}$$

Furthermore, let the velocity of the wind at a point $x\ y$ be ω so that $\omega^2=u^2+v^2$ and let ε be the angle between ω and γ, in which γ must indicate the direction of diminishing pressure. Then we have

$$cos\ \varepsilon=-\frac{u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}}{\omega\sqrt{\left(\frac{\partial p}{\partial x}\right)^2+\left(\frac{\partial p}{\partial y}\right)^2}}, \quad \ldots\ldots \quad (6)$$

or

$$\omega\,\gamma.\cos\varepsilon=-\frac{1}{\rho}\left(u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}\right). \quad \ldots\ldots \quad (7)$$

If we multiply the first of equation (5) by u and the second by v and add together, there results,

$$\omega.\,\gamma.\cos\varepsilon=k\omega^2+u\frac{\partial u}{\partial t}+v\frac{\partial v}{\partial t}+\frac{1}{2}\left\{u\left(\frac{\partial\omega}{\partial x}\right)^2+v\left(\frac{\partial\omega}{\partial y}\right)^2\right\}$$

or

$$\gamma\cos\varepsilon=k\omega+\frac{\partial\omega}{\partial t}+u\frac{\partial\omega}{\partial x}+v\frac{\partial\omega}{\partial y}$$

for which by introducing the notation

$$\frac{d\omega}{dt}=\frac{\partial\omega}{\partial t}+u\frac{\partial\omega}{\partial x}+v\frac{\partial\omega}{\partial y}$$

we can write

$$\gamma\cos\varepsilon=k\omega+\frac{d\omega}{dt}. \quad \ldots\ldots\ldots \quad (8)$$

From these equations many consequences can be drawn that lead to specially simple theorems when the velocities of the wind are so small that the term

$$u\frac{\partial\omega}{\partial x}+v\frac{\partial\omega}{\partial y}$$

can be neglected. But the following theorems will also be approximately true even if the velocities are larger.

(a) If we compare an invariable system of wind and one that is variable as to its intensity, and of which we will assume that at any given instant there prevails throughout it everywhere uniform velocities and

uniform gradients, then the angle between the direction of the wind and the gradient is smaller in the variable system than in the invariable when the intensities increase, and, inversely, larger when the intensities are diminishing.

(*b*) If in one and the same system of winds having a progressive movement we compare two points that have equal velocities and equal gradients then the deviation of the wind direction from the gradient is smaller at the point where the wind velocity is increased than where it is diminished. Therefore in general the departures from the gradient will be smaller throughout the advancing half or front of a moving cyclone than within the rear half.

(*c*) For steady motions of moderate intensity, where therefore

$$\frac{d\omega}{dt}=0$$

the velocity is proportional to the projection of the gradient on the direction of movement. Furthermore for equal gradients and equal velocity the deviation of the direction of the wind is greater in proportion as the friction is less.

Some of these theorems have been proven already for special cases by Guldberg and Mohn. The theorem expressed in paragraph (*c*) has also been attained in an entirely different way by A. Sprung.*

I pass now to the investigation of the invariable systems of wind, and therefore assume that

$$\frac{\partial u}{\partial t}=\frac{\partial v}{\partial t}=0.$$

If further we put

$$P=\frac{p}{\rho}+\frac{1}{2}(u^2+v^2) \tag{9}$$

then the equations (5) give

$$\left.\begin{aligned}\frac{\partial P}{\partial x}+ku&=-(\lambda+\zeta)v;\\ \frac{\partial P}{\partial y}+kv&=+(\lambda+\zeta)u\end{aligned}\right\}\ (10).$$

If in these we introduce for u and v expressions of the form ordinarily used in hydro-dynamics, namely:

$$\left.u=\frac{\partial\varphi}{\partial x}+\frac{\partial W}{\partial y},\qquad v=\frac{\partial\varphi}{\partial y}-\frac{\partial W}{\partial x}\right\}\ (11)$$

* See Wiedemann, *Beiblatter*, 1881, vol. v, page 240: and Sprung *Meteorologie*, Hamburg, 1885.

and furthermore put

$$\left.\begin{aligned} f_1 &= P + k\varphi - \lambda W; \\ f_2 &= kW + \lambda\varphi \end{aligned}\right\} \quad (12)$$

we thus obtain

$$\left.\begin{aligned} \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} &= -\zeta\left(\frac{\partial\varphi}{\partial y} - \frac{\partial W}{\partial x}\right) \\ \frac{\partial f_1}{\partial y} - \frac{\partial f_2}{\partial x} &= +\zeta\left(\frac{\partial\varphi}{\partial x} + \frac{\partial W}{\partial y}\right) \end{aligned}\right\} \quad (13)$$

According to the equations (1*a*) (1*b*) and (4) the functions φ and W must for the outer region satisfy the partial differential equations

$$\Delta\varphi = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14a)$$

but for the inner region the equation

$$\Delta\varphi = -c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14b)$$

and for both regions

$$\Delta W = \zeta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where we have used the abbreviation Δ for

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

For regions of pure horizontal motions solutions of these equations can be given of great generality, which will now be separately treated of.

IV. ATMOSPHERIC CURRENTS IN REGIONS OF PURE HORIZONTAL MOTION.

When in accord with the assumption of purely horizontal motions we have $\Delta\varphi=0$ throughout the whole region under consideration, then we can also put $\zeta=0$. In this case we can satisfy equation (13) if we put

$$f_1 = \text{constant}, \; f_2 = \text{constant}.$$

The second of these equations gives

$$W = -\frac{\lambda}{k}\varphi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

in which an arbitrary constant can be omitted. Then from the first of these equations, namely, for f_1, there results

$$P = \text{constant} - k\varphi\left(1 + \frac{\lambda^2}{k^2}\right) \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The component velocities are:

$$u=\frac{\partial\varphi}{\partial x}-\frac{\lambda}{k}\frac{\partial\varphi}{\partial y},\quad v=\frac{\partial\varphi}{\partial y}+\frac{\lambda}{k}\frac{\partial\varphi}{\partial x}\quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$\omega^2=u^2+v^2=\left\{\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right\}\left(1+\frac{\lambda^2}{k^2}\right)\quad . \quad . \quad . \quad (19)$$

Finally, from equation (17) we obtain

$$\frac{p}{\rho}=\text{constant}-\left(1+\frac{\lambda^2}{k^2}\right)\left\{k\varphi+\tfrac{1}{2}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]\right\}.\quad . \quad . \quad (20)$$

All these expressions still contain the as yet undetermined function φ, which is only limited by the condition $\Delta\varphi=0$. Such functions can be easily found in various ways. Thus if we bring the function of a complex variable $x+iy$ into the form

$$F(x+iy)=\varphi+i\psi,$$

then both φ and also ψ satisfy the above given differential equations. Moreover, both functions stand in the following relations to each other.

$$\frac{\partial\varphi}{\partial x}=\frac{\partial\psi}{\partial y};\quad \frac{\partial\varphi}{\partial y}=-\frac{\partial\psi}{\partial x}$$

With the assistance of these equations one can easily find the general equation for the path of the wind. We obtain this from the differential equations

$$u\,dy=v\,dx,$$

$$\frac{\lambda}{k}\left\{\frac{\partial\varphi}{\partial x}dx+\frac{\partial\varphi}{\partial y}dy\right\}=\frac{\partial\varphi}{\partial x}dy-\frac{\partial\varphi}{\partial y}dx.$$

If we introduce ψ into the right-hand side of this equation we obtain as the equation for the path described by the wind

$$\psi-\frac{\lambda}{k}\varphi=\text{constant}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The path of the wind intersects the system of lines defined by the condition $\varphi=$ constant at an angle that is everywhere the same. If we designate by ε the angle that the direction of the wind makes with the normal to the curves $\varphi=$ constant then we have

$$\tan\varepsilon=\frac{\lambda}{k}.$$

For currents of air of moderate velocity the term

$$\tfrac{1}{2}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]$$

in equation (20) can be neglected in comparison with φ. In this case the isobars, for which p equals a constant, are identical with the curves $\varphi=$ constant and we obtain the following general theorem:

In regions of pure horizontal motion, and for moderate wind velocity, the angle between the direction of the wind and the gradient is constant and depends only on the constant of rotation and the constant of friction and is independent of the direction of the isobars.

The above given relation had been found by Guldberg and Mohn * for the special cases of rectilinear and circular isobars.

The general solutions contained in equations (18, 19, and 20) can now be so applied that we may adapt the function φ to any other given system of isobars. When this is achieved, then the motions of the air are determined by the first two of these equations.

If, for instance, we have to do with a region that is under the influence of numerous but distant maxima and minima of pressure, then we can approximately put

$$\varphi = \Sigma\, c.\, \log \rho.$$

In this expression ρ indicates the distance of the point (xy) from the vertical currents of the individual regions, assuming that the dimensions of these regions are small in comparison with the distances. This value of φ would be exactly correct if all inner regions [namely, as defined on page 153] were bounded by circles. Then ρ would indicate the distance from the center of the circle. The constants c depend upon the intensity of the respective vertical currents. They are positive for the minima and negative for the maxima [*i. e.*, for areas of low and high pressure respectively]. The assumption

$$\mathrm{F}(x+iy) = (x+iy)^2 = \varphi + i\psi$$

whence

$$\varphi = x^2 - y^2 ;$$

$$\psi = 2xy$$

leads to a special example already treated of by Guldberg and Mohn.†

The potential curves

$$x^2 - y^2 = \text{constant}$$

and the stream lines

$$2xy - \frac{\lambda}{k}(x^2 - y^2) = \text{constant}$$

are systems of equilateral hyperbolas.

* See their *Études*, etc., Part I, pp. 23-26. † *Études*, Part II, pp. 51, 52.

If we assume

$$F(x+iy)=\log(x+iy)=\varphi+i\psi$$

and if we substitute

$$x=r\cos\theta;\ y=r\sin\theta$$

then follows

$$\varphi=\log r.\ \ \psi=\theta.$$

In this case the isobars consist of concentric circles. The paths of the wind are logarithmic spirals having the equation

$$\theta-\frac{\lambda}{k}\log r=\text{constant.}$$

V. STEADY SYSTEMS OF WINDS.

It is certainly at present generally assumed in meteorology that the winds at the earth's surface owe their origin and maintenance to vertical currents of air that are limited to definite regions. Let us assume that there is given such a region having any arbitrary boundary above which a current of air ascends whose velocity in the neighborhood of the earth's surface is determined by the constant (c). By this assumption the whole system of winds dependent thereon, as well as the distribution of pressure, is determined for the whole region. It is therefore the province of mathematics to determine all the quantities coming into consideration both for the inner and also for outer region.

To this end the functions φ and w are to be properly determined. The first of these is found without further difficulty from well-known theorems in the theory of the potential. Since these functions must in the outer region satisfy the partial differential equation $\Delta\varphi=0$, and in the inner region must satisfy the equation $\Delta\varphi=-c$; therefore*

$$\varphi=-\frac{c}{2\pi}\int d\sigma\log\rho \quad \dots\dots\dots\dots\dots\dots\dots (22)$$

In this ρ indicates the distance of the element of the surface $d\sigma$ from the point x, y. The integral is to be extended over the whole of the given inner region. Therefore the velocity potential is the logarithmic potential of a layer of matter having the density $-c/2\pi$ that covers the region of the ascending current of air. The function φ itself, as also its first differential quotient, varies continuously throughout the whole plane up to the boundaries of the outer and inner regions.

*See G. Kirchhoff, *Vorlesungen über Mechanik*, 1876, p. 195.

Therefore, the function W is known for the outer region and is

$$W=-\frac{\lambda}{k}\varphi.$$

In order to determine this function for the inner region also one must go back to the equations (13)

$$\frac{\partial f_1}{\partial x}+\frac{\partial f_2}{\partial y}=-\zeta\left(\frac{\partial\varphi}{\partial y}-\frac{\partial W}{\partial x}\right)$$

$$\frac{\partial f_1}{\partial y}-\frac{\partial f_2}{\partial x}=+\zeta\left(\frac{\partial\varphi}{\partial x}+\frac{\partial W}{\partial y}\right)$$

First we make the assumption that ζ is constant throughout the whole inner region: We can then write

$$\frac{\partial}{\partial x}\left(f_1-\zeta W\right)+\frac{\partial}{\partial y}\left(f_2+\zeta\varphi\right)=0$$

$$\frac{\partial}{\partial y}\left(f_1-\zeta W\right)-\frac{\partial}{\partial x}\left(f_2+\zeta\varphi\right)=0$$

These equations are satisfied if we put

$$f_1-\zeta W=\text{constant};\ f_2+\zeta\varphi=\text{constant}.$$

By considering equation (12) there follows from the last equation especially

$$k W+(\lambda+\zeta)\varphi=\text{Constant}$$

$$W=-\frac{\lambda+\zeta}{k}\varphi+\text{Constant}.$$

From the first of these equations we also obtain,

$$k\Delta W+(\lambda+\zeta)\Delta\varphi=0$$

or

$$k\zeta=c(\lambda+\zeta)$$

whence

$$\zeta=\frac{\lambda c}{k-c}\text{ and }W=-\frac{\lambda}{k-c}\cdot\varphi+\text{Constant}$$

But in general the values of W thus found merge continuously into each other at the borders of the two regions quite as little as do their differential quotients. Hence it follows that the component velocities also, and therefore both the velocity and also its direction, suffer sudden changes of finite magnitude at the boundaries of the two regions. We have therefore found only one special solution, and not one that obtains in general. This special solution is that which Guldberg and Mohn have used in the special case of a circular boundary for the inner region. Corresponding to it they find that in the outer region the di-

rection of the wind makes an angle ε with the radial gradient such that $\tan \varepsilon = \frac{\lambda}{k}$, whereas in the inner region the corresponding angle ε' is given by the equation $\tan \varepsilon' = \frac{\lambda}{k-c}$.

Still less allowable are the consequences that follow when we imagine the inner region bounded by some other curve such as an ellipse. In this case by utilizing the special solution it results that at special portions of the boundary more air flows inward from without than flows away, but at other special portions of the boundary the relation is reversed. One can easily persuade oneself of this by using the known value of the logarithmic potential of an ellipse.* When therefore W can be considered as the logarithmic potential of a stratum of the inner region still it is not to be considered as constant. Its value is to be specially determined for each given region. This computation will now be executed for the case of a circular region.

VI. CYCLONE WITH A CIRCULAR INNER REGION.

Let the region of ascending air currents be bounded by a circle of the radius R. Let the center of the circle be the origin of the system of co ordinates. We put

$$r^2 = x^2 + y^2 .$$

First the velocity potential is easily computed as follows:

For an exterior point

$$\varphi_a = -\frac{c}{2} R \log r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23a)$$

For an interior point

$$\varphi_i = -\frac{c}{4} \left\{ R^2 (2 \log R - 1) + r^2 \right\} \quad . \quad . \quad . \quad (23b)$$

Furthermore for an exterior point we have

$$W_a = \frac{\lambda c}{2k} R \log r$$

Of the functions ζ, W_i and P, which are still to be determined, it can certainly be assumed that they depend upon r only.

If we further consider that

$$\frac{\partial f(r)}{\partial x} = \frac{df(r)}{dr} \cdot \frac{x}{r}.$$

then equations (13) can be written;

$$x \frac{df_1}{dr} + y \frac{df_2}{dr} = - \zeta \left(y \frac{d\varphi}{dr} - x \frac{dW}{dr} \right)$$

$$y \frac{df_1}{dr} - x \frac{df_2}{dr} = + \zeta \left(x \frac{d\varphi}{dr} + y \frac{dW}{dr} \right)$$

*Kirchhoff, *Vorlesungen über Mechanik*, 1876, page 217.

If we multiply the first by x the second by y and add we obtain

$$\frac{df_1}{dr}=\zeta\frac{dW}{dr}$$

or if we introduce the value of f_1

$$\frac{dP}{dr}=-k\frac{d\varphi}{dr}+(\lambda+\zeta)\frac{dW}{dr} \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

If on the other hand the first of the above equations is multiplied by y the second by x and subtracted there results

$$\frac{df_2}{dr}=-\zeta\frac{d\varphi}{dr}$$

or

$$k\frac{dW}{dr}+(\lambda+\zeta)\frac{d\varphi}{dr}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Since furthermore

$$\zeta=\varDelta W=\frac{1}{r}\frac{d}{dr}\left(r\frac{dW}{dr}\right)$$

and

$$\frac{d\varphi}{dr}=-\frac{c}{2}r,$$

therefore we have in equation (25) an ordinary differential equation for the determination of W_i.

If furthermore we put

$$\frac{2k}{c}=\mu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

then equation (25) becomes

$$r\frac{d}{dr}\left(r\frac{dW}{dr}\right)+\lambda r^2=\mu\left(r\frac{dW}{dr}\right).$$

This gives the following integral where A is the constant of integration:

$$r\frac{dW}{dr}=A\,.\,r^{\mu}+\frac{\lambda}{\mu-2}\,.\,r^2.$$

This may finally be written—

$$\frac{dW}{dr}=A\,.\,r^{\mu-1}+\frac{\lambda}{\mu-2}\,r.$$

The constant A is now to be so determined that on the borders of both regions, that is to say for $r = R$, the movements pass continuously from one into the other. Since now

$$u=\frac{x}{r}\cdot\frac{d\varphi}{dr}+\frac{y}{r}\frac{dW}{dr};$$

$$v=\frac{y}{r}\cdot\frac{d\varphi}{dr}-\frac{x}{r}\frac{dW}{dr},$$

therefore at the boundary we must have

$$\frac{d\varphi_a}{dr}=\frac{d\varphi_i}{dr} \qquad \frac{dW_a}{dr}=\frac{dW_i}{dr}.$$

This condition is satisfied for the function φ. We have still to bring it about that the corresponding equation shall be satisfied by the function W. Since

$$\frac{dW_a}{dr}=\frac{\lambda c}{2k}r,$$

therefore for $r=R$ we have

$$\frac{dW_i}{dr}=\frac{\lambda c}{2k}\cdot R.$$

This latter will be the case when we put

$$A=\frac{-2\lambda}{\mu(\mu-2)}\cdot R^{2-\mu}$$

Therefore we have finally

$$\frac{dW_i}{dr}=\frac{\lambda}{\mu-2}r\left\{1-\frac{2}{\mu}\left(\frac{r}{R}\right)^{\mu-2}\right\}$$

or if for abbreviation we put

$$f(r)=1-\frac{2}{\mu}\left(\frac{r}{R}\right)^{\mu-2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

we obtain

$$\frac{dW_i}{dr}=\frac{\lambda}{\mu-2}r\cdot f(r) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

The function $f(r)$ can according to equation (26) also be written—

$$f(r)=1-\frac{c}{k}\left(\frac{r}{R}\right)^{\frac{2(k-c)}{c}}$$

This gives $f(r)=1$ for $r=0$ and $f(r)=\frac{(k-c)}{k}$ for $r=R$.

In accordance with these conditions our results are now as follows:

(*a*) for the outer region

$$\left.\begin{aligned} u &= -\frac{c}{2}\frac{R^2}{r^2}\left\{ x - \frac{\lambda}{k}y \right\} \\ \omega &= \frac{c}{2}\frac{R^2}{r}\sqrt{1+\frac{\lambda^2}{k^2}} \\ v &= -\frac{c}{2}\frac{R^2}{r^2}\left\{ y + \frac{\lambda}{k}x \right\} \\ \tan\varepsilon &= \frac{\lambda}{k} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

(*b*) for the inner portion

$$\left.\begin{aligned} u &= -\frac{c}{2}\left\{ x - \frac{\lambda}{k-c}\cdot y \cdot f(r) \right\} \\ \omega &= r\cdot\frac{c}{2}\sqrt{1+\left(\frac{\lambda}{k-c}\right)^2\left[f(r)\right]^2} \\ v &= -\frac{c}{2}\left\{ y + \frac{\lambda}{k-c}\cdot x \cdot f(r) \right\} \\ \tan\varepsilon &= \frac{\lambda}{k-c}\cdot f(r) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

In these equations ε indicates the angle between the direction of the wind and direction of the gradient, which latter coincides of course with the radius of the circle.

These expressions differ from the solutions given by Guldberg and Mohn (not to speak of some small changes in the notation) by the introduction of the function $f(r)$ in whose place the factor 1 is given by them.

The above-given expressions are subject to one limitation. It is necessary that we have $\mu > z$ or $k > c$, since otherwise for $r = o$ $f(r)$ would become infinitely great, and in the inner region a deviation of the wind from the gradient toward the left would occur instead of toward the right-hand side.

The deviation of the wind direction from the gradient is constant in the outer region, but in the inner region it increases continuously and for $\tau = o$ it attains the limiting value—

$$\tan\varepsilon = \frac{\lambda}{k-c}.$$

I pass now on to the computation of the pressure. According to equation (17) we have for the outer region—

$$P_a = \text{constant} - k\varphi_a\left(1+\frac{\lambda^2}{k^2}\right)$$

Consequently

$$P_a = \text{constant} + \frac{kc}{2}\left(1+\frac{\lambda^2}{k^2}\right)R^2 \log r.$$

For the inner region the equation (24) is to be used. According to it we have—

$$\frac{dP_i}{dr} = -k\frac{d\varphi}{dr} + (\lambda+\zeta)\frac{dW}{dr}$$

But according to equation (25) we have—

$$\lambda+\zeta = -k\frac{\frac{dW}{dr}}{\frac{d\varphi}{dr}}$$

Therefore,

$$P_i = \text{const} - k\varphi - k.\int \frac{\left(\frac{dW}{dr}\right)^2}{\frac{d\varphi}{dr}}dr.$$

The arbitrary constant can be considered as determined in that the value of P is supposed to be given for $r=0$. (For the center of the depression we have $r=0$ and $P=\frac{p}{\rho}$.) Let P_0 be this value. Then we have—

$$P_i = P_0 + F(r),$$

Where

$$F(r) = \frac{kc}{4}r^2\left\{1+\left(\frac{\lambda}{k-c}\right)^2\left(1+\frac{\zeta}{\mu^2}\left(\frac{r}{R}\right)^{2} + \frac{4}{\mu^2(\mu-1)}\left(\frac{r}{R}\right)^{2\mu-4}\right)\right\} \quad . \quad (31)$$

Since P_a and P_i must at the boundary merge continuously into each other, therefore the constant in the expression for P_a is to be determined in accordance with this condition, and we have—

$$P_a = P_0 + F(R) + \frac{kc}{2}\left(1+\frac{\lambda^2}{k^2}\right)R^2 \log\frac{r}{R} \quad . \quad . \quad . \quad (32)$$

From equation (9) we obtain the expression for the pressure—

$$\frac{p}{\rho} = P - \tfrac{1}{2}\omega^2$$

If we designate by p_0, the pressure at the center of the depression, where $\omega=0$, then in the inner region we have—

$$\frac{p-p_0}{\rho} = F(r) - \tfrac{1}{2}\omega^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

but in the outer region—

$$\frac{p-p_0}{\rho} = F(R) + \frac{kc}{2}\left(1+\frac{\lambda^2}{k^2}\right)R^2 \log\frac{r}{R} - \tfrac{1}{2}\omega^2 \quad . \quad . \quad . \quad . \quad . \quad (34)$$

VII. NUMERICAL EXAMPLE FOR A CYCLONE: NOTE ON ANTI-CYCLONES.

In order to show the applicability of the formulæ obtained in the last section to cyclones as they actually occur in nature, I have executed the following computation of a numerical example:

In this computation I have assumed

$$\lambda = 0.00012$$

This value corresponds to an average latitude of 55.5°. For k I have assumed the same value, whereby the value obtained for the influence of friction is rather large.

For the complete determination of the system of winds the constant c of the ascending current of air and the dimensions of the inner region must also be known. We can obtain this in various ways. We can assume as given, a definite difference in pressure between the center and a circle of known radius; or on the other hand, we can assume that the velocity of the wind is known at a certain distance from the center. I have chosen the last assumption.

The wind system may therefore be characterized by the assumption that at a distance of 1000 kilometres from the center the wind velocity shall be 10 metres per second.

According to equation (29) when we put $\lambda = k$ we have

$$\omega = \frac{c\ R^2}{\sqrt{2}} \cdot \frac{1}{r}$$

If in this we put $\omega = 10$ metres and $r = 1000000$ metres we then have $c\ R^2 = 10000000\ \sqrt{2}$. Since furthermore $c < k$, therefore the same equation shows that we must have $R > 343.3$ kilometres.

In the selection of appropriate values of c and R, another circumstance is to be considered. The discussion of the formulæ (30) for the velocity ω shows that under the assumption here made of $\lambda = k$, the maximum velocity of the wind occurs at the boundary of the two regions. The smaller the inner region is chosen, by so much larger results the maximum velocity ω_R. In the following table some coördinate values c, μ, R, and ω_R are given.

TABLE I.

c	μ	R	ω_R
		Kilometres.	*Metres per sec.*
$\frac{4}{5}k$	$\frac{5}{2}$	383.8	26.06
$\frac{2}{3}k$	3	420.4	23.78
$\frac{1k}{2}$	4	485.5	20.60
$\frac{1k}{3}$	6	594.6	16.82

I have also executed the further complete computation for the first case where $c=\frac{4}{5}k$; the results of this work are given in Table 2. In this computation the equations (29) and (30) were used for the determination of the velocities ω and the deviations ε of the direction of the wind from the radial gradient. Furthermore, the differences of pressure $(p-p_0)$ with respect to that at the center, in the circles of radius r, were computed according to equations (31), (32), (33) and (34). These latter are, however, converted from the units ordinarily used in hydrodynamics into differences of barometric pressure $(b-b_0)$. This latter is easily done if we recall that for $b=760$ millimetres the ratio $\frac{p}{\rho}$ is equal to the square of the Newtonian velocity of sound; therefore we have the proportion

$$(b-b_0) : 760=\frac{1}{\rho}(p-p_0);\ (279.9)^2$$

The gradients γ are in our present case the differences of barometric pressure for a horizontal distance of 100 kilometres.

TABLE II.

r	ω	ϵ	$(b-b_0)$	γ
Kilometres.	*Metres per sec.*	° ′	*Millimetres.*	*Millimetres.*
0	0	78 41	0	
				2.37
100	14.99	71 19	2.37	
				4.64
200	22.44	64 40	7.01	
				5.03
300	25.53	55 39	12.04	
383.8	26.06	45 00	15.88	4.78
400	25.00	45 00	16.82	
				4.76
500	20.00	45 00	21.58	
				3.60
600	16.67	45 00	25.18	
				2.66
800	12.50	45 00	30.50	
				1.95
1,000	10.00	45 00	34.45	

From this table we see that the cyclone includes a broad storm region from $r=200$ to $r=500$ kilometres, of which a portion is in the inner region and another portion in the outer region. Of course the gradients are greatest in the inner region; therefore there the isobars are most crowded together.

From those values of the constant c that are any way possible, it follows that the velocity of the ascending current of air is extraordinarily small; for the present example c equals 0.000096. If we assume that the formula $w=cz$ holds good to an altitude of 1,000 metres, then the vertical velocity would at that height first attain the value of about 0.1 metre per second.

Hitherto the discussion has exclusively dealt with regions of ascend-

ing currents of air and the cyclones arising therefrom. It would be easy in an entirely similar way to develop the theory of descending currents of air and the anti-cyclones resulting therefrom, and here also, as an example, to assume an inner region bounded circularly. Before the actual execution of the exact computation I had believed that this was simply a case of the change of the sign of the constant c. But in this operation we stumble upon a peculiar difficulty. The function $f(r)=1-\frac{2}{\mu}\left(\frac{r}{R}\right)^{\mu-2}$ $\left(\text{wherein } \mu=\frac{2k}{c}\right)$ which enters into the expression for the component velocities in the inner region becomes infinitely great for negative values of c and μ and for $r=o$. The same is true of the function $F(r)$ entering into the expression for the pressure. Hence it follows that the formula just given can not be applied to anti-cyclones with a reversed sign of c.

Therefore minima and maxima of pressure show a characteristic difference in their theoretical treatment. But this, as I believe, corresponds also to the real conditions of the true phenomena. Depressions are ordinarily confined to limited areas, but are of considerable intensity, while on the other hand the maxima of pressure extend with slight intensity over broad areas.

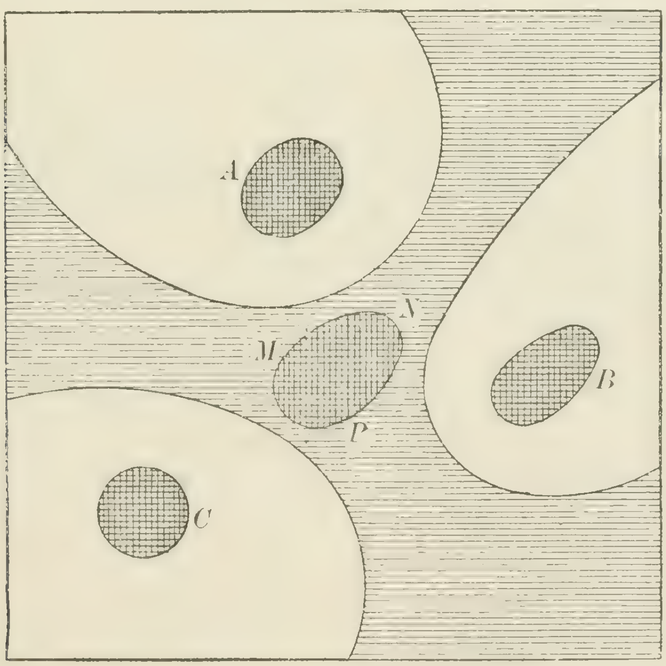

FIG. 27.

Moreover, both phenomena stand in close connection, such that one can consider the ascending currents of air as the cause of the descending currents. Hence to a complete cyclone there belong an inner region with ascending air current, a zone surrounding it of purely

horizontal movement, and at a greater distance from the center a ring-shaped region of descending currents.

If we assume that the boundaries of the three regions consist of concentric circles, it would not be difficult to compute the wind system for the whole region by the help of the potential theory as above employed. In this case, where we have to do with an annular region with a descending current of air, the use of the function $f(r)$, even with a negative sign before the μ, is allowable, and can be adopted in order to produce the necessary continuity of motion at the boundary of the two annular regions. If there are several regions of depression with ascending currents of air, as at A, B, C, fig. 27, then each of them is immediately surrounded by a zone of purely horizontal movement, which is bordered by an outside annular zone of descending movement. I have in the figure (27) distinguished the region of ascending and descending current by double and single shading. In the region where the different ring systems of ascending air currents merge into each other there will lie a region of highest pressure with anticyclonal movement of the air somewhat as within the isobar M, N, P. However, the characteristic difference between ascending and descending currents of air always consists in this, that the former consist of definite, simply connected areas; the latter, on the other hand, of a network of several complexly connected regions.

HALLE A. S., *June*, 1882.

P. S.—After sending the above treatise to the editor of the *Annalen*, I found in the May number of the *Zeitschrift* of the Austrian Association for Meteorology (vol. XVII, pp. 161–175) a review by Dr. A. Sprung of the second part of the collected memoirs by W. Ferrel, under the title of "Meteorological Researches."

From this I perceive that the views expressed by me as to regions with high pressure had been already expressed by Ferrel. Therefore, although my point of view is no longer new, still I rejoice to see that it is shared by a prominent meteorologist.

XI.

ON THE GULDBERG-MOHN THEORY OF HORIZONTAL ATMOSPHERIC CURRENTS.*

By Prof. Dr. A. OBERBECK, *of the University of Halle.*

Starting from the generally known results of recent meteorological observations in so far as these relate to the distribution of pressure and the direction and force of the wind, the author states that one of the most important problems of the mathematical theory of the motion of fluids is to explain quantitatively the connection of the above-named phenomena. The recently published investigations of Guldberg and Mohn (*Etudes sur les mouvements de l'atmosphère.* Christiania, 1876 and 1880) are to be considered as a specially successful attempt in this direction. It must be of interest also for the larger number of geographers to know the most important results to which the Norwegian scientists have attained.

In order to understand the horizontal movements of the atmosphere it is important for a moment to consider their causes. As such we consider the differences of pressure at the surface of the earth as observed with the barometer. But whence do these arise? This question has been answered a long time since. It is heat which is to be considered as the prime cause of the disturbance of equilibrium in the atmosphere. Because of the slight conductivity of the air the process of warming can progress only slowly from below upwards, so that as is well known the temperature of the air steadily diminishes as we ascend. The heated air expands. The pressure becomes less. If the heating takes place uniformly over a large area there will be at first no reason for horizontal currents. But vertical currents can certainly be brought about by this means. If we imagine a circumscribed mass of air transported into a higher region without any increase or diminution of its heat its temperature will sink because it has expanded itself proportionately to the diminished pressure. If its temperature is then equal to that prevailing in the upper stratum it will remain in equilibrium at this altitude as well as below. The atmosphere in this case exists in a state of indifferent equilibrium. If its temperature is lower the

* Translated from the *Verhandlungen des Zweiten Deutschen Geographentages.* Halle, April, 1882.

mass of air will again sink down; in the reverse case it will rise higher. The air in these cases is then in stable or unstable equilibrium respectively. In the latter case any vertical movement initiated by some accidental disturbance will not again disappear, but rapidly assume increasing dimensions. The current will also continue uniform for a long time.

This is the explanation first given by the mathematician Reye,* of Strasburg, of the ascending air currents in the whirlwinds of the tropics.

The winds of our (temperate) zone also presuppose such ascending currents whose origin must have been quite similar. The ascending current is in general restricted to a definite region that we can designate as the base. Since the ascending current consists of warmer air, therefore above its base the pressure sinks. A barometric depression is inaugurated there. The pressure increases from this region outward in all directions. The isobars therefore surround the region of ascending atmospheric currents in closed curves. At greater heights the upper cooled air flows away to one side and in other regions gives occasion to descending currents of air. At the earth's surface itself, the air flows towards the depression; its influence thus extends over an area much greater than that of the base. If we neglect the curvature of the earth's surface we find over this larger area only simple horizontal movements. Mathematical computations should now reveal to us the nature of such horizontal movements. To this end all the causes of motion, or the forces that come into consideration, are first to be collected.

The differences of pressure have already been several times spoken of. We take as the measure of these differences, the gradient which gives for any point the direction and amount of the greatest change in pressure. In horizontal movements the effect of gravity can be omitted.

On the other hand attention must be given to the rotation of the earth on its axis, since we are only interested in the paths of the winds on the rotating earth. This influence can be taken account of if we imagine at every point of the mass of air a force applied which is perpendicular to the momentary direction of motion and is equal to the product of the double angular velocity of the earth by the sine of the latitude and by the velocity of the point. In the Northern Hemisphere this influence causes a continuous departure of the path towards the right-hand side. Since the movement takes place directly on the earth's surface the direct influence of that surface, namely the friction, remains to be considered. Its influence diminishes with the distance from the earth's surface. Furthermore it depends on the nature of the earth's surface, whether sea or land, plains or wooded mountains. For this computation Guldberg and Mohn have made a convenient assumption in that they introduce the friction as a force which opposes the move-

*[This explanation is of course much older than Reye (1864), who was preceded by Espy and Henry in the United States and by Wm. Thomson in Great Britain. *C. A.*]

ment and is equal to the product of a given factor and the velocity. This factor can have different values according to the nature of the earth's surface [and will be called the friction constant].

All these forces are to be introduced into the general equations of motion of the air. If however one desires solutions of these general equations for special cases there is still needed a series of assumptions.

Let there be only one single vertical current of air present. The totality of all the atmospheric movements depending upon this one vertical current is called a wind-system. If the strength of the ascending current is variable or if the base itself changes its place, then the wind-system is variable. In the first case the system stands still, in the second case it is movable.

If on the other hand the ascending current of air retains its strength and location without change, or, which is the same, if the isobars for a long time retain their position, then the wind system is invariable.

It is evident that the last case is by far the most simple. We will therefore begin with its consideration.

In order to execute the calculation the location of the isobars must be known. Even in this respect also in a preliminary way, one must limit himself at first by simple assumptions. Let the isobars be either parallel straight lines or concentric circles.

In the first case the computation leads to the following simple results:

(1) The parallel isobars are equally distant from each other. The gradient is therefore everywhere of equal magnitude.

(2) The paths of the winds consist of parallel straight lines. The strength of the wind has everywhere the same value.

(3) The direction of the wind forms an angle with the gradient whose tangent is equal to the quotient of the factor arising from the velocity of the earth's rotation divided by the friction constant.

The deviation of the wind from the gradient is therefore greater in proportion as friction is smaller. If the earth's surface were perfectly smooth the wind would blow in the direction of the isobars.

This result, following directly from the computation and at first surprising, finds its confirmation in a variety of observations. For example, in England we observe a deviation of 61° for land winds, but of 77° for sea breezes. From this it follows that the friction on the land is more than twice as great as on the sea.

Conditions of pressure like those here considered frequently occur. In the regions of the trade winds and monsoons they ordinarily prevail either during the whole or about the half of the year.

The circular isobars to the consideration of which we now pass produce systems of wind that can be considered as the simplest types of cyclones and anti-cyclones according as the pressure in the interior is a minimum or maximum. We confine ourselves here to the consideration of cyclones.

As already remarked cyclones are not conceivable without an ascend-

ing current of air, whose area in our case is defined by a circle. Outside of this circle horizontal movements prevail exclusively; inside of it there is also the vertical movement to be considered. Therefore the computations for the outer and inner regions are different. In this way we obtain the following results:

(1) The pressure increases from all sides outward from the center; the gradient increases also from the center out to the limit of the inner region; thence outward it diminishes and at a great distance becomes inappreciable.

(2) The wind-paths in both regions are curved lines, logarithmic spirals, which cut the isobars everywhere at the same angle or make everywhere the same angle with the radial gradient. Therefore the movement of the air can be considered as consisting of a current toward the center and a rotation around the center, the latter in direction opposite to the hands of a watch. This departure from the gradient is of different magnitudes in the outer and inner regions. For the former the departure has the same value as for straight-line isobars, that is to say, it depends alone upon the rotation of the earth and the friction. For the inner region the departure is greater, and depends besides upon the intensity of the ascending current of air. If both regions were separated from each other by a geometrical cylindrical surface then the wind-paths in these would not continuously merge into each other, but would form an angle with each other. This of course can never occur in nature. We must therefore assume a transition region in which the wind is continuously diverted from one into the other direction. At any rate accurate and comparative observation of the wind direction in the inner and outer region of a cyclone would be of great interest. From these one could draw a conclusion as to the limitation of the ascending current of air. This limit is moreover also notable because at it the winds reach their greatest force.

There are no other arrangements that have been discussed theoretically as yet except the straight line and the circular and nearly circular forms of the isobars.

We have as yet only spoken of the invariable systems of wind. In fact however their duration is relatively short. No sooner is a depression formed than it fills up. Furthermore the central region of depression generally does not remain long in the same place but wanders often with great velocity, drawing the whole system of winds with it. We must look to the density of the horizontal current flowing in towards the ascending current of air as the cause of these changes. The system of winds remains unchanged only when, as has hitherto been silently assumed, the temperature and density of the horizontal and vertical currents are alike. If the inflowing air is warmer the depression increases in depth; in the opposite case it becomes shallower.

Finally, if the inflowing air is not of the same temperature on all sides, but has on the one side higher and on the other side lower

temperature than the ascending air, then it will on the one side be strengthened and its area increased, on the other side enfeebled and its area diminished. The consequence of this is that the current of air or the region of depression moves along; the cyclone progresses. Since in the cyclones of our north temperate zone the air entering on the east side comes from more southern—therefore in general—warmer regions, while the air entering on the west side comes from the north and is generally colder, therefore the cyclone progresses from west to east or from southwest to northeast. This is in fact the path of most cyclones in northern Europe. For a moving cyclone the isobaric curves must have a different shape than for one that is stationary; therefore one can inversely from the shape of the isobars infer the direction of motion. If the region of ascending air has a circular form the computation can be rigorously executed. Without going into the details of this interesting problem in this place I will only remark that the isobars consist in closed curves similar to an ellipse. There is one direction from the center outward in which the isobars are most crowded together, while in the opposite direction they are furthest apart. The movement of the cyclone is in a direction at right angles to this line. With the solution of this problem we now stand about at the limits of what analysis has thus far accomplished. Still there is hope that it will make further progress so far as concerns the relation between the pressure and the motion of the air at the earth's surface.

XII.

ON THE PHENOMENA OF MOTION IN THE ATMOSPHERE.*

(FIRST COMMUNICATION.)

By Prof. A. OBERBECK, *of the University of Greifswald, Germany.*

I.

The meteorological observations of the last ten years have given a series of notable laws that principally relate to the connection between the currents of air and the pressure of the air in the neighborhood of the earth's surface.

Of course one can only hope to obtain a complete insight into the complicated mechanism of the motion of the air when one understands more accurately the condition of the atmosphere in its higher strata. But difficulties that are perhaps never to be overcome oppose the observation of these strata. On the other hand, the completion of this and many other gaps in the theory of the motion of the air is certainly to be expected from a comprehensive mechanics of the atmosphere. The Treatise on Meteorology, by A. Sprung, Hamburg, 1885, gives a summary of what has hitherto been accomplished in this field, from which summary it is seen that only special individual problems have found a satisfactory solution.

The principal features of a rational mechanics of the atmosphere are given in the memoir by W. Siemens, "The conservation of energy in the earth's atmosphere."† It appears to me worth while to follow out mathematically the questions there treated of and to develop a theory of the motions of the air as general as possible. The results thus far attained by me, are collected in this present memoir.

On account of the magnitude and difficulty of the problem to be solved, I have at first confined myself to the determination of the currents of the air. A corresponding investigation of the distribution of pressure will follow hereafter. Moreover the phenomena of motion

* Read before the Royal Prussian Academy of Sciences, at Berlin, March 15, 1888. Translated from the *Sitzungsberichte Königl. Preus. Akad. der Wissenschaften.* 1888, pp. 383–395.

† See *Berlin Sitzungsberichte*, 1886, pp. 261–275.

will here be considered as "steady motion." On the other hand I have labored so to arrange the calculation that it can be applied to any condition of the atmosphere and to the general currents between the poles and the equator, or the atmospheric circulation, as well as also to individual cyclones or anticyclones.

In order to test the applicability of the formula thus obtained, the first of the problems just mentioned is completely solved.

I begin with an enumeration of the factors upon which the movement of the atmosphere depends, and with a description of the manner in which I have introduced these into the calculation.

II

(1) Since the ultimate cause of the motion of the air is to be sought in the effect of gravity and in the differences of temperature in the atmosphere, therefore the attraction of the earth must enter into the equations of motion as the moving force. But it is entirely sufficient here to consider the earth as a homogeneous sphere.

(2) The temperature of the atmosphere is to be considered as a function of the locality, but entirely independent of the time. The last condition is necessary if one confines himself to steady motions. For the temperature T, the analytical condition

$$\Delta T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

must be satisfied.

This equation, as is well known, follows from the assumption that the heat is distributed through the medium in question according to the laws of the conduction of heat. Although I am by no means of the opinion that the conduction of heat principally determines the flow of heat from the earth's surface through the atmosphere into the planetary space, still it is very probable that the totality of all the phenomena here coming into consideration (conduction, radiation from the earth's surface with partial absorption in the atmosphere, vertical convection currents, etc.) will bring about a distribution of temperature analogous to that due to the conduction of heat.

(3) According to the rules of mechanics, the influence of the rotation of the earth can be expressed by a deflecting force, so that after its introduction the earth can be considered as at rest.

(4) Friction is furthermore to be considered, since without it the atmospheric currents under the continuous influence of accelerating forces would attain to indefinitely great velocities. In my opinion, the attempts made hitherto to give a correct theory of the motions of the air, especially one that can be developed analytically, have failed because of the insufficient or incorrect introduction of friction. I have adhered to the simplest assumption, namely, that the same law of friction holds good for atmospheric currents that has also been shown

to be correct in the motion of liquids.* But I would not hereby assert that the same numerical coefficient is to be used as is given by the laboratory experiments on the internal friction of the air made under the exclusion of all attendant disturbing circumstances. More likely is it that along with the greater horizontal currents there will arise small vertical currents of a local nature which will increase the friction. The air can either be held fast at the earth's surface or glide with more or less resistance. This fact, as is well known, is expressed in the boundary equations of condition by a number, the coefficient of slip, whose value may lie between zero and infinity.

(5.) The density of the air must be considered as dependent upon the temperature, since the effective cause of the currents results from this. But I have not objected to use, as the equation of continuity, that simpler expression that obtains for incompressible liquids. The error introduced hereby can be eliminated if, at places where the density is less than the average, one increases to a corresponding extent the velocity found for that locality, but considers the velocity as diminished at locations where the density exceeds the average.

(6) A hydro-dynamic problem is only perfectly definite when the fluid occupies a definite space, and its behavior is known for all limiting boundary surfaces. I have therefore assumed that the atmosphere is bounded both by the earth's surface and by a second spherical surface concentric therewith. The distance of the two spherical surfaces, which I will briefly designate as the height of the atmosphere, can remain undetermined. But this is quite small in comparison with the earth's radius. The above assumption just made however, only expresses the idea that for a given altitude above the earth's surface the radial or vertical currents are very small, or rather that when they are present they exert an inappreciably small influence on the remaining motions. This is certainly the case, since at very large altitudes the density is very small. Since moreover it is assumed that the air can glide without resistance on the upper spherical surface, therefore in my opinion no limitation of the motions of the atmosphere, contradictory to the real phenomena, results from the introduction of such an upper boundary surface.

III.

The following notation will be used for the principal equations of the problem. The position of a point in the atmosphere is determined by the rectangular coördinates x, y, z. The center of the earth is the origin of coördinates and the earth's axis in the direction of the North Pole is the positive axis of z. The positive directions of the two other axes are to be so chosen that the axis of y as seen from the North Pole must be turned through an angle of 90° in the direction of the motion of the hands of a watch in order to be made to coincide with the axis of x.

*[The term friction as here used therefore includes viscosity and slip, but excludes the resistance due to wave motion and to vortex motion and all the resistances implied in turbulent flow of fluids.—*C. A.*]

Let there be furthermore—

u, v, w, the components of velocity;

p, the pressure;

μ, the density;

k, the coefficient of friction;

G, the acceleration of gravity;

R, the radius of the earth;

r, the distance of any point from the center of the earth;

ε, the angular velocity of the earth.

Then we have—

$$\left.\begin{aligned}
\frac{du}{dt}&=GR^2\frac{\partial\frac{1}{r}}{\partial x}-\frac{1}{\mu}\frac{\partial p}{\partial x}+\frac{k}{\mu}\Delta u+2\varepsilon v,\\
\frac{dv}{dt}&=GR^2\frac{\partial\frac{1}{r}}{\partial y}-\frac{1}{\mu}\frac{\partial p}{\partial y}+\frac{k}{\mu}\Delta v-2\varepsilon u,\\
\frac{dw}{dt}&=GR^2\frac{\partial\frac{1}{r}}{\partial z}-\frac{1}{\mu}\frac{\partial p}{\partial z}+\frac{k}{\mu}\Delta w,\\
&\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0.
\end{aligned}\right\} \quad \ldots\ldots\ldots (1)$$

Since according to the law of Mariotte and Gay-Lussac

$$\frac{p}{\mu}=\frac{p_0}{\mu_0}(1+\alpha\, T)$$

we may put

$$\frac{1}{\mu}\cdot\frac{\partial p}{\partial x}=\frac{p_0}{\mu_0}(1+\alpha T)\frac{\partial \log p}{\partial x}.$$

The zero point of temperature is arbitrary. It is most appropriate to assume for it the average temperature of the atmosphere.

If c is the Newtonian value of the velocity of sound, then we have

$$\frac{p_0}{\mu_0}=c^2$$

After the introduction of these expressions into the above principal equations, imagine the latter divided throughout by $1+\alpha\, T$. Excepting in that member in which the gravity occurs, one can omit from consideration the influence of the factor $\frac{1}{1+\alpha T}$. In the term just mentioned one can, as a first approximation, put $(1-\alpha\, T)$ for the value of this factor. Furthermore let

$$\frac{k}{\mu}=\varkappa$$

The first of the equations of motion now becomes

$$\frac{du}{dt} = (1 - \alpha T)\, GR^2 \frac{\partial \frac{1}{r}}{\partial x} - c^2 \frac{\partial \log p}{\partial x} + \varkappa \Delta u + 2 \varepsilon v.$$

If the temperature of the atmosphere depended only on the altitude above the earth's surface and were therefore only a function of r, then would these equations be fulfilled by putting u, v, w respectively $= 0$; the atmosphere would then be in equilibrium. Therefore put

$$T = T_0 + T_1$$

wherein T_0 is a function of r only, but T_1 is also a function of the longitude and latitude; therefore

$$T_0 \frac{\partial \frac{1}{r}}{\partial x} = - \frac{\partial}{\partial x} \int \frac{T_0}{r^2} dr$$

$$T_1 \frac{\partial \frac{1}{r}}{\partial x} = \frac{\partial \frac{T_1}{r}}{\partial x} - \frac{1}{r} \frac{\partial T_1}{\partial x}$$

Finally one may put

$$p = p_1 \cdot (1 + \nu).$$

The quantity ν in this latter equation expresses those changes of pressure that are caused by the phenomena of motion. Since ν is small in comparison with unity, therefore instead of $\log (1 + \nu)$ the quantity ν itself can be substituted. By this means the first principal equation becomes

$$\frac{du}{dt} = GR^2 \frac{\partial}{\partial x} \left\{ \frac{1 - \alpha T_1}{r} + \alpha \int \frac{T_0}{r^2} dr \right\} - c^2 \frac{\partial \log p_1}{\partial x} - c^2 \frac{\partial \nu}{\partial x} + \varkappa \Delta u + 2 \varepsilon v$$

After transforming the two other principal equations in the same manner we can put

$$c^2 \log p_1 = \text{constant} + GR^2 \left\{ \frac{1 - \alpha T_1}{r} + \alpha \int \frac{T_0}{r^2} dr \right\} \quad . \quad . \quad (2)$$

This equation gives the diminution of pressure at larger altitudes above the earth's surface, and can for smaller differences of altitude easily be transformed into the ordinary equation of barometric hypsometry.

The following system of equations relating to the phenomena of motion proper now remains:

$$\left.\begin{aligned}
\frac{du}{dt}&=\frac{\alpha G R^2}{r}\cdot\frac{\partial T_1}{\partial x}-c^2\frac{\partial \nu}{\partial x}+\varkappa\Delta u+2\varepsilon v,\\
\frac{dv}{dt}&=\frac{\alpha G R^2}{r}\cdot\frac{\partial T_1}{\partial y}-c^2\frac{\partial \nu}{\partial y}+\varkappa\Delta v-2\varepsilon u,\\
\frac{dw}{dt}&=\frac{\alpha G R^2}{r}\cdot\frac{\partial T_1}{\partial z}-c^2\frac{\partial \nu}{\partial z}+\varkappa\Delta w,\\
\frac{\partial u}{\partial x}&+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0.
\end{aligned}\right\} \quad \ldots\ldots\ldots (3)$$

One can now compute first those components of the current that depend only on temperature differences; after that those that are brought about by the rotation of the earth. If we put $u=u_1+u_2$; $v=v_1+v_2$; $w=w_1+w_2$; $\nu=\nu_1+\nu_2+\nu_3$, then will the following two systems of equations be those that are first to be discussed:

$$c^2\frac{\partial \nu_1}{\partial x}=\frac{\alpha G R^2}{r}\cdot\frac{\partial T_1}{\partial x}+\varkappa\Delta u_1$$

$$c^2\frac{\partial \nu_1}{\partial y}=\frac{\alpha G R^2}{r}\cdot\frac{\partial T_1}{\partial y}+\varkappa\Delta v_1$$

$$c^2\frac{\partial \nu_1}{\partial z}=\frac{\alpha G R^2}{r}\quad\frac{\partial T_1}{\partial z}+\varkappa\Delta w_1$$

and

$$c^2\frac{\partial \nu_2}{\partial x}=2\,\varepsilon v_1+\varkappa\Delta u_2;$$

$$c^2\frac{\partial \nu_2}{\partial y}=-2\,\varepsilon u_1+\varkappa\Delta v_2;$$

$$c^2\frac{\partial \nu_2}{\partial z}=\varkappa\Delta w_2.$$

Thus there still remain the following equations which are no longer linear and which will serve principally in the computation of the variations in pressure produced by the motion:

$$c^2\frac{\partial \nu_3}{\partial x}+u\,\frac{\partial u}{\partial x}+\,v\,\frac{\partial u}{\partial y}+w\,\frac{\partial u}{\partial z}=2\,\varepsilon v_2;$$

$$c^2\frac{\partial \nu_3}{\partial y}+u\,\frac{\partial v}{\partial x}+\,v\,\frac{\partial v}{\partial y}+w\,\frac{\partial v}{\partial z}=-2\,\varepsilon u_2;$$

$$c^2\frac{\partial \nu_3}{\partial z}+u\,\frac{\partial w}{\partial x}+\,v\,\frac{\partial w}{\partial y}+w\,\frac{\partial w}{\partial z}=0.$$

The first two systems of equations are linear. When therefore T_1 consists of a sum of terms we shall obtain corresponding sums for the

component velocities. The solution will be quite simple when T_1 is developed into a series of spherical harmonics.

If we put

$$T_1 = \Sigma \left\{ A_n r^n + \frac{A'_n}{r^{n+1}} \right\} p_n$$

and for brevity

$$\beta = a G K^2.$$

and indicate by Q any term of the series with its corresponding constant then the solutions of the first two systems of equations are as follows:

$$\left.\begin{aligned} u_1 &= \frac{\beta}{\varkappa} \left\{ E \frac{\partial Q}{\partial x} + \frac{\partial (QF)}{\partial x} \right\} \\ v_1 &= \frac{\beta}{\varkappa} \left\{ E \frac{\partial Q}{\partial y} + \frac{\partial (QF)}{\partial y} \right\} \\ w_1 &= \frac{\beta}{\varkappa} \left\{ E \frac{\partial Q}{\partial z} + \frac{\partial (QF)}{\partial z} \right\} \\ c^2 \nu_1 &= \beta \{ \varDelta (QF) + a Q \} \end{aligned} \right\} \quad . \; . \; . \; . \; . \; . \; . \quad (4)$$

In this E and F are functions of r only, and must satisfy the differential equations

$$\left.\begin{aligned} &\left(\frac{d^2 E}{dr^2} + \frac{2}{r} \frac{dE}{dr} \right) \frac{\partial Q}{\partial r} + 2 \frac{dE}{dr} \frac{\partial^2 Q}{\partial r^2} = \frac{\partial Q}{\partial r} \left(-\frac{1}{r} + a \right) \\ &\left(\frac{d^2 F}{dr^2} + \frac{2}{r} \frac{dF}{dr} \right) Q + \frac{\partial Q}{\partial r} \left(2 \frac{dF}{dr} + \frac{dE}{dr} \right) = 0 \end{aligned} \right\} \quad . \; . \quad (5)$$

The constant a must be added in order to obtain the number of constants needed in the consideration of the boundary conditions. The terms depending upon the earth's rotation are

$$\left.\begin{aligned} u_2 &= \frac{2\varepsilon\beta}{\varkappa^2} \left\{ -\left(J \frac{\partial Q}{\partial y} + \frac{\partial (QH)}{\partial y} \right) + \frac{\partial K}{\partial x} \right\} \\ v_2 &= \frac{2\varepsilon\beta}{\varkappa^2} \left\{ +J \frac{\partial Q}{\partial x} + \frac{\partial (QH)}{\partial x} + \frac{\partial K}{\partial y} \right\} \\ w_2 &= \frac{2\varepsilon\beta}{\varkappa^2} \cdot \frac{\partial K}{\partial z} \\ c^2 \nu_2 &= \frac{2\varepsilon\beta}{\varkappa} \varDelta K \end{aligned} \right\} \quad . \; . \; . \quad (6)$$

Here also J and H are functions of r only, and must satisfy the differential equations

$$\left.\begin{aligned} &\left(\frac{d^2 J}{dr^2} + \frac{2}{r} \frac{dJ}{dr} \right) \frac{\partial Q}{\partial r} + 2 \frac{dJ}{dr} \frac{\partial Q}{\partial r} = \frac{\partial Q}{\partial r} (E - b) \\ &\left(\frac{d^2 H}{dr^2} + \frac{2}{r} \cdot \frac{dH}{dr} \right) Q + 2 \frac{dH}{dr} \cdot \frac{\partial Q}{\partial r} = Q (F + b). \end{aligned} \right\} \quad . \; . \; . \quad (7)$$

The constant b must also here be added for the same reason as above given.

The function K is to be computed from the equation

$$\Delta K+\frac{dJ}{dr}\left(\frac{\partial Q}{\partial y}\cdot\frac{x}{r}-\frac{\partial Q}{\partial x}\cdot\frac{y}{r}\right)=0 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

From this last equation it follows that the introduction of the function K can be omitted when the temperature of the atmosphere is assumed symmetrical with reference to the earth's axis. In this case $w_2=0$ and the [atmospheric] movement resulting from the rotation of the earth consists exclusively in a movement of rotation depending on the geographical latitude and the altitude above the earth's surface.

In order to present in the ordinary manner the currents of air for a given point in the atmosphere, the following components are to be introduced instead of u, v, w:

V, the vertical component computed positively upwards;

N and O, the two horizontal components, of which the first indicates movement toward the north, the latter, movement toward the east;

θ, the complement of the geographical latitude of a given place;

ψ, the longitude counted from an arbitrary meridian;

then we have

$$\left.\begin{aligned} V&=+(u\cos\psi+v\sin\psi)\sin\theta+w\cos\theta \\ N&=-(u\cos\psi+v\sin\psi)\cos\theta+w\sin\theta \\ O&=-u\sin\psi+v\cos\psi. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (9)$$

The formulæ (4, 6, and 9) contain the general solution of the problem so far as this is at present intended to be given, assuming the distribution of temperature to be given and that the functions E, F, J, H, K are determined in accordance with the boundary conditions.

IV.

When one attempts to represent the distribution of temperature on the earth's surface by a series of harmonic functions then the most important term is a harmonic function of the second order. Therefore as a first approximation we put

$$T_1=\left(Ar^2+\frac{A'}{r^3}\right)(1-3\cos^2\theta).$$

This function, with a proper determination of the constants, expresses the great contrast in temperature between the equator and the pole. If now one would take into account the variation with the seasons one must next introduce harmonic functions of the first order. The consideration of the various peculiarities of the earth's surface will of course demand further terms that depend on the geographical longitude also.

I have at first limited myself to the computation for the above given distribution of temperature, and put

$$Q = Ar^2\,(1-3\cos^2\theta)$$
$$Q' = \frac{A'}{r^3}\,(1-3\cos^2\theta).$$

The functions E, F, H, J are now to be computed with the help of this Q, and the corresponding E', F', H', and J' with the help of this Q'. We first obtain the general expressions:

$$V = \frac{\alpha GR^2}{\varkappa}(1-3\cos^2\theta)\left[A\left\{r^2\frac{dF}{dr}+2r\,(F+E)\right\}\right.$$
$$\left.+\frac{A'}{r^4}\left\{r\frac{dF'}{dr}-3\,(E'+F')\right\}\right]$$

$$N = -\frac{\alpha GR^2}{\varkappa}\,6\cos\theta.\sin\theta\left\{Ar(F+E)+\frac{A'}{r^4}(F'+E')\right\}$$

$$O = \frac{\alpha GR^2 2\varepsilon}{\varkappa^2}\sin\theta\left[(1-3\cos^2\theta)\left\{Ar\left(r\frac{dH}{dr}+2\,(H+J)\right)\right.\right.$$
$$\left.\left.+\frac{A}{r^4}\left(r\frac{dH'}{dr}-3\,(H'+J')\right)\right\}+6\cos^2\theta\left\{Ar\,(H+J)+\frac{A'}{r^4}(\mathrm{H}'+\mathrm{J}')\right\}\right]$$

The actual computation, having due reference to the boundary conditions, of the functions here introduced, gives results that are difficult to be discussed. But this is simplified when we make use of the circumstance that the atmosphere fills a very thin shell in comparison with the terrestrial sphere, wherefore the distances from the earth's surface are all small in comparison with the earth's radius. If we put

$$r = R\,(1+\sigma)$$

then is σ small with respect to unity. If we introduce these quantities in the above given equations and put

$$r\frac{dF}{dr}+2\,(E+F)=Rf(\sigma), \qquad F+E=R\varphi(\sigma);$$

$$r\frac{dF'}{dr}-3\,(E'+F')=Rf'\,(\sigma), \qquad F'+E'=R\varphi'\,(\sigma);$$

$$r\frac{dH}{dr}+2\,(H+J)=R^3g\,(\sigma), \qquad H+J=R^3\gamma\,(\sigma)$$

$$r\frac{dH'}{dr}-3\,(H'+J')=R^3g'\,(\sigma), \qquad H'+J'=R\nu^3\gamma'\,(\sigma);$$

then by restricting ourselves to the terms of the lowest order, we can obtain simple expressions for these functions. Primarily we find that the functions f and f', φ and φ', g and g', γ and γ' are identical.

Moreover the two constants A and A', which occur in the combination

$$A\ R^2 + \frac{A'}{R^3}$$

can be expressed in terms of the temperatures of the earth's surface at the equator, T_a, and at the pole, T_p. We have

$$\frac{1}{3}\ (T_a - T_p) = A\ R^2 + \frac{A'}{R^3}$$

Finally we put

$$C = \frac{\alpha\ G\ R^2}{\varkappa}\ \tfrac{1}{3}\ (T_a - T_p)$$

$$D = \frac{\alpha\ G\ R^4}{\varkappa^2}\ 2\varepsilon.\ \tfrac{1}{3}\ (T_a - T_p).$$

The numerical value of these two last constants can not be given, since, as before remarked, the coefficient of friction, $\varkappa$, will not agree with that determined from laboratory experiments. In any case D is considerably larger than C, since in D the fourth power of the radius of the earth occurs, but in C only the second power. The components of motion of the atmosphere are, therefore:

$$V = C\ (1 - 3\ \cos^2\ \theta).\ f\ (\sigma)$$
$$N = -C.\ 6\ \cos\ \theta\ \sin\ \theta.\varphi\ (\sigma)$$
$$O = D\ \sin\ \theta \left\{ (1 - 3\ \cos^2\ \theta)\ g\ (\sigma) + 6\ \cos^2\ \theta.\gamma\ (\sigma) \right\}$$

If we take $R.h$ for the altitude of the atmosphere as above defined, then the four functions, f, φ, g, γ, are to be so determined that they satisfy the prescribed boundary conditions for $\sigma = 0$ and $\sigma = h$. I have executed this computation for the most general case, namely, that in which at the upper limit slipping occurs without friction, but at the lower limit sliding with friction. Undoubtedly however the condition of the atmosphere on the earth's surface is much more nearly that of adhesion than that of free slipping, so that I will here communicate only the solutions for this latter case. For this case the motion at the earth's surface is everywhere zero. But for this motion one can easily substitute the motion at a slight altitude, that is to say, for small values of σ. For the four functions we find the following expressions:

$$f\ (\sigma) = \frac{\sigma}{8}(h - \sigma)\ (3h\sigma - 2\sigma^2)$$

$$\varphi(\sigma) = \frac{\sigma}{48} \left\{ 6h^2 - 15h\sigma + 8\sigma^2 \right\}$$

$$g\ (\sigma) = \frac{\sigma}{480} \left\{ -9h^5 + 15h^2\sigma^3 - 15h\sigma^4 + 4\sigma^5 \right\}$$

$$\gamma(\sigma) = \frac{\sigma}{960} \left\{ 20h^2\sigma^2 - 25h\sigma^3 + 8\sigma^4 \right\}$$

According to this solution the following gives a picture of the atmospheric circulation, which in its principal points agrees with that of W. Siemens.

(1) *Currents on a spheroid without rotation.*

These currents consist of currents in the meridian, and of vertical movements.

(*a*) The meridional current in the northern hemisphere is southerly below, but northerly above, since the function φ changes its sign when σ increases from zero to h. It attains its largest value at 45°, and disappears at the equator and at the poles.

(*b*) The vertical circulation is zero at the earth's surface and at the upper limit of the atmosphere. From the equator to 35° 16′ north and south latitudes the flow of air is positive—that is to say, ascending—but in higher latitudes it is descending. Its velocity at the poles is twice as great as that at the equator.

By the comparison of the expressions for $f\ (\sigma)$ and $\varphi\ (\sigma)$, it appears that the former function contains the fourth powers of the small quantities h and σ; the latter function contains their third powers. Therefore, the vertical flow is to the horizontal flow, so far as magnitude is concerned, as h is to 1, or as the altitude of the earth's atmosphere is to the radius of the earth. From this we can scarcely assume that we should be successful in the direct observation of the vertical current. The great effect of the vertical current arises from this, that it rises or sinks over a very extensive area.

(2) *Currents in consequence of the rotation of the earth.*

Under the assumption here made as to the distribution of temperature on the earth's surface, these currents consist exclusively of movements along the parallel circles of latitude. As in the case of the two terms in the component O, so here we distinguish the two following.

(*a*) The movement depending on the function $g\ (\sigma)$. Since this function is invariably negative; therefore to begin with at the equator the motion is directed toward the west. It changes its sign at latitude 35° 16′, and then becomes a motion directed toward the east.

(*b*) The second current is zero at the equator; becomes a maximum at 54° 44′, and is exclusively directed toward the east. Both currents disappear at the poles.

The two motions (*a*) and (*b*) differ from each other fundamentally in that $\gamma\ (\sigma)$ differs from zero first when σ has larger values. It is therefore a current that only occurs in the higher strata of the atmosphere. But thereby the function g is of a higher order than γ for the small quantities h and σ. Therefore at great altitudes the current (*b*) must greatly exceed the current (*a*) in velocity.

The components 1*a* and 2*a* combine at the earth's surface to form the regular movement of the air that we designate as the lower trade wind.

On the ocean where this system of winds can freely develop in the manner here assumed, without the influence of continents, their course is in good agreement with the conclusions of theory. Thus, on the northern hemisphere, between 0° and 35° latitude, east and northeast winds prevail; at 35° nearly north or in general only feeble winds; in higher latitudes northwest and west winds.

It results from the preceding that the two currents (1*a*) and (1*b*) are of the same order of magnitude and give moderate winds in the lower strata of atmosphere. Since now the current (2*b*), in comparison with (2*a*) is of a different order of magnitude, therefore the former is by far the most intense of all currents of air, but only in the upper strata of the atmosphere.

In so far as this component combines with the upper current (1*a*), it forms in the tropics the southwest or upper trade wind. In higher latitudes the purely westerly current prevails. So far as is known to me, the observations of the highest clouds which show prevailing west winds agree herewith. That the just-mentioned rotation-currents attain a great velocity has its reason in this that they can circulate around the whole earth without being hindered by the friction of a lower opposite current, as for instance is the case with the meridional currents. I consider it probable (as also W. Siemens has already announced) that in this powerful upper current we have to seek for the principal source of the energy found in the wind system of the lower strata.

XIII.

ON THE PHENOMENA OF MOTION IN THE ATMOSPHERE.*

(*SECOND COMMUNICATION.*)

By Prof. A. OBERBECK, *of Greifswald.*

I.

A comparison of the highest and lowest atmospheric temperatures at the surface of the earth shows permanent differences of 70°C. If the pressure were uniform everywhere these would correspond to differences of density of the air of more than 20 per cent. Since, however, pressure and density mutually influence each other one should therefore expect minima of pressure at places of highest temperature and maxima of pressure at places of low temperature of a corresponding intensity.

Instead of this the average differences of pressure on the earth's surface attain only 6 or 7 per cent., and even the largest rapidly passing barometric variations scarcely exceed 10 per cent. We explain the relatively small value of these differences of pressure by the formation of corresponding currents; a lower current at the earth's surface in the direction of the increasing temperature and an opposite upper current. Still the above-mentioned rule as to the connection between temperature and pressure must be true in general. But this is by no means always the case. While the equatorial zone of highest temperature shows a feeble minimum of pressure there occurs a maximum of pressure between the twentieth and fortieth degree of latitude from which toward either pole, and especially markedly in the southern hemisphere, the atmospheric pressure very decidedly sinks.

It appears to me not to be doubted that we can explain this remarkable phenomenon only by the influence of the rotation of the earth upon the currents of air that originate in temperature differences. In a previous memoir † I have endeavored to carry out an analytical treatment of these phenomena of motion under certain assumptions which

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* Read before the Royal Prussian Academy of Sciences at Berlin, November 8, 1888. Translated from the *Sitzungsberichte Königl. Preus. Akad. der Wissenschaften* zu Berlin. 1888, pp. 1129–1138.

† [See the previous number (XII) of this collection of Translations.—*C. A.*]

are there given in detail. In that memoir the pressures were not explained; this is done in the present treatise. I have arrived thus at the result that the distribution of pressure just described finds its explanation completely in the currents of the atmosphere, and that from the observed values of the pressure a conclusion can be drawn as to the intensity of the atmospheric currents.*

II.

In conformity with the notation of my first memoir the temperature of the atmosphere will be expressed by

$$T = T_0 + T_1$$

where T_0 depends only upon r, the distance of the point in question from the center of the earth, while T_1 is a function of r and of θ, the polar distance.

Let the pressure at the given point be

$$p=p_0(1+\nu)$$

In this expression p_0 also depends only upon r, while ν is a function of r and θ. So far as the observations of atmospheric pressure show, ν can be considered as a small numerical quantity in comparison with unity. For determining p_0 the following equation holds good:

$$c^2 \log p_0 = \text{constant} + GR^2\left(\frac{1}{r}+\alpha\int\frac{T_0}{r^2}dr\right)$$

from which the diminution of pressure as a function of the altitude above the earth's surface can be computed when the law of the diminution of temperature with the altitude, that is to say, the value of T_0 as a function of r is known.

Let us further put

$$\nu=\nu_0+\nu_1+\nu_2+\nu_3$$

in which

$$\nu_0=-\frac{GR^2\alpha T_1}{r}$$

while ν_1, ν_2, ν_3 shall indicate the values determined in the previous memoir (pages 180 and 181).

The first two terms of this summation $\nu_0+\nu_1$ give those changes in pressure which result directly from the differences of temperature on the earth's surface; that is to say, without considering the rotation of the earth.

If the temperature diminishes uniformly on both hemispheres from the equator toward the poles; or, in other words, if the temperature

* [Ferrel had published similar conclusions in 1859 but Oberbeck's independent confirmation is none the less valuable.—*C. A.*]

depends only on the geographical latitude (and not also on the longitude), then the motion of the air can only consist in vertical and meridional currents, and which (corresponding to the above given component velocities u_1, v_1, w_1) consist of one lower current toward the equator and of one upper current toward the poles. The distribution of pressure $\nu_0+\nu_1$ existing in connection with this furnishes (by means of the equation (4), page 182 of the previous memoir) the anticipated result that on the surface of the earth the pressure increases from the equator toward the pole, while at a medium altitude the differences of pressure disappear, but that finally, at greater altitudes, the pressure is greatest at the equator and least at the poles.

Since as above remarked, the actual distribution of pressure in nowise agrees with the above, it must be concluded that the influence of the term $\nu_0+\nu_1$ on the pressure can only be slight.

From the previous developments it results that the term ν_2 disappears under the assumption of a uniform distribution of temperature symmetrical with the earth's axis, so that as was already indicated in the first memoir, ν_3 will be the most important term.

III.

In the computation of this quantity ν_3 the system of equations previously given is to be used, namely:

$$c^2\frac{\partial \nu_3}{\partial x}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=2\varepsilon v_2$$

$$c^2\frac{\partial \nu_3}{\partial y}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-2\varepsilon u_2$$

$$c^2\frac{\partial \nu_3}{\partial z}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=0$$

Since according to the accordant opinion of meteorologists, as also according to my previous deductions, it is very probable that the intensity of the rotatory currents of the atmosphere materially exceeds that of the meridional currents, therefore I have only introduced into the further computation the rotation currents, whose components are designated by u_2 and v_2.

Since we have to do with a movement of rotation about the axis of z therefore we can put

$$u_2=-\chi y,\ v_2=+\chi x,\ w_2=0,$$

and these values can also be used for u, v, and w, in the above-given system of equations.

The relative angular velocity χ is to be deduced from the expression for the easterly component O (see equation (9), page 183). This is a func-

tion of θ and of r or also of σ the altitude above the earth's surface. The first system of equations is therefore transformed into the following:

$$c^2 \frac{\partial \nu_3}{\partial x} = (2\varepsilon + \chi)\, \chi x,$$

$$c^2 \frac{\partial \nu_3}{\partial y} = (2\varepsilon + \chi)\, \chi y.$$

$$c^2 \frac{\partial \nu_3}{\partial z} = 0.$$

Since χ is a function of r and θ, or of ρ and z if we put

$$z = r \cos \theta$$

$$\rho = r \sin \theta;$$

therefore, we can not find one function ν_3 that shall satisfy the three equations. If χ were independent of z we should find

$$c^2 \nu_3 = \text{constant} + \int (2\varepsilon + \chi)\, \chi \rho \, d\rho.$$

Since however this is not the case we must therefore conclude that the above-given system of equations still needs a supplement; that therefore a movement of rotation of a fluid to the exclusion of all other movements can only exist when the angular velocity in the direction of the axis of rotation is everywhere the same. If this is not the case then further currents occur perpendicular to the rotary motion. In our case these latter would consist of vertical and meridional movements. Their components may be designated by $u_3\ v_3\ w_3$. These are to be introduced into the above system of equations as was done in the corresponding fundamental equations (3) of the first memoir which now become

$$\left.\begin{aligned} c^2 \frac{\partial \nu_3}{\partial x} &= (2\varepsilon + \chi)\, \chi x + \varkappa \Delta u_3 \\ c^2 \frac{\partial \nu_3}{\partial y} &= (2\varepsilon + \chi)\, \chi y + \varkappa \Delta v_3 \\ c^2 \frac{\partial \nu_3}{\partial z} &= \varkappa \Delta w_3 \\ \frac{\partial u_3}{\partial x} + \frac{\partial v_3}{\partial y} &+ \frac{\partial w_3}{\partial z} = 0. \end{aligned}\right\} \quad \ldots\ldots \quad (2)$$

If the component motions indicated by the subscript 3 that directly depend on the movements subscript 1 are materially less in intensity than the movements of rotation, then in any computation of the pressure their introduction ought not to be omitted. The former memoir gave

a rather complicated value for the angular velocity χ. I have introduced a simplified expression for this in that, while retaining the dependence upon the polar distance θ, as there given, I have temporarily introduced a constant average value instead of the dependence upon the distance above the surface of the earth. According to this, one can put

$$\chi=\chi_1 \cos^2 \theta-\chi_2 \tag{3}$$

or with a slight difference

$$\chi=\frac{1}{R^2}\left\{ \chi_1 z^2-\chi_2 r^2 \right\} \tag{4}$$

In these equations χ_1 and χ_2 are considered as constants. Therefore, as before found, the movement of rotation of the air in higher latitudes is positive, that is to say, has the same sign as the axial rotation of the earth. For a specific latitude the average value is 0, and at the equator the movement has the opposite sign.

Further computation shows that the relative angular velocity χ is small in comparison with that of the earth ε, so that the simpler equations to be solved are as follows:

$$\left.\begin{aligned} c^2\frac{\partial \nu_3}{\partial x} &= 2\varepsilon\chi x + \kappa \Delta u_3 \\ c^2\frac{\partial \nu_3}{\partial y} &= 2\varepsilon\chi y + \kappa \Delta v_3 \\ c^2\frac{\partial \nu_3}{\partial z} &= \kappa \Delta w_3 \\ \frac{\partial u_3}{\partial x}+\frac{\partial v_3}{\partial y}+\frac{\partial w_3}{\partial z} &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots \tag{5}$$

In solving these we first determine a function $\mathfrak{F}$ that is of such form as to satisfy the conditions $\dfrac{\partial \mathfrak{F}}{\partial x}=2\varepsilon\chi x, \dfrac{\partial \mathfrak{F}}{\partial y}=2\varepsilon\chi y.$

These conditions give

$$\mathfrak{F}=\frac{\varepsilon r^2}{R^2}\left\{ \chi_1 z^2-\frac{\chi_2}{2} r^2 \right\} \tag{6}$$

Furthermore we put

$$u_3=\frac{\partial L}{\partial x},\ v_3=\frac{\partial L}{\partial y},\ w_3=\frac{\partial L}{\partial z}+M \quad \ldots\ldots \tag{7}$$

where L and M are two new functions of x, y, and z, we can then write the system of equations as follows:

$$c^2\frac{\partial \nu_3}{\partial x}=\frac{\partial \mathfrak{F}}{\partial x}+\kappa\frac{\partial}{\partial x}(\Delta L)$$

$$c^2\frac{\partial \nu_3}{\partial y}=\frac{\partial \mathfrak{F}}{\partial y}+\kappa\frac{\partial}{\partial y}(\Delta L)$$

$$c^2\frac{\partial \nu_3}{\partial z}=\frac{\partial \mathfrak{F}}{\partial z}+\kappa\frac{\partial}{\partial z}(\Delta L)-\frac{\partial \mathfrak{F}}{\partial z}+\kappa\,\Delta M.$$

The equation of continuity now becomes

$$\Delta L = -\frac{\partial M}{\partial z} \quad \text{(8)}$$

The three first equations lead to the two following:

$$c^2 \nu_3 = \text{Constant} + \mathfrak{F} - \varkappa \frac{\partial M}{\partial z} \quad \text{(9)}$$

$$\Delta M = \frac{1}{\varkappa} \cdot \frac{\partial \mathfrak{F}}{\partial z} \quad \text{(10)}$$

If the functions L and M are so determined that they satisfy the boundary conditions then the problem is to be considered as solved and equation (9) gives the desired distribution of pressure. As boundary conditions I have retained those previously laid down, viz. adhesion to the earth's surface, slipping on an upper boundary surface at an altitude $R.\,h$ above the earth whereby h is to be considered as a small number in comparison with unity.

For further calculation it is expedient to introduce the vertical and meridional components of the current or V and N. These are connected with L and M by the equations

$$\left.\begin{aligned} V &= \frac{\partial L}{\partial r} + M \cos \theta \\ N &= -\frac{1}{r}\frac{\partial L}{\partial \theta} + M \sin \theta \end{aligned}\right\} \quad \text{(11)}$$

The equation of continuity now becomes

$$\frac{\partial V}{\partial r} + \frac{2}{r} V = \frac{1}{r}\left\{ \cot \theta \,.\, N + \frac{\partial N}{\partial \theta} \right\} \quad \text{(12)}$$

The elimination of L gives the further equation

$$\frac{\partial (Nr)}{\partial r} + \frac{\partial V}{\partial \theta} = r\frac{\partial M}{\partial r} \sin \theta + \frac{\partial M}{\partial \theta} \cos \theta \quad \text{(13)}$$

The calculation gives the following values:

$$V = \frac{2\varepsilon}{\varkappa} R^3 \left\{ \chi_1 + 2\chi_2 - 6\,(4\chi_1 + \chi_2) \cos^2 \theta + 35\chi_1 \cos^4 \theta \right\} . f(\sigma) \quad \text{(14)}$$

$$N = \frac{2\varepsilon}{\varkappa} R^3 \sin \theta \cos \theta \left\{ -\chi_1 - 2\chi_2 + 7\chi_1 \,.\, \cos^2 \theta \right\} . \varphi(\sigma) \quad \text{(15)}$$

In these $f(\sigma)$ and $\varphi(\sigma)$ have a signification similar to that in the previous memoir, namely,

$$\left.\begin{aligned} f(\sigma) &= \frac{\sigma^2}{48}(h-\sigma)(3h-2\sigma) \\ \varphi(\sigma) &= \frac{\sigma}{48}\left\{ 6h^2 - 15h\sigma + 8\sigma^2 \right\} \end{aligned}\right\} \quad \text{(16)}$$

Moreover, σ is determined by the same equation as before,

$$r = R\,(1+\sigma)$$

Finally, from the equation (9)

$$c^2 \nu_3 = \text{const} + \aleph - \epsilon \frac{\partial M}{\partial z}$$

there results the following:

$$c^2 \nu_3 = \text{const} + \epsilon R^2 \left\{ \left(\frac{3\chi_1}{7} + \chi_2 \right) \cos^2\theta - \chi_1 \cos^4\theta \right\} \quad . \quad . \quad (17)$$

This last equation allows of a direct comparison with the above-mentioned observations of the distribution of pressure.

IV.

The average values of the pressure of the air in the Southern Hemisphere are given in the following table (under the column of observations) as a function of the latitude.*

Air pressure at the earth's surface.

Latitude.	Observed.	Computed.
°	*mm.*	*mm.*
0	758.0	758.0
S. 10	759.1	758.9
20	761.7	760.5
30	763.5	762.0
40	760.5	760.5
50	753.2	755.3
60	743.4	747.1
70	738.0	738.0
80	......	730.9
S. 90	......	727.2

These pressures are fairly represented by an expression of the form

$$p = p_a + a\cos^2\theta - b\cos^4\theta.$$

If we determine the constants a and b from the observed values for two different polar distances, for which I have used $\theta=50°$ and $\theta=20°$, then we obtain

$$p = 758 + 31.295\cos^2\theta - 61.094\cos^4\theta.$$

By the means of this formula the values given in the second column, under "computed," have been obtained.

* See A. Sprung, *Lehrbuch der Meteorologie*, p. 193; J. van Bebber, *Handbuch der Witterungskunde*, II, p. 136. [These figures are taken originally from Ferrel, "Meteorological Researches," I, 1880.—*C. A.*]

Furthermore, if we make the very probable assumption that the variations in pressure here considered depend exclusively on the movement of rotation, that therefore

$$p=p_a(1+\nu_3)$$

where p_a represents the pressure at the equator, then is

$$\nu_3=\frac{p-p_a}{p_a}.$$

Therefore

$$\nu_3=\frac{\cos^2\theta}{758}\left\{31.295-61.094\cos^2\theta\right\}$$

$$=0.0413\cos^2\theta-0.0806\cos^4\theta \quad . \quad . \quad . \quad . \quad . \quad (19)$$

But the computation of ν_3 had already given

$$\nu_3=\frac{\varepsilon R^2}{c^2}\cos^2\theta\left\{\frac{3\chi_1}{7}+\chi_2-\chi_1\cos^2\theta\right\}$$

wherein the appended constant can be omitted.

Hence, the two expressions for ν_3 can be put equal to each other, and for the computation of the motion of rotation we obtain the two equations

$$\frac{\varepsilon R^2}{c^2}\chi_1=0.0806$$

$$\frac{\varepsilon R^2}{c^2}\left(\frac{3\chi_1}{7}+\chi_3\right)=0.0413$$

If in these we put

$$R=6379600^{m};\ c=280^{m};$$
$$\varepsilon=0.00007292$$

then we shall obtain

$$\chi_1=0.0292\ \varepsilon$$
$$\chi_2=0.0836\ \chi_1.$$

Hence, the relative angular velocity of the rotary motion of the air is

$$\chi=0.0292\ \varepsilon\left\{\cos^2\theta-0.0836\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

This is small in comparison with ε, the angular velocity of the earth, therefore it nowhere leads to improbably large movements of the atmosphere. If we form the product $\chi_1 R$, we obtain for it the value 13.58 metres per second. But the true linear velocity corresponding to the rotatory motion is

$$O=\chi.\ R.\ \sin\theta.$$

The maximum value of this occurs at S. latitude 56° 27′ and amounts to 4.59 metres per second. From the S. pole to 16° 49′ S. latitude the average

value of the rotatory motion is positive, that is to say, directed toward the east; thence to the equator the value is negative, therefore directed toward the west.

These results can easily be combined with the conclusions of my previous memoir, according to which the motion of rotation can be considered as the sum of two terms that are of entirely different natures. Of the second term it was remarked especially that the current corresponding to it first attains sensible values at great altitudes. This therefore becomes at that altitude materially larger than the above deduced average value. The first term gave a movement entirely confined to the lower strata of the atmosphere: it is directed toward the east from the pole down to 35° latitude, but directed toward the west exclusively in the equatorial zone and less in velocity than the first component movement. The numerical computation leads to the same conclusion, since χ_2 is small in comparison with χ_1 Since from 35° of latitude down to the neighborhood of the equator there are two currents of opposite signs flowing over each other, therefore the place where the average movement of rotation is 0° will lie nearer to the equator than to 35°.

Therefore the conclusion of W. Siemens, which gave the first stimulus to the present investigation, has to be subjected to a modification only in so far as we must consider that the westward movement of the upper regions and higher latitudes has a predominance over the easterly movement of the lower regions and lower latitudes, because the former loses a much smaller fraction than the latter of its living force in consequence of friction.

The vertical and meridional components V and N are to be added to the corresponding components that were computed in my first memoir. The vertical component is positive at the equator and at the pole, it therefore gives an ascending current at both places, whereas V is negative throughout a broad central zone. *Therefore at the equator the ascending current is strengthened, at the pole the descending current is enfeebled.*

The meridional component N is zero at the surface of the earth at the equator; it is negative, *i. e.*, it is directed toward the south from thence to about 24° latitude; thence to the pole, where it is again zero, it has a northerly direction. Therefore in the tropics it strengthens the equatorial current and in higher latitudes it enfeebles it. Perhaps this explains the occurrence of northwest winds which frequently occur in the southern hemisphere between 50° and 60° south latitude.

Finally it may be remarked that the formula above used for the distribution of pressure agrees still better with the observations if a third term with a 6th power of cos θ is introduced. This term would also find its explanation by the analytical development, since the newly found meridional current should properly be again evaluated, in order to further compute the movements of rotation that are to be added

to the first approximation, and which will bring about a corresponding change in the formula for pressure.

In other words, by a series of approximations one seeks the true solution in a manner similar, for instance, to that used in the computation of mutual inductive effects of two conductors, in which computation we imagine the total influence developed into a series of individual influences of the first conductor upon the second and then again of the second upon the first, and so on. It is easy to foresee that the further prolongation of the computation must afford a corresponding term in the expression for the pressure. By this means the expression for the rotatory motion will suffer some change; still it is to be seen that the order of magnitude of this is already correctly established. After the execution of the further computations just indicated, I expect then to elaborate in a similar manner the average distribution of pressure in summer and in winter in order to determine more precisely the changes of the rotatory motion with the seasons. The formula above found is only to be applied with caution to the northern hemisphere, since in this hemisphere the fundamental condition that the temperature is a function of the geographical latitude applies much less truly than in the southern hemisphere.

XIV.

A GRAPHIC METHOD OF DETERMINING THE ADIABATIC CHANGES IN THE CONDITION OF MOIST AIR.*

By Dr. H. Hertz.

The theoretical meteorologist daily has to discuss considerations as to the changes of condition that take place in moist air that is compressed or expanded without the addition of any heat. Hence he desires to attain answers to these questions with the least possible expenditure of time, and he does not care to use any of the complicated formulæ of thermo-dynamics. Actually he generally uses the small practical table that Professor Hann communicated in the year 1874 (*Zeit. der Oest. Ges. f. Met.*, 1874, IX, p. 328). Still it appears that with at least an equal convenience one may attain a greater completeness if one makes use of the graphic method, and the table accompanying this paper presents an attempt in this direction. This contains nothing theoretically new except in so far as that it also completely considers the peculiar behavior of moist air at 0° C., which, so far as I know, has hitherto not been treated of.† In the following I will now in Section I, collect together the exact formulæ of the problem, since a complete collection of such appears to be wanting. Under Section II, the presentation of the formulæ by the graphic table is described. Finally under Section III, I explain completely, although purely mechanically, the application of the latter to a numerical example. If one follows this example with the diagram in the hand, one attains a judgment as to the use of the table and a knowledge of the method of using it without the necessity of going through the computations of Sections I and II.

I.

In a kilogram of a mixture of air and aqueous vapor let λ represent the proportional weight of dry air and μ the proportional weight of unsaturated aqueous vapor contained therein. Let the pressure of the mixture be p and its absolute temperature be T. Our problem is: What conditions will the mixture pass through when its pressure is di-

* Translated from the *Meteorologische Zeitschrift*, 1884, vol. I, pp. 421–431.

† See, however, Guldberg and Mohn, "Studies on the movement of the atmosphere," part 1, pp. 9–16, and, also, by the same authors, *Oest. Zeit. f. Meteorologie*, 1878, xiii, p. 117.

minished indefinitely without addition of heat? We must distinguish different stages.

First stage.—The vapor is unsaturated; liquid water is not present. We assume that the unsaturated vapor follows the laws of Gay-Lussac and Mariotte. Let e be the partial pressure of the aqueous vapor; $p - e$ be that of the dry air; v the volume of a kilogram of the mixture. We then have $p - e = \lambda \frac{R\,T}{v}$; $e = \mu \frac{R_1\,T}{v}$ where R and R_1 are constants of well known meaning and value.

Since now the total pressure p is the sum of these two values, therefore

$$pv = (\lambda R + \mu R_1)\,T$$

and this is the so-called equation of condition [equation of elasticity] for the mixture. If further, c_v is the specific heat of air at constant volume and c^1_v the same for aqueous vapor, then in order to bring about the changes dv and dT, the quantity of heat to be added to the air must be

$$dQ_1 = \lambda \left\{ c_v\,dT + A\,R\,T\frac{dv}{v} \right\}$$

On the other hand, the quantity of heat to be added to the aqueous vapor must be (see Clausius *Mechanische Wärmetheorie.* 1876, vol. I, p. 51.)

$$dQ_2 = \mu \left\{ c'_v\,d\,T + A\,R_1\,T\frac{dv}{v} \right\}.$$

Therefore for both together, the quantity of heat is

$$dQ = (\lambda\,c_v + \mu\,c'_v)\,d\,T + A\,(\lambda\,R + \mu\,R_1)\;T\frac{dv}{v}$$

But this quantity of heat must be zero for the adiabatic changes now investigated by us. In order to integrate the differential equation arising from putting dQ equal to 0, we divide it by T. From the mechanical theory of heat we know beforehand that by this operation the equation becomes integrable, and we find this confirmed *a posteriori.* If we carry out the integration and eliminate v by means of the equation of elasticity, in that we recall that $c_v + AR$ is equal to c_p or the specific heat under constant pressure there follows

$$(\lambda\,c_p + \mu\,c'_p)\log\frac{T}{T_0} - A\,(\lambda\,R + \mu\,R_1)\log\frac{p}{p_0} = 0 \quad . \quad . \quad . \quad (1)$$

The quantity that forms the left-hand side of this equation has a physical significance. It is the difference of the entropy of the mixture in the two conditions that are characterized by the quantities $p\,T$ and $p_0\,T_0$. Moreover the mixture evidently behaves exactly like a gas

whose density and specific heat have values midway between those of the aqueous vapor and the air.

We now have to compute the limit of p up to which the equation (1) may be used. Hereafter let e be the pressure of the saturated aqueous vapor at the temperature T; e is a function of T, but of T only. The mass ν of saturated aqueous vapor that is present in the volume v at the temperature T amounts to

$$\nu = \frac{ve}{R_1 T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

and this quantity must be greater than μ so long as the vapor is unsaturated. Therefore the limit occurs when $\mu = \nu$. If we substitute for v its value from the equation of elasticity, then this latter condition ($\mu = \nu$) takes the form

$$p = \frac{\lambda R + \mu R_1}{\mu R_1} e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1b)$$

As soon as T and p attain values that satisfy this equation, we must relinquish the use of equation (1) and pass over to the equations for the second stage.

Second stage.—The air is saturated with aqueous vapor and contains also additional fluid water. We neglect the volume of the latter. We can therefore here also consider the air on the one hand and the water, with its vapor, on the other hand, each as though the other were not present. To both are to be ascribed the same volume v and the same temperature T as that of the mixture; on the other hand, the pressure p of the mixture is equal to the sum of the partial pressures, $p_1 = \frac{\lambda RT}{v}$ for the air and $p_2 = e$ for the aqueous vapor.

$$\text{The equation } p = \lambda \frac{R\,T}{v} + e$$

or

$$(p - e)\, v = \lambda\, R\, T$$

is therefore now the equation of elasticity of the mixture. The quantity of heat that we must communicate to the air in order to bring about the changes dT and $d\nu$ is as before

$$dQ_1 = \lambda \left\{ c\, dT + ART \frac{dv}{v} \right\}.$$

On the other hand, the quantity of heat that must be communicated to the water in order to bring about the change dT, and to simultaneously increase by $d\nu$ the quantity ν of aqueous vapor, while pressure and volume change correspondingly, is

$$dQ_2 = Td\left(\frac{\nu r}{T}\right) + \mu c dT.$$

This equation is deduced in Clausius *Mech. Wärmetheorie*, vol. I, section vi, art. 11; and in it c is the specific heat of liquid water, r the external latent heat of vapor, both of them expressed in units of heat. Therefore the total heat communicated to the mixture is

$$dQ = \lambda \left\{ c_v d\ T + ART\frac{dv}{v} \right\} + dT\left(\frac{\nu r}{T}\right) + \mu c dT.$$

Here also we have to put $dQ = 0$, then divide by T and integrate. With the help of the equation of elasticity and equation ($1a$) we eliminate the quantities v and ν from the integral equation, and thus obtain

$$\left(\lambda c_p + \mu c\right) \log \frac{T}{T_0} + \lambda AR \log \frac{p_0 - e_0}{p - e}$$
$$+ \lambda \frac{R}{R_1} \left\{ \frac{r}{T} \frac{e}{p-e} - \frac{r_0}{T_0} \frac{e_0}{p_0 - e_0} \right\} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Here also the quantity on the left hand that is equated to zero represents the difference of the entropies between the final and the initial conditions of a kilogram of the mixture. The equation thus obtained can be used until the temperature attains the freezing point, then we arrive at the third stage.

Third stage.—In this case, in addition to the vapor and the liquid water, the air contains also ice. By further expansion of the air, the temperature will now not sink immediately further, for the latent heat of the freezing water will, even without a lowering of temperature, furnish the force necessary for overcoming external pressure. But the heat of liquefaction must not be applied to this purpose only, but also to the evaporation into vapor of a part of the already condensed water. For since the volume increases during the expansion without allowing the temperature to sink, therefore at the end of the process again, more water is become vapor than before, therefore the weight of the ice that is formed will be less than that of the fluid that was present.

Let now, again, ν be that portion of μ that is in the form of aqueous vapor, σ the part that exists as ice, and q the latent heat of liquefaction of a kilogram of ice. T, e, r are constants. Since therefore $dT=0$, we have now only to communicate to the air the quantity of heat $\lambda\ ART\frac{dv}{v}$ and to the water that we evaporate the quantity of heat $rd\nu$, and to the water that we allow to freeze the quantity $-qd\sigma$. Therefore the quantity of heat given to the whole mixture is

$$dQ = \lambda\ ART\frac{dv}{v} + rd\nu - qd\sigma.$$

If we put $dQ=0$, divide by T and integrate, there follows

$$\lambda\ AR \log \frac{v}{v_0} + \frac{r}{T}(\nu - \nu_0) - \frac{q}{T}(\sigma - \sigma_0) = 0$$

The division by T was necessary in this case only in order to give the left-hand side of the equation the form of a difference of entropy. With the help of the equation of elasticity and the equation ($1a$) we can eliminate v and ν, and introduce instead of them the pressure p. The equation then shows us how the quantity σ of ice that is formed varies with the change of pressure. The details of this process however interest us less than the limits within which it takes place. Therefore we let the subscript index figure 0 refer to the condition in which the mixture just reaches the temperature 0° in which therefore ice is not present, and where $\sigma_0=0$. On the other hand we let the subscript index figure 1 refer to the condition in which the last particles of water are freezing, in which therefore the temperature just begins to fall below zero. In this condition, evidently, $\sigma=\mu-\nu$, since only ice and vapor are now present. If now we substitute these values after introducing the pressure, there results

$$\lambda\, A R \log \frac{p_0-e}{p_1-e} + \lambda \frac{R}{R_1} \cdot \frac{e}{p_1-e} \cdot \frac{r+q}{T} - \lambda \cdot \frac{R}{R_1} \cdot \frac{e}{p_0-e} \cdot \frac{r}{T} - \mu \frac{q}{T} = 0 \quad . \quad . \quad (3).$$

This equation connects the pressures p_0 and p_1, at which respectively the third stage is attained and relinquished.

It was not necessary to append an index figure to the quantities e and T since they are alike for the initial and final conditions.

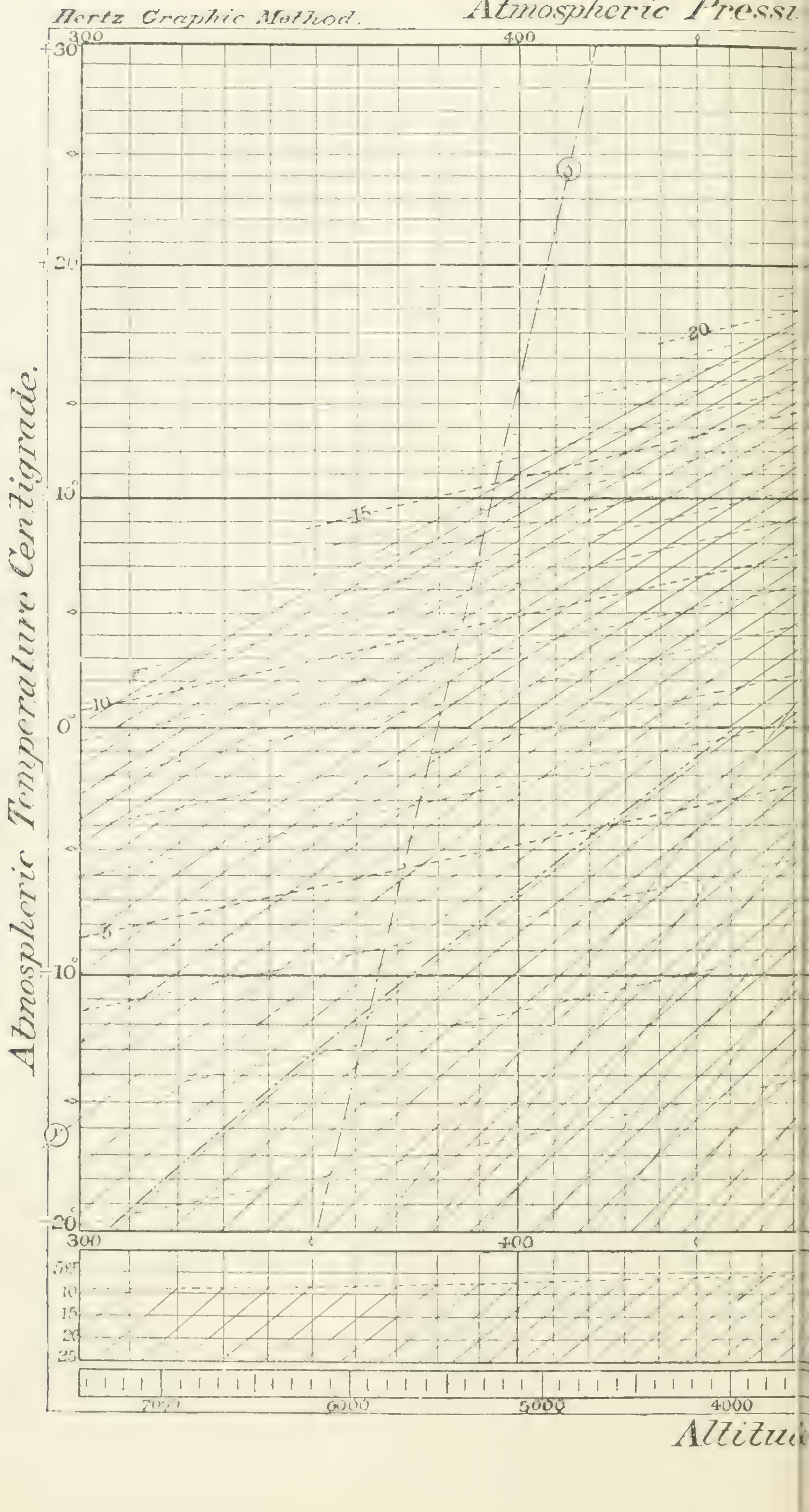
Fourth stage.—If now the temperature sinks lower, we have then only vapor and ice. The relations that we have to consider are the same as in the second stage, and the final formula is also the same. Only here the specific heat of evaporation has another value from that there given. Here, namely, it is equal to $r+q$ since the heat that is necessary to immediately change ice into vapor must exactly equal the heat that is needed to first melt the ice and then change the water into vapor. If we would be perfectly rigorous we ought not to assume q as constant, but must consider it as slightly variable with the temperature, but the differences are so small that here they may remain out of consideration. In this fourth stage we may attain to those low temperatures at which the air itself can no longer be considered as a permanent gas.

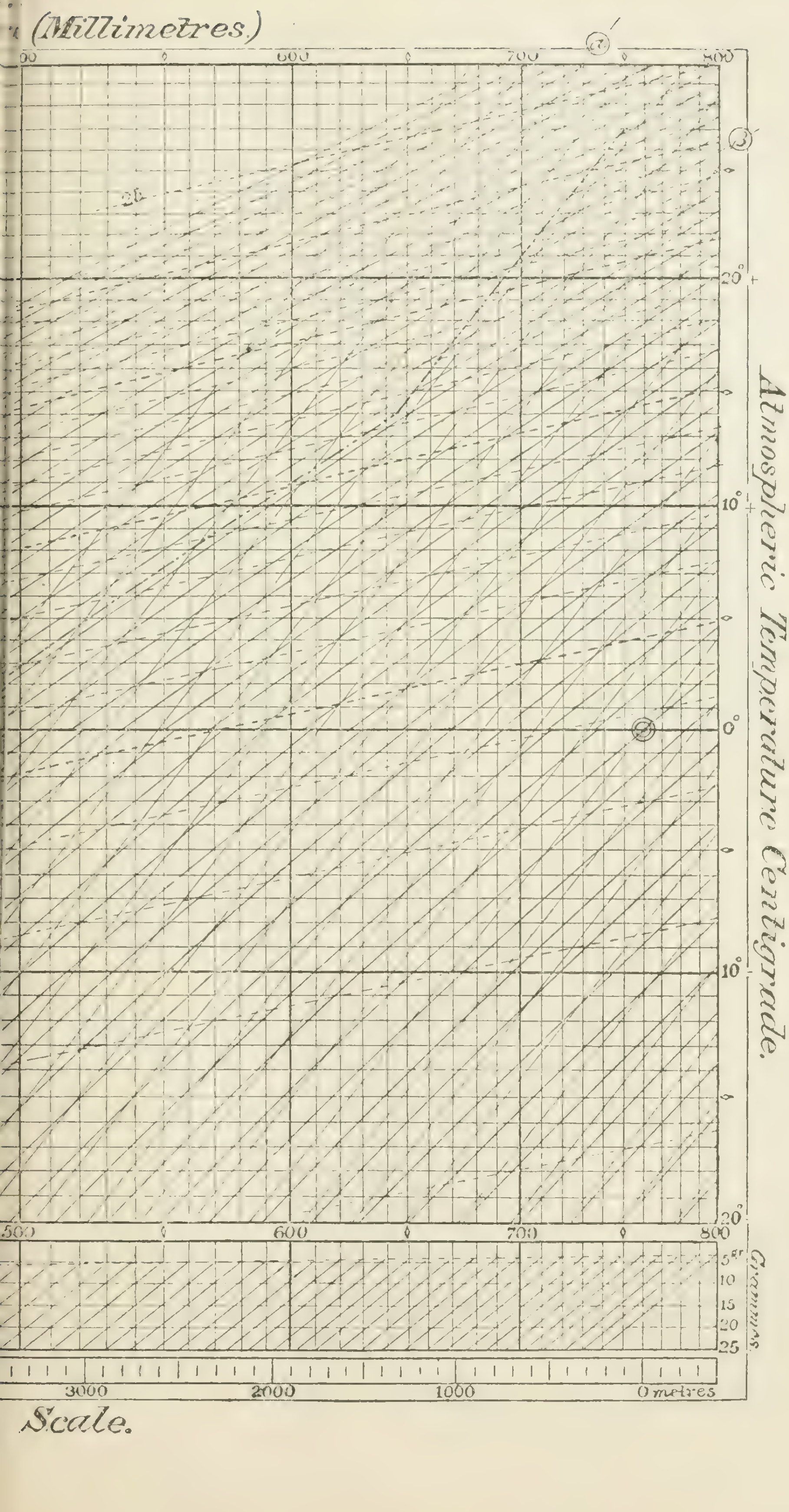
The four stages that we have here distinguished, one can very properly designate as the dry, the rain, the hail, and the snow stage.

If one is now in a position such that he is obliged to exactly follow the changes that a mixture containing a considerable percentage of water must undergo, then nothing further remains than to abide by these more complicated formulæ. In that case one proceeds in the following manner: First we substitute the values of λ and μ in all the equations. Then we substitute the quantities p_0 and T_0 for the given initial condition in equation (1). We then consider the resulting equation and the equation ($1b$) as two simultaneous equations with the two unknown quantities p and T. Solving those equations with reference to these quantities, we obtain that condition through which we must go in pass-

Fig. 2

Atmospheric Pressu





(Millimetres.)
600
700
800
20°
10°
0°
10°
20°
500
600
700
800
5gr
10
15
20
25
Grammes.
3000
2000
1000
0 metres
Atmospheric Temperature Centigrade.
Scale.

ing from the first to the second stage. The values thus obtained are then to be substituted as p_0 and T_0 in equation (2). By substituting $T = 273°$ in the equation thus obtained, we obtain that p_0 which occurs in the equations of the third stage. If now we further determine from the equation (3) the final pressure p_1 of the third stage then this pressure and the temperature 273° form the p_0 and T_0 of the equations of the fourth stage. It will frequently happen that the temperature down to which the first stage holds good will lie below the freezing point; in that case one passes directly over to the fourth stage, omitting the second and third. After we have thus determined for all the equations the coefficients and the limits for which each equation holds good we can use them in order to determine the T belonging to any given p or inversely. All these computations can however only be executed by successive approximations, and one would do well to take the necessary approximate values from the accompanying diagram. If we have determined p and T for any special condition then the remaining characteristics are easily found. The density of the mixture follows from the corresponding equation of elasticity. The equation (1a) gives the quantity of water still present in the form of vapor, and therefore also the quantity of water already liquefied. Frequently one desires to know the difference in altitude h that corresponds to the different conditions p_0 and p_1 under the assumption that the whole atmosphere is found in the so-called condition of adiabatic equilibrium. If one desires the exact solution of this problem, it must be attained by the laborious mechanical evaluation of the integral

$$h = \int_{p_1}^{p_0} v dp ;$$

but since it is precisely with regard to this point that an exact determination never has a special value, therefore here one may always abide by the accompanying convenient diagram.

II.

If we had to deal only with one mixture whose composition is *exactly* known for which we therefore can have only *one* value of the ratio $\mu : \lambda$, then we could exactly re-produce the formulæ above developed by a graphic table that would enable us to directly perceive the adiabatic changes of the mixture for any condition.

We should represent pressure and temperature by coördinates in one plane and cover this plane with a system of curves that should connect all those conditions together that can adiabatically pass from one to the other. It would then only be necessary to glide from a given initial condition along the curve going through the corresponding point in order to perceive the behavior of the mixture as it passes through all these stages.

Since however the meteorologist must necessarily deal with mixtures

of very various proportions, therefore by this method a great number of tables would be required. But it can now be shown that one can also manage with only one graphic table, if first we confine ourselves to those cases in which the weight and pressure of the aqueous vapor is small in comparison with the weight and pressure of the air, and if secondly we do not require of the results any greater accuracy than corresponds to the neglect of those quantities in comparison with these. If we neglect μ and e as compared respectively with λ and p, then the form of the curves to be drawn is the same for all the absolute values of μ, therefore the same curve can be used for all the different mixtures. But the points at which the different stages pass into each other will be located very differently for different mixtures, and special devices will therefore be needed by means of which this point may be determined. The graphic table, Fig. 28, is therefore constructed in accordance with the following principles.

The pressures are laid off as abscissas on the adopted scale for the interval between 300 millimetres to 800 millimeters of the barometer: the temperatures are laid off as ordinates for the interval between -20° Cels. and $+30^\circ$ Cels. But as we see by the diagram, a uniform increase in the length of either of these coördinates does not indicate an equal increase of pressure or of temperature; on the contrary the diagram is so constructed that an equal increase of distance corresponds to an equal increase in the logarithm of the pressure and in the logarithm of the absolute temperature. The advantage of this arrangement consists in the fact that thus the curves with which we have to do become, some of them exact, and some of them approximate straight lines, which brings an important advantage in the accurate construction and use of the table.

Now the adiabatics of the first stage (if we neglect μ with respect to λ) are given by the equation

$$c_p \log T - AR \log p = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\alpha)$$

In this diagram the logarithms are always those of the natural system. With Clausius we put

$$c_p = 0.2375 \frac{\text{Calorie}}{\text{Cels. degree} \times \text{kilogr.}}$$

$$A = \frac{1}{423.55} \frac{\text{Calorie}}{\text{Kilogrammetre}}$$

$$R = 29.27 \frac{\text{Kilogrammetre}}{\text{Cels. degree} \times \text{kilogr.}}$$

These adiabatics appear in our diagram as straight lines. One of them is distinguished by the letter alpha (α) and the whole of this system may be called by this letter. The individual lines are so drawn that

from one to the next the value of the constant (which is the entropy) increases by the quantity

$$0.0025 \frac{\text{Calorie}}{\text{Cels. degree} \times \text{kilogram}}.$$

These lines therefore appear at equal distances apart from each other. One of them is drawn to the point 0° Cels., and the pressure 760 millimetres.

The curves of the adiabatics in the second stage must satisfy the equation*—

$$c_p \log T - AR \log p + \frac{R}{R_1} \cdot \frac{r}{T} \frac{e}{p} = \text{constant} \quad . \quad . \quad . \quad . \quad (\beta)$$

In this equation $\frac{R}{R_1}$ is the density of aqueous vapor in reference to the air, and therefore is equal to 0.6219. According to Clausius,

$$r = 607 - 0.708 \, (T - 273) \frac{\text{calorie}}{\text{kilogram}}.$$

I have taken the value of e for the different temperatures from the table computed by Broch (*Travaux. du Bur. Internat. des Poids et Mesures*, tome I). The curves run along with feeble curvature from the right hand above to the left hand below. One of these is distinguished by the letter beta (β). They also are so drawn that the entropy increases from one to the next by a constant value of—

$$0.0025 \frac{\text{Calorie}}{\text{Cels. degree} \times \text{kilogram}}$$

or the same as before for the alpha system, and so that one of them passes through the point 0° C., 760 millimetres.

The curves that correspond to the third stage coincide with the isotherm of 0° C.

Finally the curves of the fourth stage are entirely similar to those of the second stage, but are not exactly the same, for their formula is derived from that belonging to the second system by substituting $r + q$ for r where q is equal to 80 calories per kilogram. They are distinguished by the letter gamma (γ), and are drawn according to the same rules as alpha (α) and beta (β) curves. In general the gamma curves are not precise prolongations of the β system.

We have now to find some means by which the points of transition

* Although μ is neglected in comparison with λ, still it is questionable whether $c\mu$ is negligible in comparison with $c_p\lambda$, since c is four times larger than c_p. Even although within the limits of the diagram μ does not exceed $\frac{1}{40}\,\lambda$, yet the $c\mu$ is $\frac{1}{10}\,c_p\lambda$. But in meteorologic applications we recall that in these extreme cases the liquid water is not generally wholly carried up with the air. Frequently so large a fraction of it falls from this air as rain that we keep nearer the truth when we entirely neglect the specific heat of the liquid water, rather than to introduce it with full value into the computation.

can be found for the different stages. The dotted lines serve to show the end of the first stage. These lines give the greatest quantity of water, expressed in grams and computed according to the formula $\nu = \frac{R}{R_1} \cdot \frac{e}{T}$ that a kilogram of the mixture in the different conditions can contain as vapor. Thus, for instance, the curve designated by 25 connects all those conditions in which one kilogram of the mixture when saturated contains 25 grams of vapor. These curves are drawn from gram to gram. If a mixture contains n grams of vapor in every kilogram of mixture, then evidently we have to follow the curve of the first stage up to the dotted line n, but then we must pass either to the second or fourth stage.

The limit of the second stage, with respect to the third, is given by the intersection of the corresponding adiabatic beta with the isotherm of 0° C. By the pressure p_0, that corresponds to this intersection, and by the quantity μ of water, is determined the pressure p_1, at which the transition takes place from the third to the fourth stage. The small auxiliary diagram that is given beneath the main table of Fig. 28 serves for the graphic determination of p_1. This auxiliary diagram contains as abscissa the pressure arranged as in the larger diagram, and as ordinate the total quantity μ of the water in all conditions expressed in grams per kilogram of the mixture. The oblique lines of this small table are the curves that correspond to the equation (3) of the third stage, when in this equation we consider p_0 as constant, but p_1 and μ as the variable coördinates. These lines are not perfectly straight, but are not to be distinguished from such in a diagram on this scale. The highest point of each of these lines corresponds to the case $p_1 = p_0$. The corresponding μ is not zero, but is equal to the least value, ν, that μ must have in order that the mixture may be saturated at 0° C., and the auxiliary table come into use. If one wishes to find the p_1 belonging to a definite value of p_0 and μ, then we seek that oblique line whose highest point lies on the abscissa p_0, and then we pass along this line downwards to the ordinate μ. The pressure at which we attain this ordinate is the desired pressure p_1. In this pressure we have the point of transition from the third to the fourth stage.

Having in this way determined the totality of the stages through which the mixture runs, we find the remaining desired quantities for each stage in the following manner:

(1.) The dotted line which one selects, (corresponding to the condition given,) indicates directly the number of grams of water still remaining in the form of vapor. If we subtract this quantity from the original total quantity μ, we obtain the quantity of water that has already been condensed.

(2.) The density δ of the mixture can under the adopted approximations be computed for all conditions by the formula

$$\delta = \frac{p}{RT}; \text{ or } \log \delta = \log p - \log T - \log R.$$

These can also be read off graphically if the diagram is covered with another system of lines of equal density. We see that these lines will constitute a system of parallel degrees of density.

Only one of these lines is in reality drawn on the accompanying diagram, namely, the line marked δ (delta), in order not to confuse the diagram. But with the assistance of this one we can also compare the densities in any two conditions C_1 and C_2, according to the following rule: From the points 1 and 2, representing these conditions on the diagram, draw two straight lines, respectively, parallel to δ, until they intersect the isotherm 0° C., and read off the pressures p_1 and p_2 for these points of intersection. The densities for the conditions C_1 and C_2 are in the ratio of the pressures $p_1 : p_2$; as is seen from the considerations that the densities for the condition (p_1, 0°), and for (p_2, 0°) are according to Mariotte's law in the ratio of p_1 to p_2, and are equal to the densities for the conditions C_1 and C_2 since they lie on the same line of equal density with these.

(3.) The difference of altitude h that corresponds under the assumption of adiabatic equilibrium to the passage from the condition p_0 to the condition p is given by the equation

$$h = \int_p^{p_0} v dp = R \int_p^{p_0} T \frac{dp}{p}.$$

In using this equation we take T as a function of p from the diagram and then perform the integration mechanically. Actually however the assumption of adiabatic equilibrum is always so imperfectly fulfilled that it is not worth while to trouble about an exact development of its consequences. On the other hand, for moderate altitudes, we commit a relatively very unimportant error when we give T an average value, and consequently consider it as constant. Within the limits of the diagram T ranges only between the values 253 and 303; if therefore we give it the constant value $T_0 = 273$, then the error in h will scarcely exceed one-ninth of the whole value. If we are satisfied with this error, then we have

$$h = \text{constant} - RT_0 \log p,$$

and we now can, along with the pressure, directly introduce the altitude as abscissa. Consequently an equal increase in the length of the abscissa will everywhere correspond to an equal increase in altitude. The scale of altitudes is introduced at the base of the diagram. Its zero point is put at the pressure 760, because this is usually taken as the normal pressure at sea-level.

III.

In order to illustrate the use of the table by an example, we propose to ourselves the following concrete problem: Given a mass of air at sea-level under the pressure of 750 milimetres, the temperature 27 degrees

centimetre, and relative humidity 50 per cent., it is desired to find what conditions this mass of air will pass through when it is carried without change of heat into the higher strata of the atmosphere, and therefore into a lower pressure, and at what approximate altitudes above the sea-level the different conditions will be attained.

We first seek from the diagram the point that corresponds to the initial stage. We find it as the intersecting point of the horizontal isotherm 27 and the vertical isobar 750. We remark that it lies almost exactly on the dotted line 22. This indicates that our mass of air must contain 22.0 grams of aqueous vapor in each kilogram of its own weight in order to be saturated. Since however it has only a relative humidity of 50 per cent., therefore it contains 11.0 grams of water per kilogram. We note this for future use. Furthermore, we go along down the isobar 750 to the scale of altitude that is found at the lowest edge of the diagram, and here we read off 100 metres. The 0 point of the scale of altitude therefore lies about 100 metres below the sea-level adopted by us as a base, and therefore we have to subtract 100 metres always from all the direct readings on the altitude scale, in order to obtain the altitude above sea-level. If now we raise our atmospheric mass upward, then the series of conditions which it runs through will be directly given by that line of the Alpha system that passes through the initial condition.* An engraved line not being given for this case we therefore interpolate such an one (*i. e.*, the — . . — . . line of the diagram). If the number of intersecting lines appears to be bewildering, then we take a strip of paper and lay it parallel to the system under consideration, when all confusion disappears. In order now to recognize the condition in the neighborhood of the altitude 700 metres we seek for the point $700 + 100 = 800$ on the scale of altitudes, and go perpendicularly up until we intersect our Alpha line. The intersection gives this point at pressure 687 milimetres, and temperature 19.3° C. But we ought to use the Alpha line only to that point in which it itself intersects the dotted line 11 (or the line of absolute weight of contained water). The attainment of this line indicates that we have arrived at a condition in which the air is only just able to contain 11 grams of water per kilogram in the form of aqueous vapor. Since now we have 11 grams per kilogram, therefore with any further cooling condensation begins. The pressure for the point at which precipitation commences is 640 milimetres; the temperature is 13.3° C. This is not the temperature of the original dew-point, but it is lower. The dotted line, eleven, intersects the isobar 750 at 15.8° C., and this is the initial dew-point. But since besides cooling our air has also experienced an increase in its volume, therefore the vapor has remained volatile to a

*The letters α, β, γ, that designate the systems are to be found in the small circles at the edge of the diagram. For each of these there corresponds one line of the system that it designates. A line of special dots and dashes in the diagram indicates the change of condition of the air in our illustrative example.

temperature 13.3. The altitude at which we now find ourselves corresponds to the lower limit of the formation of clouds, and is about 1,270 metres. In order to follow the conditions further we draw a curve of the Beta (β) system through the point of intersection.

This curve is inclined much more slowly toward the axis of abscissas than the Alpha line hitherto used, therefore the temperature now changes with the altitude much more slowly than before, which is due to the evolution of the latent heat of the aqueous vapor. We have now risen 1,000 metres since the commencement of condensation, but the temperature has sunk only to 8.2°, or only 0.51° to each 100 meters. We now find ourselves on the dotted line 8.9, and perceive that 8.9 grams of water are still in the state of vapor; that therefore in this first 1,000 metres of the cloud layer 2.1 grams of water have been condensed per kilogram of air. We attain the temperature zero degrees C. at the pressure 472 millimetres, and at the altitude 3,750 meters, whereas if the air has been dry, and we had not been obliged to leave the Alpha line, this temperature would have been attained at an altitude of 2,600 metres. It now appears that by this time 4.9 grams of water, or 0.45 per cent. of the total contents, have been condensed, and during further expansion this portion begins to freeze and form hail [the reader will recall that although 45 per cent. has been condensed into visible cloud, yet it has not separated from its original air and been precipitated as rain, but is still rising with the air and of course cooling with it]. But the temperature can not sink further until the last particle of water is frozen, and we therefore must retain the temperature 0° uniformly during a certain distance of further ascent.

In order to ascertain this distance we make use of the auxiliary diagram between the scale of altitude and the larger diagram, we pass down the isobar 472 millimetres to the dotted line of this diagram; we draw through this intersection a line parallel to the inclined line of the auxiliary table, and go along this line until we reach that horizontal line that is characterized by the number 11, or the total weight of the contained water, and which we easily interpolate between the engraved lines 10 and 15. As soon as we have attained this line we read off the pressure $p = 463$ millimetres, and turn back to the larger diagram. At the pressure thus found the process of freezing is finished, and the layer within which it all takes place has a thickness of about 150 metres. It must surprise one that, according to the dotted line, the quantity of water in the form of aqueous vapor has again increased a little during the process of freezing. But this is quite correct; in fact, the volume has increased without lowering the temperature. We leave the temperature 0° C. at the pressure 463 millimetres. The water which hereafter is precipitated passes directly over into the solid condition. Since there is now but little water as aqueous vapor, therefore the temperature again begins to sink more rapidly with the altitude. We ascertain the different conditions in that we make use of

that special Gamma line that can be drawn through the point 463 millimetres on the isotherm 0° C. The temperature -20 down to which our table can be used is attained at the altitude 7,200 metres, and at the pressure 305 millimetres, at which only two grams of water per kilogram remain as vapor, the other nine having been condensed. If it interests us to know how the density in this condition is related to the density in the initial condition, we draw through the corresponding points two lines parallel to the Delta line. These intersect the isotherm of 0° C. at the pressures 330 and 680 millimetres. The densities are to each other as these pressures, namely, as 33 to 68; and as 33 and 68 are to 76, so they are related to the density of the air in its normal condition of 0° C. temperature and 760 millimetre pressure.

All these items are directly read off from the diagram. Errors that could be injurious certainly occur only in the altitudes. These latter refer strictly speaking to ascent in an atmosphere of a uniform temperature of 0° C. But it would have been generally better to have assumed that the temperature of the atmosphere is everywhere the same as that of the ascending mass of air. The resulting error can be materially reduced by a very little computation. Thus we found that the point where condensation began, is at the pressure 640 millimetres. To this corresponds an altitude of 1,270 millimetres, provided that the temperature is 0°, but in our case this is between 27° and 13°, therefore on the average about 20°. For this temperature the altitude must be about $\frac{20}{273}$ or $\frac{1}{14}$ greater, since the density of the air is by this same fraction smaller than for 0°. Therefore the altitude really lies between 1,350 and 1,400 millimetres.

We must still supplement the above example by the mention of special cases:

(1) We assume in the above that during the hail stage the total quantity of water originally present in the air, namely, 11 grams, was still contained therein. This will certainly only be an appropriate assumption in the case of very rapid ascents. In other cases perhaps the greater part of the condensed water falls as rain, and therefore only a fraction of it remains to be frozen. If one has any estimate as to how great this fractional part is, then the diagram will always allow us to ascertain the correct conditions. Thus if in our example one had reason to assume that half of the water condensed at 0° were removed, then on attaining the isotherm of 0° only 8.5 grams of water per kilogram of air would be present. We should then in using the auxiliary table not descend to the horizontal 11, but only to the horizontal 8.5, and should have started from the temperature line of 0° at the point corresponding to the pressure 466 millimetres (instead of 463 millimetres); this would have been the only difference.

(2) If we had assumed not 50 per cent. but 10 per cent. relative humidity in our example we should then have been able to use the Alpha line only to the dotted line 2.2. This point of intersection occurs

at pressure 455 millimetres, and at temperature $-13.6°$ C., therefore considerably below 0. Therefore there would have been no formation of liquid water and therefore no stage for the formation of hail but only sublimation of water from the vaporous into the solid condition. We should then from the intersection of the Alpha line with the dotted line 2.2 have followed directly the line of the Gamma system that might have passed through this intersecting point.

The question is not uninteresting—what dew point is the highest that our mixture could have possessed in its initial condition as to pressure and temperature, in order that the condensation of liquid water, that is to say, the condensation at temperature above 0° C. should be just avoided? In order to answer this we follow the Alpha line to the isotherm 0° and here find the dotted line 5.25. We therefore at the highest could have had 5.25 grams of water per kilogram of air. In order now to ascertain at what temperature the air would then have been saturated under a pressure 750 millimetres, we slide along the line 5.25 up to the isobar 750 and intersect it at the temperature 4.8° C., and this is the desired maximum value of the dew point.

KIEL, *October*, 1884.

XV.

ON THE THERMO-DYNAMICS OF THE ATMOSPHERE.*

(FIRST COMMUNICATION.)

By Prof. WILHELM VON BEZOLD.

In the application of the mechanical theory of heat to the processes going on in the atmosphere we have hitherto almost exclusively confined ourselves to those cases in which one can disregard the increase or loss of heat during the expansion or compression.

The so-called convective equilibrium of the atmosphere, the unstable equilibrium in cyclones, the phenomena of the *foehn* winds have all hitherto been treated of under the assumption that we have to do with adiabatic changes of condition.

In fact, especially in the last-mentioned phenomena, the quantity of heat used or produced by expansion and compression as also by the changes in the physical condition of the water, are so prominent in comparison with those that, in these rapidly executed processes, are introduced or taken away by other sources that the above-mentioned assumption may be said to be thoroughly allowable. In the investigation of the convective equilibrium we obtain, under this assumption, at least a glimpse of the special case that lies as a limiting case between the two greater groups that correspond to the loss or increase of heat. Notwithstanding these extremely restrictive assumptions, still through the above-mentioned investigations, the comprehension of meteorological processes has been furthered to such an extent that we must consider their introduction as one of the characteristic features of modern meteorology. But the more valuable are the results that are already attained in this manner, so much the stronger must be the desire to free ourselves from the above-given limitations, and to extend the application of the mechanical theory of heat to those atmospheric processes in which the increase and diminution of heat from without can be no longer neglected. That this generalization had not already been long before taken is certainly because the formulæ are extremely complicated, so

* Translated from the *Sitzungsberichte der König. Preuss. Akademie der Wissenschaften zu Berlin:* Berlin, April 26, 1888. pp. 485–522.

that one always runs in danger of losing the leading thought in the midst of the notation and signs.

But in consideration of the fundamental importance that the application of the mechanical theory of heat in the most comprehensive manner possesses for the development of meteorology, one evidently ought not to be frightened by these extreme difficulties. This has induced me to make the attempt to introduce a method into meteorology that has proved so remarkably fruitful in the application of the mechanical theory of the heat to the theory of machines: I mean the graphic method that Clapeyron* has invented in order to make the ideas first expressed by Sadi Carnot† visible and comprehensible. Already, some years ago, a step in a similar direction was taken by Hertz‡ in a highly meritorious work on a graphic method for the determination of the adiabatic changes in moist air; but the problem that Hertz had before him, as also the method which he adopted, were materially different from those that I have now in mind. On the one hand, Hertz confined himself, as his title states, exclusively to the consideration of the adiabatic changes, and on the other hand, his object was only by means of a simple graphic process to avoid the complicated computations that one has to execute in following these changes. My object, on the other hand, has been to give a method of presentation that can serve as a guiding thread in the still more complicated formulæ with which one has to compute as soon as we disregard the restrictive assumption of adiabatic change, and that also allows one to draw certain important conclusions even from the form of the geometrical figures. To attain these objects however, scarcely any mental presentation is so appropriate as that introduced into science by Clapeyron, of course with such extensions as are required by the condition that in meteorological problems we have not as there to consider only two independent variables, but three, or in special cases, even still more.

But before I enter upon the subject itself I must touch upon another point on which notwithstanding its fundamental importance, remarkable to say, still perfectly clear views do not prevail. This has respect to the true reason of the cooling that occurs in the ascent of air to higher regions as well as the corresponding warming for descending air. While Sir William Thomson,§ Reye,|| Hann,¶ Peslin,** and with these investigators probably also the greater part of all physicists and meteorologists, correctly consider the cooling of ascending air as a consequence of the expansion occurring therein, on the other hand, Guldberg and

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*Poggendorff's *Annalen*, vol. 59, pp. 446–566.

†*Réflexions sur la puissance motrice du feu.* Paris, 1824.

‡*Meteorologische. Zeit.*, 1884, I, pp. 421–431. [See No. XIV of this collection.]

§ *Proc. of Manchester Soc.*, 1862, II, 170–176.

|| *Die Wirbelstürme*, Hannover, 1872.

¶ *Zeitschrift d. Oesterr. Ges. f. Met.*, 1874, Bd. IX, pp. 321, 337. Smithson. Rep. 1877, p. 397.

** *Bull. hebd. de l'Assoc. scientif. de France*, 1868, Tome III, p. 299.

Mohn* find the reason therefor in the work that is done in raising the air, and that is balanced by an equivalent quantity of heat taken from the air. Since by both methods of consideration the same value is found for the diminution of temperature with the height, therefore in the well-known excellent treatise of Sprung † both methods of consideration are presented beside each other as equally proper. But in fact only the first of these two is allowable, while that of Guldberg and Mohn contains in itself an error as to which one can only wonder that it could have escaped two such thoughtful investigators, and evidently also has hitherto not been remarked by others.

In order to obtain perfect clearness on this point one must first recall how it is that the ascending and descending currents in the atmosphere come to exist at all. This is, however, always brought about by differences in specific gravity that cause an ascent at certain places, while a corresponding mass descends at other places. The work that is required to raise the air at the one place is therefore always obtained by the falling of an equally great mass at another place. If no friction occurs the corresponding rising and falling movements once started would continue without any further addition of energy to infinity, and such an external addition of energy is only needed in order to overcome these frictions. These latter, however, are left out of consideration in all the discussions that are here considered, and this will also be done in the present memoir. We can consequently then compare the process with which we have to do, with movements in closed systems of tubes, such as a closed series of hot water pipes, or the movements of a continuous chain that hangs freely upon a roller. But it would never occur to any one to consider that the ascending water in the warmer half of a conduit, or the ascending portion of an endless chain must cool because of the work done in raising it. Similarly in the case of the ascending or descending currents in lakes or in the ocean, we must expect cooling or warming in consequence of these motions, if the ascent is accomplished at the expense of the heat latent in the fluid. The temperature changes occurring in the vertical motions of the air are therefore exclusively to be attributed to the work of expansion and compression, which is to be done or acquired respectively, and they would occur to precisely the same extent if the corresponding changes in pressure and volume occurred within a horizontal cylinder where rising and sinking was entirely out of the question.

On the other hand if we have air compressed within a vertical cylinder whose base is fixed, but which is closed above by a movable piston, and if we should now by a proper change in the load cause an expansion of the air then, besides the work of expansion, it would be necessary also to consider the work necessary in order to raise the center of gravity of the inclosed mass of air, and thus the cooling would be more

* *Zeit. Oesterr. Ges. Met.*, 1878, XIII, p. 113.

† *Lehrbuch d. Meteorologie*, Hamburg, 1865, p. 162.

considerable than when the whole change of condition took place with a horizontal position of the cylinder.

If the piston were without weight and without any loading, and if it were only at the beginning held fast but then suddenly loosed, and first held fast again at some other position at a greater distance from the base, then indeed the cooling would be attributable alone to the work which was necessary to be done in order to raise the center of gravity of the mass of air, since in this case no work of expansion is accomplished. By the explanations that I have made in such detail, in consideration of the fundamental importance of the question, it certainly ought to be perfectly clear that the cooling and warming in ascending and descending currents of air in the atmosphere are to be considered only as consequences of the work of expansion and compression; not of the work that is consumed in raising the air or that is gained by its descent, unless the ascending and descending masses belong permanently to one system. Since however the work of expansion and compression ought never to be left unconsidered, therefore in Guldberg and Mohn's method of consideration these, under all conditions, should have been further taken into consideration, and there would then have resulted for the rate of change of temperature with altitude a value exactly double that given by them. This being premised I will now pass to the problems mentioned in the opening paragraphs.

For our purpose it is first necessary to establish the fundamental quantities that come into consideration in investigations into the change of condition of a mixture of air and water or aqueous vapor. If in this I do not accord wholly with the steps that Hertz has chosen, this is because he has made various simplifying assumptions that are appropriate to the attainment of the end that he had in view, but that are not allowable in the general theoretical investigation that I contemplate. For the same reason I must again review the equations for the various conditions through which the mixture of air and water can pass, and which Hertz has developed in such a perspicuous manner, since not only by reason of the somewhat different notation, but also by the consideration of certain points intentionally neglected by Hertz, some material differences result.

Hertz and others in their investigations have made the assumption ordinarily used in the mechanical theory of heat that the unit of mass of the substance under consideration is given, and that it in succession passes through the different conditions. This assumption can not be rigorously adhered to in the case of atmospheric processes. A kilogram of moist air retains its mass unchanged only so long as during the expansion no condensation of aqueous vapor occurs, but suffers a diminution as soon as the formation of precipitation begins and rain, snow, or hail falls from it. When therefore a mass of moist air that is rising within a depression, or on the windward side of a mountain during a foehn on the lee side, is followed on its way through the atmos-

phere until it finally, either within an anti-cyclone or under the well known conditions of the foehn wind on the lee side of a mountain, comes again to its initial level, it is not the whole mass that we again find there present, but only a portion, although it may be a very considerable fraction, since a part of the water has been lost.

One can therefore begin the computation with the unit of mass of the mixture, but must consider the loss in mass that may occur in the course of the processes (a gain only occurs when the air passes over moist surfaces). But in this we have to combat the difficulty that, according to the point of departure that we choose, or according to the prevailing absolute humidity of the air at the point of departure, we have present, not only different quantities of vapor, but also different quantities of dry air, since the sum of the two must be equal to unity. It is therefore more appropriate to consider the unit of mass of dry air as given, and the water as an additional variable mixture.

This being premised, we will now indicate by M_a, M_b, M_c, M_d the masses of the mixture in the four stages so well distinguished by Hertz, namely, *the dry*, *the rain*, *the hail*, and *the snow* stage, and will also attach to the other quantities similar subscript letters as indices, in so far as a distinction of the respective stages may be necessary. But in computations that relate throughout to only one stage these indices may be dropped, in order not to overburden the formulæ too much. This being premised, we next find for the four stages the accompanying equations that may be temporarily designated as the equations of mixture.

(A).—The dry stage:

$$M_a = 1 + x_a$$

or abbreviated

$$M = 1 + x$$

where x or x_a designates the mass of aqueous vapor that is mixed with the unit mass (one kilogram) of dry air. In this it is assumed that the air is not saturated with aqueous vapor, and therefore x_a indicates always the mass of unsaturated (overheated) vapor that is contained in the mixture. This mixture remains, in general, constant in the free atmosphere, since in this stage precipitation is excluded and an appreciable introduction of aqueous vapor is only possible at the surface of the earth, and again since the quantity of aqueous vapor that is exchanged in the atmosphere between masses of air of different absolute humidities can certainly at first be wholly neglected.

(B).—The rain stage.

$$M_b = 1 + x_b + x_b'$$

or, when confined to one stage as before,

$$M = 1 + x + x'.$$

In this x_b indicates the mass of saturated aqueous vapor that is contained in the air, x'_b is the additional mass of water liquid that is present.

If we assume that by cooling, as for example through adiabatic expansion, the air has passed from the dry stage to the rainy stage, then will

$$M_b \overline{\overline{<}} M_a$$

wherein the equality sign is the limiting case but in general the inequality is to be considered as the characteristic sign. The quantity x'_b is always very small and can only assume a somewhat greater value in exceptional cases, as for instance in the case of a remarkably strong ascending current of air that hinders the fall of the rain or rather that carries the drops upward with itself. How large this value may become we have as yet no indications whatever.

(*C*).—The hail stage: for this case

$$M_c = 1 + x_c + x'_c + x''_c$$

wherein x_c is the quantity of saturated vapor; x'_c the quantity of water present in the fluid condition; x''_c the quantity of ice that is present. Here as above, under the corresponding assumptions, we have

$$M_c \overline{\overline{<}} M_b$$

this stage can, in general, only occur when fluid water is mixed with the air and this mixture is cooled to 0° Cent.

(*D*).—The snow stage; for this case

$$M_d = 1 + x_d + x''_d$$

where the notation is easily understood by what precedes and where again so far as the mixture can be considered as coming from the previous stage, we must have

$$M_d \overline{\overline{<}} M_c.$$

In the most common case, where an ascending mass of air p by cooling gradually goes through all the different conditions, x' and x'' are generally exceedingly small, so that the hail stage is entirely passed over, and in all formulæ only one independent variable x appears. In this case M steadily diminishes.

Hertz in his investigation has not considered the change of M, but has considered this quantity as constant. This was allowable in view of his object, but here as already stated in the beginning, this limitation must be avoided. The present more general consideration leads first of all to the recognition of the fact that here we have to do with a class of processes which so far as I know have not yet been considered in the mechanical theory of heat; such namely, as are reversible in their smallest parts but are not reversible as a whole.

So long as the quantities x' and x'' are not equal to zero but possess a finite value even though exceedingly small, then can the quantity of

vapor that is condensed by cooling, as in expansion, be again evaporated by warming or compression. But as soon as the small quantity of water is evaporated, then by a further warming, the air enters again into the dry stage, but with a different quantity of vapor than it originally had, so that now it will pass through other conditions than at first, when the air expanded under continued loss of water. In order now to be able to determine perfectly the condition of the mass of air, we need beside the variables that occur in the equations of mixture to know also the volume v that the mass M occupies and the pressure p.

The latter we measure by the pressure in kilograms per square metre, wherein we now have to understand by kilogram the weight that a kilogram of mass has at 45° Lat. The simple relation

$$p = 13.6\beta$$

exists between the pressure p thus measured or the so-called specific pressure and the barometric pressure β expressed in millimetres of mercury; whereas expressed in atmospheres it has the value

$$P = \frac{p}{10333}$$

so that one can without difficulty pass from one mode of measurement to the other.

This much being prefaced, we can now establish the equations for the gaseous condition [equations of elasticity] for the different stages. Their general form is

$$f(v, p, t, x) = 0$$

therefore they contain one variable more than we generally find in the equations of elasticity. The quantities x' and x'' do not appear in these since in general they are so small that they exert no influence on p and v.

If now we would geometrically picture a condition of mixture we must (besides p and v which will be represented in the ordinary method by ordinates and abscissas in a rectangular system of coördinates with the axes OP and OV) make use further of a third coördinate; as such we advantageously choose the value of x, and lay this off parallel to the axis OX in a direction perpendicular to the plane PV. In this method of presentation, all conditions that correspond to any value of x find their representation in one and the same plane, which only slightly differs from the PV plane if we adopt the atmospheric pressure as the unit of pressure, and adopt lines of equal length in the direction of the axes of V and X as expressing the units of volume (one cubic metre) and of mass (one kilogram).

If now we imagine successive planes lying above each other, on which conditions are represented that differ progressively from gram to gram (that is, by a thousandth of the adopted unit), then these will lie

like sheets above each other, and in the study of the changes in condition one can simply adhere to the consideration of the curves described by the projection of the represented points upon the PV plane. Therefore, this plane will frequently hereafter be briefly designated as the coördinate plane. We can therefore execute the mental presentation of these processes in this plane, if only certain artifices are used, of which mention will be made hereafter, and when we consider the resulting curves after a manner similar, as it were, to the lines on a Riemann surface. The most important result is, that thereby the external work consumed or expended finds its mental representation precisely as in the simple method of Clapeyron. The formula

$$d\,Q = A\ d\ U + A\ pdv$$

expresses the quantity of heat to be added for an infinitely small change of condition, under the notation* here adopted, and the special assumptions here considered; or if we pass from an initial condition over to the final condition

$$Q = A\ [U_2 - U_1] + A\int_{v_1}^{v_2} pdv.$$

In this equation the quantities x, x', x'' are contained in the values for the energy, and indeed play a very important part therein; moreover, x is implicitly contained in v, but notwithstanding this $\int_{v_1}^{v_2} pdv$ will be represented by the area included between the curved portion (more accurately, the projection on the PV plane of the curve) representing the change of condition, the initial and the final ordinate and the portion of the axis of abscissas lying between these ordinates.

In the following sections the equations of condition for the individual stadia will now be considered, from them those of the characteristic curves (isotherms, adiabatics, and curves of constant quantities of saturation) will be deduced, and finally the course of these in the geometrical form of presentation will be investigated.

A. THE DRY STAGE.

If we indicate by p_λ the partial pressure exerted by the dry air, by p_δ the pressure resulting from the vapor and in general distinguish all quantities relating to the air and vapor, in an analogous manner by the same indices then we obtain directly

$$p = p_\lambda + p_\delta = \frac{R_\lambda\,T}{v} + x\,\frac{R_\delta\,T}{v}$$

or,

$$p\ v = (R_\lambda + x\ R_\delta)T \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

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* I adopt Zeuner's method of writing as more familiar to me; that is, I assume that the energy is expressed in units of work.

where R_λ and R_δ are the constants of the equations of elasticity* for air and vapor, namely, $R_\lambda = 29.272$ and $R_\delta = 47.061$. If now x remains constant, then for a constant T this equation becomes that for an equilateral hyperbola. The isotherm for moist but not saturated air is therefore, as for dry air, an equilateral hyperbola or a portion of one; but for certain values of v this equation loses its meaning.

The fundamental condition of the dry stage consists simply in this: that the pressure p_δ shall be smaller than the pressure e corresponding to that of saturated vapor at the same temperature, or expressed algebraically

$$x\frac{R_\delta T}{v} < e,$$

where e is a quantity depending upon T and rapidly increasing with T. The equation (1) holds good, however, only when

$$v > \frac{x R_\delta T}{e} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and, therefore, the effective portion of the hyperbola begins with the point whose abscissa is

$$v_s = \frac{X R_\delta T}{e} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Since e increases more rapidly than T, therefore this initial abscissa diminishes with increasing temperature. The initial points of all isotherms belonging to one and the same quantity of vapor x lie therefore on a curve whose course is to be seen approximately on the figures to be given hereafter, in which such curves are designated by $S\ S$ with corresponding indices.

Hence the area within which are represented the conditions of the dry stage for constant quantities of vapor x is bounded by this curve on the side toward the co-ordinate axes. If this curve is so intersected by another curve, representing any change of condition that one passes from the side that is concave away from the axis over to its convex side then one leaves the dry stage and arrives in that of condensation, that is to say, in the rain or snow stage. I will therefore designate this curve as the curve of saturation or of dew-point. Points on this saturation curve are, in accord with the considerations just developed, determined by the hyperbolic isotherm and the initial abscissa. We can, however, equally well utilize also the corresponding ordinates and the initial abscissa. In the dry stage the following equation holds good for the ordinate:

$$p = p_\lambda + p_\delta < p_\lambda + e$$

consequently for the initial ordinate of the isotherm T, which will be

* Throughout this work, the "equation of elasticity" is used as a translation of the German *Zustandsgleichung*, as being preferable and more general than the ordinary expression "Equation for a gas" or "equation of condition."—*C. A.*

designated by p_s as being located on the curve of saturation, the equation is

$$p_s = p_\lambda + e$$

or

$$p_s = \frac{R_\lambda\, T}{v_s} + e$$

or finally after substituting the value of v_s

$$p_s = \frac{R_\lambda + x\, R_\delta}{x\, R_\delta} . e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

It is therefore easy to determine the correlated values of v_s and p_s for any constant quantity of moisture x and for any given temperature. On the other hand, only with the greatest difficulty and even then only by the use of empirical formulæ is it possible to bring the curve of saturation into the ordinary form:*

$$F\,(v_s,\, p_s) = 0.$$

We also will therefore entirely relinquish all attempts in this direction. By so much the more important is it therefore to show that from the curve of saturation for a given value of x one can with ease deduce such curve for any other quantity of moisture. If T and hence also e is constant, then it directly follows from the equation

$$v_s = x \frac{R_\delta\, T}{e}$$

that the initial abscissas of isotherms corresponding to equal temperatures but different quantities of moisture are proportional to these quantities of moisture themselves, or if we indicate by v_1 and v_2 the initial abscissas belonging to the quantities of moisture x_1 and x_2, we have

$$v_1 : v_2 = x_1 : x_2.$$

If therefore we have any point such as N_1 of the dew-point curve S_1 corresponding to a given temperature T this will be the initial point of the isotherm (T, x_1) if as in the above given manner we indicate the point corresponding to the temperature T and the quantity of vapor x_1; now draw the isotherm (T, x_2) for the same temperature T but for another quantity of vapor x_2, then we have only to increase or diminish the abscissa of N_1 in the ratio $x_2 : x_1$ in order to obtain the x_2 of the

* We see this from the following consideration: Since according to equation (4) $e = \varphi\,(p_s,\, x)$, and since again $e = F(T)$, and moreover $T = \psi\,(p_s,\, x)$; since further the equations (3) and (4) give $v_s\, p_s = (R_\lambda + x\, R_\delta)\, T$, therefore $v_s\, p_s = (R_\lambda + x\, R_\delta)\, .\ \psi\,(p_s,\, x)$, or if we omit x from under the functional sign as being constant,

$$v_s\, p_s = (R_\lambda + x R_\delta).\ \psi(p_s)$$

an equation which contains only v_s and p_s, but not explicitly, as variables.

initial point N_2 of the isotherm (Tx_2) originally considered as being unlimited; that is to say, in order to obtain a point in the dew-point curve S_2 corresponding to the quantity of moisture x_2.

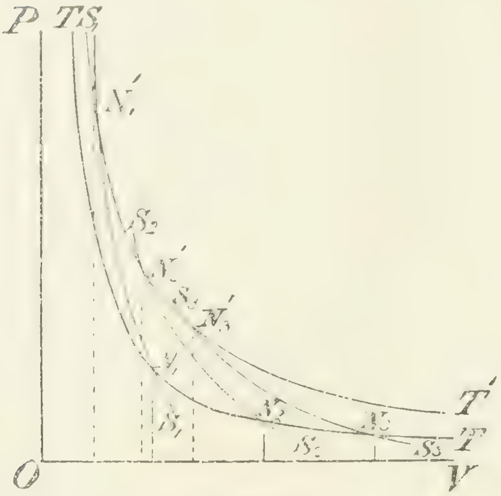

Fig. 29.

The dew-point curves S_2, S_3, of figure 29 therefore correspond respectively to quantities of vapor $x_2 = 2x_1$; $x_3 = 3x_1$ when S_1 corresponds to the quantity of vapor x_1.

The isotherms (T, x_1) and (T, x_2) run so near each other that they can only appear separated in a figure drawn to a very large scale,* since between the ordinates p_1 and p_2 of the two isotherms belonging to a given v, the following relations exist:

$$p_1 - p_2 = (x_1 - x_2)\,\frac{R_\delta\, T}{v}$$

or also

$$\frac{p_1}{p_2} = \frac{R_\lambda + x_1\, R_\delta}{R_\lambda + x_2\, R_\delta}$$

But this quotient is always very near unity, since all the values of x that here come into consideration lie between zero and 0.03. In the majority of cases one can consider all the isotherms (T, x) corresponding to a given value T as coinciding with each other and have then only to remember that according to the value of x they have their initial points at different places on the same hyperbola. Therefore from any one dew-point curve S_1 we obtain another one S_2 in that as already done in figure 29 we simply go with a constant ratio of expansion or compression further along an equilateral hyperbola drawn through S_1.

If we confine our consideration still to that portion of the plane of a constant quantity of vapor x that lies to the right (that is to say, on that side of the dew-point curve that is distant from the coördinate axes) that is to say to the dry stage, then in this region the same theorems will hold good for the characteristic curves as for the so called perfect gas, and particularly as for air, with such very small changes in the constants as depend on the mixing ratio [or the quantity x].

* It must here be expressly remarked that all the diagrams occuring in this memoir have a purely illustrative character. If we should introduce the separate quantities as they result from the computation the diagrams would lose perspicuity. The method here given therefore will need special modifications (as is hereafter to be shown) before it can be applied to graphical computations.

In this stage the isodynamic lines are also equilateral hyperbolas, and moreover the equation

$$pv^{\kappa} = p_1 v_1^{\kappa}$$

holds good also for the adiabatic lines, when p_1 and v_1 relate to a definite initial condition, but p and v to an arbitrary final condition.

The constant κ can be adopted without notable error the same as for dry air, namely, $\kappa = 1.41$. The quantity of vapor therefore disappears entirely from the formula and the adiabatics have the same course in all the planes corresponding to the different values of x. If now the adiabatic curves are considered as lines of constant entropy and we therefore take the equation $S - S_1 = 0$ as the fundamental condition where $-S$ is the entropy, then the equation of the adiabatic lines receives the following form

$$(c_p + xc_p^*) \log \frac{T}{T_1} - A\ (R_\lambda + x\, R_\delta) \log \frac{p}{p_1} = 0$$

where the capacity for heat of superheated aqueous vapor under constant pressure is indicated by c_p^*.

If one knows the path of any one adiabatic in the dry stage, then it is easy to construct any given number of others by means of it. To this end we consider that for any further progress along one and the same isotherm, according to well-known propositions, the following formula holds good for the quantity of heat needed in the expansion from v_1 to v_2:

$$Q_{1,\,2} = A\ R^*\ T \log \frac{v_2}{v_1}$$

where, for the sake of simplicity, we put $R_\lambda + xR_\delta = R^*$

Therefore we have

$$\frac{Q_{1,\,2}}{T} = A\ R^* \log \frac{v_2}{v_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

But the quotient $\frac{Q_{1,\,2}}{T}$ is nothing else than the diminution of the entropy in the isothermal expansion from the volume v_1 to v_2. If, therefore, we start from a line of constant entropy (an adiabatic), and proceed along various isotherms that cut this curve, so that the ratio of expansion remains constant, then we attain to points on a second adiabatic.

If now we put $v_1 = v$ and $v_2 = v + \Delta v$, and then make $\Delta v = \nu v$, where ν is a constant (an appropriate proper fraction), and if in a corresponding manner we put ΔQ for Q and ΔS for the difference of the entropy, we find

$$\Delta S = \frac{\Delta Q}{T} = AR^* \log\,(1 + \nu)$$

Therefore as soon as the course of one adiabatic line is known (just

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* For the problems here presented, as is done by Zeuner in the application of the mechanical theory of heat to machines, it is recommended to give the positive sign to

as in the case of the dew-point curve) one can by a simple method of construction cover the plane of coördinates with a series of such adiabatics, each of which, with reference to its neighbor, shows a constant difference in the entropy by the amount $\varDelta S$.

B. THE RAIN STAGE.

For the rain stage, as already stated, there obtains the equation of mixture

$$M=1+x+x',$$

where x' is in general very small, but x, except in exceptional cases, can only diminish. The equation of elasticity, on the other hand, is

$$p=\frac{R_\lambda T}{v}+e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where e is the vapor pressure, which in this stage, that is to say in the condition of saturation, depends simply and alone on the temperature T. Moreover, there obtains also the equation developed as a limiting condition in Art. 3 above, viz:

$$e=\frac{x R_\delta T}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

This last formula shows at once the above suggested fact, that here we have in general to do with changes that are reversible to only a very limited extent. If, for instance, T is put constant while v increases, then the equation can only be fulfilled when x increases. This same holds good (because e increases rapidly with increasing T) when v is kept constant and T increases, or, as expressed still more generally, it holds good for all changes in condition that are represented in the diagram by a movement toward the concave side of the dew-point curve.

But an increase of x is only to a very limited extent possible in general in the free atmosphere, namely, only when liquid water, in addition to the vapor, is suspended in the air, and only so long as this store of liquid holds out. The latter in most cases is soon exhausted, since it is precisely the liquid drops of water that fail as rain as soon as their mass becomes considerable.

Therefore in the rain-stage, changes of condition toward the concave side of the dew-point curve are possible only to a very limited extent and only until the condition of supersaturation comes to its end and

the quantity of heat communicated to the air. Therefore an increase of the quotient $\frac{Q}{T}$ corresponds to a diminution of the entropy according to the definition of entropy as given by Clausius (see Clausius's Collected Memoirs, Brunswick, 1884) Memoir IV, page 140, and Memoir VI, page 276.

becomes that of simple saturation.* This occurs as soon as the curve of change of condition attains the dew-point curve $x + x'$. Having in mind the geometrical presentation one can express this proposition as follows:

In the rain or snow stage, changes of condition are only reversible when and so long as they find their representation above the dew-point surface. If they find this in the dew-point surface itself, then only those changes are possible by which the representative point approaches the quasi horizontal coördinate plane, that is to say slides down toward the surface or in the limiting case becomes the dew-point curve itself. An ascent to the dew-point surface is in the free atmosphere only imaginable in exceptional cases (as for instance in case of the falling of rain through other layers or the mixing of other layers with moist air); a further progress toward the concave side of the dew-point curve or toward the lower side of the dew-point surface indicates a transition over into the dry stage.

Therefore in making use of the graphic presentation one must always keep in mind that in the rain and snow stages the curves in general can only be travelled over in one direction best represented by arrows and that a backward movement on the same curve is an impossibility. Nevertheless for the forward progress in the one possible direction exactly the same formulæ are applicable as for the reversible changes of condition. Therefore the case here occurring may with propriety be designated as "limited reversible."

We now turn to the consideration of the isotherm and the adiabatic for the rain stage. The equation of the isotherm we obtain at once as soon as we consider the temperature T as constant in the equation of elasticity

$$p = \frac{R_\lambda T}{v} + e.$$

Since in this case e is also constant, therefore this curve as in the dry stage is an equilateral hyperbola, one of whose asymptotes, as in the dry stage, coincides with the axis of p, but the other is by the small quantity e shoved from the axis of v toward the side of positive p. At the same time however, in so far as we exclude super-saturation and starting from a given initial condition, this equation holds good only for diminishing values of v.

Moreover a glance at the equations of the isotherms in the dry and the rain stages suffices to show us that the two curves for any given temperature differ from each other only very little and that in the transition from the dry to the rain stage only a very small indentation

* In a certain sense the case where liquid water or ice is mixed with the air should certainly also be called that of super-saturation, but of course with the reservation that any confusion with the condition of super-saturation properly so called, in which the excess above the quantity needed for saturation is present in gaseous form, shall be excluded.

can be seen with the vertex toward the right and above. This results from the circumstance that the isotherm for the rain stage contains the initial points, of all isotherms for the dry stage, which points correspond to values of x_a that are smaller than the value of x_b from which one starts out.

In order to obtain the equation of the adiabatic we must know the quantity of heat, dQ, that is to be communicated for a very small change in the condition. This dQ is composed of the quantity of heat dQ_λ that is given to the dry air and of the quantity dQ_δ that is communicated to the intermingled water or aqueous vapor. The following equations hold good for these quantities:*

$$dQ_\lambda = C_v dT + AR_\lambda\, T\frac{dv}{v}$$

and $$dQ_\delta = Td\left(\frac{xr}{T}\right) + (x + x')\,dT$$

[Where r is the quantity of heat required to vaporize a unit mass of water at the temperature T and the pressure p.]

In these x' has values that lie between 0 and $x_a - x$ where x_a indicates the quantity of vapor that was given to the original kilogram in its passage from the dry stage to the rain stage. x' is equal to 0 when all the condensed water immediately falls down and is thus separated from the mass; it is equal to $x_a - x$ when all such water is carried along with the mass. The two limiting cases will occur relatively quite seldom in nature, but since at present we have no basis for determining to what extent liquid water is suspended in the air or can be carried along with it, therefore one must in the theoretical investigation confine himself to these limiting cases. Expressed in the language of the graphic presentation one must content himself with investigating those cases in which the indicating point either remains in the same plane as in the dry stage or on the other hand goes further on over to the dew point surface itself. Hitherto the first case only has been taken into consideration, although in general the second better agrees with the conditions occurring in nature.

Therefore the above given equation for dQ_δ assumes different forms, according as we consider the one or the other limiting case and we have, either

$$dQ_\delta = Td\left(\frac{xr}{T}\right) + x_a dT$$

for the case where x_a is constant when all the water formed by condensation remains suspended,

or $$dQ_\delta = Td\left(\frac{xr}{T}\right) + x dT$$

where $$x = \frac{ev}{R_\delta T}$$

for the case when all this water immediately separates from the mass.

* See Clausius Collected Memoirs, Brunswick, 1884, Memoir V, page 174, or Hirst's translation of Clausius, pages 153 and 353.

The first case corresponds to a super-saturation limited only by the original amount of water, or, as I will briefly call it, the "maximum super-saturation;" the second case corresponds to the "normal saturation," rejecting any supersaturation.

For the quantity of heat $dQ=dQ_\lambda+dQ_\delta$ communicated to the mixture we obtain therefore two equations, namely:

(1) For "maximum super-saturation:"

$$dQ=(c_v+x_1)dT+Td\left(\frac{xr}{T}\right)+AR_\lambda T\frac{dv}{v}. \quad . \quad . \quad . \quad . \quad (8)$$

(2) For the "normal saturation:"

$$dQ=c_v dT+x dT+Td\left(\frac{xr}{T}\right)+AR_\lambda T\frac{dv}{v}. \quad . \quad . \quad . \quad (9)$$

If we put $dQ=0$ then we obtain the differential equations of the adiabatics for the two limiting cases. But in doing this we ought not to overlook the fact that strictly speaking in satisfying the condition $dQ=0$ we have to do with an adiabatic in the ordinary sense of the word only in one of these limiting cases, namely, that of maximal supersaturation. For if we establish for the adiabatic the single condition that for the given change of condition heat shall be neither gained nor lost, then we have in both cases true adiabatics to deal with. If however we define the adiabatic change of condition as one in which not only all exterior work shall be done at the cost of the energy, but also where the whole loss of energy shall be consumed in exterior work then will the definition for the second limiting case and also for all intermediate cases corresponding to values of $x'>0$ and $x'<x_1-x'$ equally agree with changes of condition that satisfy the condition $dQ=0$.

When, namely, the condensed water separates from the mass the energy diminishes not only by the quantity needed for the performance of exterior work, but also further by that quantity which is carried away by the water that has precipitated at a given temperature. I will therefore call those changes of condition for which $dQ=0$ but $x+x'<x_0$, that is to say, those changes for which the water wholly or partly separates the "pseudo-adiabatic," and especially that curve which obtains for the complete discharge of the water of condensation, the "pseudo-adiabat."

Corresponding to this method of distinction the equation

$$(c_v+x_0)\,dT+Td\frac{xr}{T}+AR_\lambda\,T\frac{dv}{v}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

obtains for the adiabat and the equation

$$(c_v+x)\,dT+Td\left(\frac{xr}{T}\right)+AR_\lambda\,T\frac{dv}{v}=0 \quad . \quad . \quad . \quad . \quad (10)$$

obtains for the pseudo-adiabat.

From these two equations we see, first of all, that the pseudo-adiabat descends more rapidly than the adiabat. Since for $dv > 0$ we always have $dT < 0$ and since moreover $x < x_a$, therefore the absolute value of dT in the case of pseudo-adiabatic expansion must be larger than for adiabatic; that is to say, the temperature must sink more rapidly when all the condensed water is immediately discharged than when it remains still suspended.

Furthermore, both curves must sink more rapidly than the dew-point curve, or, in other words, for $dv > 0$ we must always have $dx < 0$. This follows directly from the circumstance that in expansion along the dew-point curve heat is to be added as also is shown from the manner in which the adiabatics of the dry stage intersect this curve. On the other hand, changes of condition with increase of heat are always represented by curves that descend less rapidly toward the axis of abscissas than do the adiabatics.

Therefore in the expansion of air the adiabatics depart from the dew-point curve toward the axis of abscissas and therefore x diminishes.

The equation (8) is easily integrated and thus gives the following equation of condition for the adiabat:

$$AR_\lambda \log\frac{v_2}{v_1}+(c_v+x_a)\log\frac{T_2}{T_1}+\frac{x_2r_2}{T_2}-\frac{x_1r_1}{T_1}=0 \quad . \quad . \quad . \quad . \quad (10)$$

or if v is expressed in terms of p, e, and T with the help of the equation of elasticity;

$$AR_\lambda \log\frac{p_1-e_1}{p_2-e_2}+(c_p+x_a)\log\frac{T_2}{T_1}+\frac{x_2r_2}{T_2}-\frac{x_1r_1}{T_1}=0 \quad . \quad . \quad . \quad (11)$$

or finally by consideration of equation (7) and by the substitution of the corresponding values of x_1 and x_2;

$$AR_\lambda \log\frac{v_2}{v_1}+(c_v+x_a)\log\frac{T_2}{T_1}+\frac{e_2v_2r_2}{R_\delta T_2^2}-\frac{e_1v_1r_1}{R_\delta T_1^2}=0 \quad . \quad . \quad (12)$$

or

$$AR_\lambda \log\frac{p_1-e_1}{p_2-e_2}+(c_p+x_a)\log\frac{T_2}{T_1}+\frac{R_\lambda}{R_\delta}\left[\frac{e_2r_2}{T_2(p_2-e_2)}-\frac{e_1r_1}{T_1(p_1-e_1)}\right]=0 \quad . \quad (13)$$

If we consider the final condition as variable and corresponding to this drop the subscript index 2, then the equations become the following:

$$AR_\lambda \log v+(c_v+x_a)\log T+\frac{xr}{T}=C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10a)$$

$$-AR_\lambda \log(p-e)+(c_p+x_a)\log T+\frac{xr}{T}=C \quad . \quad . \quad . \quad . \quad (11a)$$

$$AR_\lambda \log v+(c_v+x_a)\log T+\frac{evr}{R_\delta T^2}=C \quad . \quad . \quad . \quad . \quad . \quad . \quad (12a)$$

$$-AR_\lambda \log(p-e)+(c_p+x_a)\log T+\frac{R_\lambda}{R_\delta}\cdot\frac{er}{T(p-e)}=C \quad . \quad . \quad (13a)$$

Simple as are these collected equations in certain respects, still none of them allow us to express the relation between v and T or p and T or even between p and v explicitly, and in using them we are obliged to proceed by trials.

On the other hand one can, in comparatively simple manner, construct the curves in question when we remember that the left-hand side of equations (10) to (13), in all cases, even when they are not equal to 0, must still always give the value of

$$\int_{(1)}^{(2)} \frac{dQ}{T}$$

when we take this integral from the initial condition $v_1 p_1$ to the final condition $v_2 p_2$, and thereby apply the notation of the limits as here given, and as is easily comprehended.

But this value is nothing else than the diminution of the entropy during the passage from the initial to the final condition.

If therefore we compute this quantity for various properly chosen pairs of v_2 and p_2 we thus obtain the value of the entropy for the corresponding points, excepting only a constant that holds good for the whole system. Thus we shall be enabled to interpolate the corresponding values for intermediate points and thus to draw lines of equal entropy, namely, adiabatics. It is especially desirable to so choose these points that they come to lie in regular succession on the isotherms.

Then we have for the difference of the entropy due to the passage from a point 1 to a point 2 of the same isotherm, that is to say, for $T_1 = T_2 = T$

$$\int_{(1)}^{(2)} \frac{dQ}{T} = \frac{Q_{1,2}}{T} = A\, R_\lambda \log \frac{v_2}{v_1} + (v_2 - v_1).\ \tau \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where $\tau = \dfrac{er}{R_\delta T^2}$ that is to say, a quantity that remains constant for the same isotherm. This equation also teaches that the isentropic curves in the rain stage cut the isotherms at more acute angles than in the dry stage, for which latter the equation (5) holds good, namely,

$$\frac{Q_{1,2}}{T} = AR^* \log \frac{v_2}{v_1}$$

From the comparison of both equations, (5) and (14), it follows that a given change of the entropy in the dry stage corresponds to a greater change of v than in the rain stage. Since now the isotherms in both stages can be considered as having very nearly the same course and, when we consider a very small part of the coördinate plane, can be considered as parallel straight lines, therefore for the given change of entropy in the dry stage one has to go a greater distance along the isotherm than in the rain stage.

Since, however, on the other hand, the dew-point curves descend more rapidly than the isotherms toward the positive side of the axis of abscissas, therefore the adiabatics must experience a bend at the dew-point curve in the manner shown in the figure 30. In this $S\ S$ presents a part of a dew-point curve; $A\ A$, $A'\ A'$, etc., adiabatics; $T\ T$, $T'\ T'$, etc., isotherms.

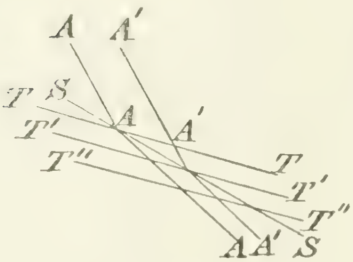

Fig. 30.

The differential equation of the pseudo-adiabatic can be treated in a similar manner to that of the adiabatic, but whereas in the adiabatic the integration was possible even when the connection of the independent variables was not explicitly given, on the other hand this is not the case for the pseudo-adiabatic. That is to say, instead of equation (10) we have for the pseudo-adiabatic the following:

$$AR_\lambda \log \frac{v_2}{v_1} + c_v \log \frac{T_2}{T_1} + \int_{(1)}^{(2)} \frac{x\,dT}{T} + \frac{x_2 r_2}{T_2} - \frac{x_1 r_1}{T_2} = 0,$$

or, preferably,

$$AR_\lambda \log \frac{v_2}{v_1} + (c_v + x_a) \log \frac{T_2}{T_1} - \int_{(1)}^{(2)} \frac{(x_a - x)\,dT}{T} + \frac{x_2 r_2}{T^2} - \frac{x_1 r_1}{T_1} = 0 \quad . \quad (15)$$

If therefore the point (1) is at once located in the dew-point curve then will $x_1 = x_a$; and if then we consider the point (2) alone as variable, that is to say, omit the subscript index 2 entirely, we obtain

$$AR_\lambda \log \frac{v}{v_1} + (c_v + x_a) \log \frac{T}{T_1} - \int_{(1)}^{(2)} \frac{(x_a - x)\,dT}{T} + \frac{xr}{T} - \frac{x_a r_1}{T_1} = 0 \quad . \quad . \quad (16)$$

or after further modifications

$$AR_\lambda \log v + (c_v + x_a) \log T + \frac{xr}{T} - \int_{(1)}^{(2)} \frac{(x_a - x)\,dT}{T} = C \quad . \quad . \quad (17)$$

We omit the development of formulæ entirely analogous to equations (11) etc., and it suffices to say that in them all the integral occurs as a correcting term. Happily its value remains always within very moderate limits, so that in the computation one can be satisfied with more or less perfect approximations. One can therefore omit the further consideration of the pseudo-adiabatic process and only call attention to the fact that it follows from equation (16) that the pseudo-adiabatic curve descends more rapidly than the adiabatic as was already pointed out above. For since when $v_2 > v_1$ we always have $dT < 0$ therefore the definite integral that still occurs in the equation has always a negative

value and because of the minus sign before the integral it therefore always exerts its influence in the same direction as the term $AR_\lambda \log \frac{v_2}{v_1}$. Therefore for the same starting point and for equal values of T_2, we must have v_2 in the case of the pseudo-adiabatic smaller than if we had gone along on the adiabatic.

C. THE HAIL STAGE.

The above given equations hold good for the value $T > 273°$; as soon as the temperature 0° C. or the absolute temperature $T=273$ has been attained, then very different equations replace these but only when liquid water is present. In this last case the following equation of mixture holds good, namely:

$$M=1+x+x'+x'',$$

an equation that can only be true for the temperature 0° C. since only at this temperature can water and ice occur together. The equation of elasticity therefore then acquires the simple form

$$p=\frac{aR_\lambda}{v}+e_0$$

while the equation $x=\frac{ev}{R_\delta T}$ becomes $x=\frac{e_0 v}{aR_\delta}$ . . . (18)

wherein $a=273$, $e_0=62.56$. But the one possible change of condition in this stage consists in an isothermic expansion. For this case therefore, the dT also falls out of the equation for the transfer of heat and this takes the form,

$$dQ=r_0 dx-l dx''+AR_\lambda a\frac{dv}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

[r_0=latent heat of evaporation at 0° C.; l=latent heat of liquefaction of ice.]

In this equation the first term on the right-hand side must be positive, the second must have a negative sign when dx and dx'' are considered as positive, since an increase in the quantity of vapor x makes an addition of heat necessary, but an increase in the formation of ice demands a withdrawal of heat.

If we put $dQ=0$ then we have the differential equation of the adiabatic which in this case coincides with the isotherm and is moreover always a pseudo-adiabat, since the ice that is formed falls away under all circumstances.

If we consider that

$$dx=\frac{e_0 dv}{aR_\delta}$$

then the differential equation of the adiabat takes the form

$$AR_\lambda a\frac{dv}{v}+\frac{r_0 e_0}{aR_\delta}dv-l dx''=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

hence we obtain by integration

$$AR_\lambda a \log\frac{v_2}{v_1}+\frac{r_0e_0}{aR_\delta}(v_2-v_1)-lx_2''=0 \quad . \quad . \quad . \quad . \quad . \quad (21)$$

where we assume the integral to be taken throughout the whole stage from the initial value v_1 that corresponds the entrance into this stage to the final value v_2 that refers to the exit therefrom, and remember that the initial value of x'' namely, x_1'' is equal to 0 under these conditions. If however the integral extends only up to a value of v lying between these two limits and which v can then be considered as variable, then the equation can be again brought into a form analogous to that above given and we obtain

$$AR_\lambda a \log v+\frac{r_0e_0}{aR_\delta}v-lx''=C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

This equation allows us to see directly that for increasing values of v that is to say for continued progressive expansion the quantity of hail also steadily increases whereas on the other hand from [equation (18) or] the expression

$$dx=\frac{e_0}{aR_\delta}dv$$

it follows that an evaporation goes hand in hand with the freezing of the water, so that at the end of the hail stage the quantity of vapor present is greater than it was at the entrance upon this stage.

With the help of the above described geometrical presentation we represent these results in the following manner.

The condition that must exist at the entrance upon the hail stage finds its representation at the termination N' of a straight line N_0N' perpendicular to the chief plane of coördinates and which rises up above the dew point surface. The length of this straight line is $x+x'$. It cuts the dew-point surface at a point N that is distant from the plane of PV by the quantity x. If now the mixture expands along the isotherm then N rises along the dew-point surface slowly upwards, while the foot N_0 of the straight line advances along an equilateral hyperbola. But at the same time, the total quantity $x+x'$ diminishes in consequence of the discharge of the ice and N' sinks correspondingly down until N and N' coincide in a single point N_2 and with this the hail stage has reached its end.

It is now of especial importance to learn how much water is thrown down in the form of hail; this question is answered by the following consideration. At the beginning of this stage we have only water and vapor, at the end only ice and vapor while the sum of these in the first and in the second case remain the same, if we take the precipitated ice also into the computation. Let x_1' be the quantity of liquid water present

at the entrance into the hail stage, then according to what has just been said,

$$x'_1+x_1=x''_2+x_2$$

or

$$x''_2=x'_1-(x_2-x_1)$$

or finally, making use of the equation (18),

$$x''_2=x'_1-\frac{e_0}{aR_\delta}(v_2-v_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

If we substitute this value in equation (21) then after an easy transformation we find

$$AR_\lambda a \log \frac{v_2}{v_1}+\frac{(r_0+l)e_0}{aR_\delta}(v_2-v_1)=lx'_1 \quad . \quad . \quad . \quad . \quad (24)$$

From this we can now first find v_2 by trial; the value thus found can be substituted in equation (23), whence in this manner x''_2 is found.

If we are justified in the assumption that all the vapor of water originally present is also after the condensation carried along until the entrance upon the hail stage, as appears to be the case in heavy hail-storms, then we have $x'_1=x_a$ and this is certainly large with respect to x_1 and x_2, and therefore so far as concerns the absolute value of x''_2 we may briefly put $x'_1=x''_2$, since the difference x_2-x_1 no longer comes into consideration. In cases in which this difference is appreciable, as for instance in the determination of v_2, one can of course not make use of the above approximation.

The equation (23) also shows in a very clear manner that in general the hail stage can only occur when liquid water is suspended in the air, that is to say, when $x'_1>0$ and that it acquires a greater extent the greater this value of x'_1, that is to say, the greater the quantity of suspended water that is present. Already, many years ago, Reye showed that on days of thunder storms the conditions are present in a conspicuous degree for the suspension and carrying up of water.

D. THE SNOW STAGE.

If the air, saturated with aqueous vapor, be cooled below 0° C., then a part of this vapor must be precipitated as snow. The same formula can be applied to this process as that which we have used in the rain stage if only in place of the heat of evaporation r there be inserted the sum $r+l$ where l as above indicates the heat of liquefaction of ice. Therefore we can after small modifications apply to this stage all the equations developed in Section B. I confine myself to the re-writing in this modified form the two equations (10a) and (17); they thus become for the adiabatic

$$AR_\lambda \log v+(c_v+cx_c) \log T+\frac{x(r+l)}{T}=C \quad . \quad . \quad . \quad (25)$$

and for the pseudo-adiabatic

$$AR_\lambda \log v+(c_v+cx_c) \log T+\frac{x(r+l)}{T}-\int_a^T \frac{c(x_c-x)dT}{T}=C \quad . \quad . \ (26)$$

where x_c is the quantity of vapor at the beginning of the snow stage and the limits a and T are introduced into the integral, because in the hail stage, as in the beginning of the snow stage, $T=a=273$; c is the specific heat of ice. Since x is always smaller with diminishing T and finally approximates to 0, therefore in the snow stage the deeper the temperature falls the more does the adiabatic approximate to that of the dry stage.

In the investigation just finished, attention has been especially directed to the course of the adiabatics, as had also been done in the above-mentioned older investigations. But in truth the adiabatic expansion and compression constitutes only a rare, exceptional case, as is already shown by the fact that the vertical temperature diminution computed under this assumption (according to the so-called convective equilibrium) results considerably larger than is given on the average by observations. It is therefore important to deduce the quantity of heat absorbed or emitted for given changes of condition, as determined by the values simultaneously observed of pressure, temperature, and moisture. In this process the method of geometrical presentation here developed is applied with great advantage. First, a glance at the manner in which the curve representing any given change of condition cuts the adiabatic suffices to give a decision as to whether in this change one has to do with a gain or loss of heat. Moreover the curve puts one in a position to deduce the quantity of heat exchanged by graphic planimetric methods or by a combination of computation with planimetric measures. According to what was said in the beginning the equation

$$Q=A[U_2-U_1]+A\int_{(1)}^{(2)} pdv$$

holds good also for the processes here considered with three independent variables, and therefore also for a closed cyclic process

$$Q=AF,$$

where F is the surface inclosed by the projection of the points that are imagined to be upon the PV plane. Assuming that any change of condition is given by its projection on this plane and is represented by the line between the points a and b in Fig. 31, then we obtain the quantity of heat $Q_{a,b}$ involved in this change easily in the following manner: One draws through a (Fig. 31) any curve of change of condition for which it may be easy to compute the increase or diminution of heat; also draw

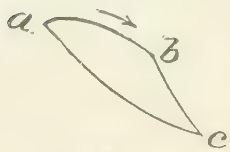

Fig. 31.

through b an adiabatic and prolong both curves until they cut each other in a point, c; then is $Q_{b,c}=0$, and the quantity of heat is given by—

$$Q_{a,b}+Q_{c,a}=AF,$$

or,

$$Q_{a,b}-Q_{a,c}=AF,$$

and therefore, also,

$$Q_{a,b}=AF+Q_{a,c}.$$

When now $Q_{a,c}$ is determined by computation, but F is found by planimetric method, this formula gives the value of $Q_{a,b}$.

If the curve ac is the curve of constant energy (or isodynamic), then $Q_{a,c}=AL$, where L is the exterior work and is therefore also directly obtained as a surface from the diagram, and then we have to execute the well-known graphic construction for the determination of the quantity of heat gained or lost by a given change of condition. But the method here given possesses the advantage of greater generality and much easier applicability.

This consideration also holds good when we have to do with limited reversible changes, only one has then to remember that the *closed* curve projected upon the plane of PQ must also be the projection of a *closed* curve in space. If the curve in space that represents the change in condition is not closed, but if it only has the peculiarity that at the initial and final condition the coördinates p and v have equal values, then it indeed gives a closed projection, but the quantity of heat computed by the above-given method is erroneous, and that too by the quantity which corresponds to the increase in internal energy at the passage from the initial to the final point, that is to say, by the addition of the necessary quantity of vapor.

The circumstance that one and the same point of the PV plane can correspond to very different conditions appears at first sight to exclude the general presentation of the processes in this plane alone, and thereby to materially diminish not only the applicability of the last-given construction but in general to detract from the whole conception here described. But by a closer consideration this is seen not to be the case; rather does it specially apply when for every point in the plane of PV one has given the corresponding dew-point curve. An example will elucidate this: Let us assume that one desires to obtain an idea of the difference in the internal energy that is present in the dry stage for equal values of p and v, but different quantities of vapor. If, in Fig. 32, P is the point having the coördinates p and v, but the quantity of vapor is in one case x_m and in the other x_n, then these latter correspond to two different dew-point curves, S_m and S_n. One can now convert the whole internal energy as it existed in the initial condition into external work by moving from the point P forwards adiabatically to the absolute

zero point, which of course would demand a continuation of the adiabatic to infinity. If we do this in the case when the quantity of moisture is x_m, then will the projection of the adiabatic be represented by the line $PM\,M_2$, but by the line $PN\,N_2$ when the quantity of vapor is x_n, because in the first case under the pressure $M\,M_{\prime}$, in the second case under the pressure $N\,N_{\prime}$, the air passes out of the dry stage into the rain stage, and therefore the adiabatic descends according to another law, and in fact less precipitously. But the difference in the internal energy corresponding to the quantity of vapor belonging to the condition represented in P, and which by a self evident notation is expressible as $A\,[U_m - U_n]$, is graphically represented by the surface $M_2\,M\,N\,N_2$, in so far as we imagine M_2 and N_2 extended to infinity and there united together.

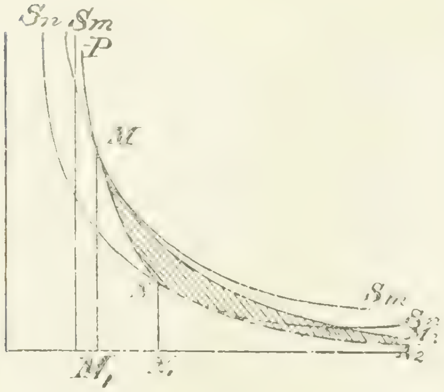
Fig. 32.

When expressed analytically we find for this difference the expression—

$$A\,[U_m - U_n] = x_m\,t_m - x_n\,t_n + x_m\,\rho_m - x_n\,\rho_n,$$

wherein ρ expresses the internal latent heat, and one has to remember that for given values of p and v the temperature varies with the intermixed aqueous vapor. However, this difference is so slight that in most cases it may be neglected, and one can therefore be satisfied with the approximation—

$$A\,[U_m - U_n] = (x_m - x_n)\,(t + \rho).$$

By this simplification the application of the above-described combination of planimetric measures and computations to the determination of the quantity of heat interchanged is very much lightened. If the temperatures are below 0° then the last formula must be slightly modified, which here need only to be referred to.

After having thus explained and established in general terms this new method of presenting the thermo-dynamic processes peculiar to the atmosphere their applicability will now be elucidated by a few examples.

(1) *The foehn.*

Moist air expands during its rise up the side of a mountain chain, and is then again compressed in its descent without having any heat added or withdrawn.

This is represented by a diagram, as shown in Fig. 33. Let a be the initial condition, the corresponding dew-point curve S_a, then the air expands according to the adiabatic for the dry stage until it cuts the curve S_a in a point b, the curve ab thus lies in a plane parallel to that of PV distant therefrom by x_a. A glance at the course of the isotherms (of which only the one corresponding to the initial temperature is drawn and designated by T_a) shows that in this passage from a over to b the temperature sinks rapidly. As soon as the condition b is reached the representative point [the indicator] slides down on the dew-point surface, the adiabatic of the dry stage goes over into bc, or that of the rain stage, and forms at b an obtuse angle with the former. The temperature, with continued uniform progressive expansion, sinks much more slowly than before, water is condensed, since the curve bc prolonged cuts the dew-point lines of lower quantities of vapor. The condensed water is deposited first as rain, afterwards as snow, and therefore bc is the projection of the pseudo adiabatic.

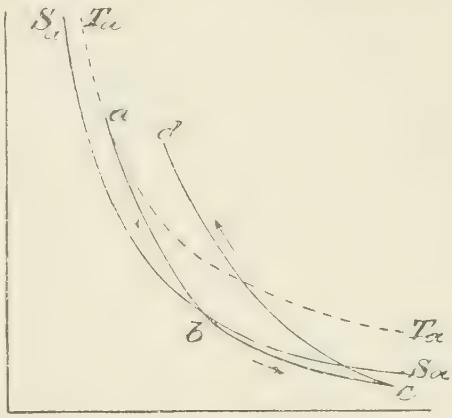

Fig. 33.

In this case the hail stage is entirely wanting, and although the cooling due to the continued expansion goes on beyond the freezing point, still this does not make itself so strongly felt in the course of the pseudo-adiabatic as that this transition should be perceptible in a drawing like the present diagram.

Let expansion continue up to a condition c, and now let compression occur, that is to say, the air reaches the summit or ridge of the divide and the ascent now becomes a descent on the other side. *Now*, all depends upon whether the condensed water was really completely precipitated or not. If not precipitated then during the compression there will be a retrogression of the indicator along the curve bc in the direction from c to b, and so much the farther along in proportion as more water has been carried with the air. If all the condensed water has remained suspended, then the change of condition in the retrograde direction continues back to b, and thence beyond to a, and we find on reaching the same level on the other side of the mountain again the same relations as in the beginning. This is always the case whenever the curve of saturation is not reached in the expansion, that is to say, when the whole process is entirely transacted in the dry stage in which case also the characteristic peculiarities of the *foehn* are wanting.

If however the rain stage is attained, and if in it the condensed water is actually precipitated then the process can not be reversed, and

then by the compression the change of condition from c onward goes further along the adiabatic cd of the dry stage. In this case a glance at the diagram shows immediately that for this change of condition the initial temperature will be attained even at a pressure that lies far below the initial pressure, and that in the farther progress towards pressures that are near the initial pressure, that is to say, in the descent to the old original level, much higher temperatures will be attained. At the same time the quantity of moisture is much less since the dew-point curve S_c (which however is not drawn in order not to confuse the diagram), lies nearer the coördinate plane than the curve S_a, and since the curve of condition cd remains with S_c in the same plane which is parallel to the plane PV. The quantity of moisture which in the initial condition was x_a is now at the end $x_d = x_c < x_a$, while for the temperature the equation $T_d > T_a$ holds good. Therefore after the passage over the mountain one has warm dry air, whereas at first it was cool and damp.

At the same time we see directly from the diagram that the characteristic peculiarities of the foehn must stand out so much the plainer in proportion as the point a is nearer to the curve of saturation, that is to say, the warmer and moister the air is before its ascent and again, the longer the portion $b\ c$ is, that is to say, the more extensive is the expansion in the rain stage, or in other words, the higher the summit is that has to be surmounted.

Therefore we understand also at once why it is that in the Alps, independent of the prevailing conditions of atmospheric pressure, northerly foehns are so much rarer than the southerly foehns, as also why descending winds that have surmounted no summit, but have only passed along over a plateau, as for example the bora, have not the characteristic warmth of the foehn.

(2) *The interchange of air between cyclone and anti-cyclone in summer.*

Between an anti-cyclone and the cyclones that feed it, similar relations exist as between the masses of air on the two sides of a mountain range to be surmounted by them. In cyclones one has to do with an ascending current of air that afterwards descends in the anti-cyclone. Hence arises the precipitation in the region of the cyclone, the dryness and the clear sky in the region of anti-cyclone. But, whereas in the foehn the ascent and descent occur at points in the neighborhood of each other, so that in the short path there scarcely remains time for gain or loss of heat, but the whole process may in fact be considered as adiabatic; on the other hand very different relations obtain for the ascent and descent in cyclone and anti-cyclone. These two opposite processes in general occur at places so distant from each other that in the transit from one to the other extended opportunity is offered to take up or give out heat. In this process during the summer season the increase of heat prevails, but during the winter time the loss of heat; the day-time also

in its relations follows more or less closely the summer, while the night-time is like the winter.

Under the assumption of a prevailing increase of heat the process presents itself somewhat as shown in the diagram (Fig. 34); starting with the condition *a* (in a cyclonic area) the expansion with a diminution of temperature proceeds according to the curve *a b*, which descends rather less steeply than does the the adiabatic curve. Corresponding to this, and also without reference to the initial quantity of moisture, the dew-point curve is first attained later, that is to say, at a greater altitude above the earth's surface than it would be in adiabatic expansion.

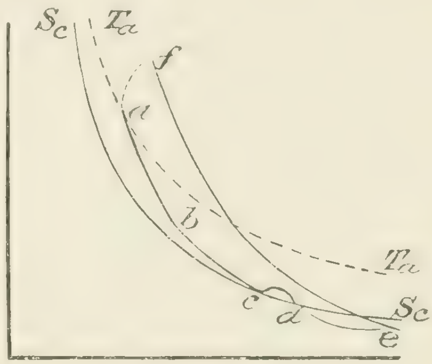

FIG. 34.

In the rain stage, therefore, the curve of change of condition experiences a deflection toward the upper side of the adiabatic, and therefore remains nearer the curve of saturation.

If now there occurs a still further greater addition of heat, as must be the case during the period of insolation and at great altitudes, where the condensation is less and the density of the clouds is correspondingly diminished, then the air can again pass over into the dry stage as is indicated in the portion *c d* of the curve.

Thus the upper limit of the first layer of clouds then would be at *c*. At this limit, during the summer days, more intense warming is in fact to be expected, which through a further expansion, that is to say at a greater altitude, on account of the diminished absorptive power of the atmosphere, again passes over into the approximate adiabatic *c d*, by which process, however, the dry stage is finally left and the snow stage *d e* is entered.

To this greater increase of heat at the upper limit of the clouds the fact is certainly to be ascribed that the cirrus (or snow) clouds are not directly continuous with the (lower or) water clouds, but generally separated from them through a wide space such as corresponds to the expansion from *c* to *d*.

During the descent in the anti-cyclone or by reason of the compression the process must take place according to the curve *e f*, which in general nearly agrees with the adiabatic of the dry stage. As we approach the earth's surface however, on account of the strong absorption of heat occurring there, then and for that reason this curve can depart to the right upwards from the adiabatic. This latter can however only occur temporarily, since in such a case we should have to do with a condition of unstable equilibrium.

The final pressure p_f, with which the sinking air reaches the ground in the anti-cyclone, is greater than the initial pressure p_a that prevails at the ground within the cyclone, and correspondingly f is higher above the axis of abscissas than a. In this case it may occur that the point f comes to lie not only (as is self evident) above, but also to the right of a, so that $v_f > v_a$ or in other words that the air at the base of the anti-cyclone, notwithstanding the higher pressure, is specifically lighter than in the cyclone, because the temperature more than compensates for the influence of the pressure.

This shows in a very clear manner that in the exchange of air between cyclone and anticyclone we have to do not only with the specific weight of the mass of air, but that here dynamic relations are of first importance, a point to which Hann has called attention lately in the discussion of the observations taken on the Sonnblick.* It will be well in the more accurate investigation of this question to give increased attention to the processes above the aqueous clouds especially at their upper boundary surfaces.

As to the relations of the humidity to the processes just considered these are nearly the same as those in the case of the foehn. Here also, that is to say in the anticyclone, the air arrives in the neighborhood of the ground warm and dry, but in the immediate neighborhood of the ground the evaporation stimulated by unrestrained insolation will rapidly add moisture to the air, so that the indicator, which moving from b nearly to e has steadily approached the PV plane and from e on the way towards f has remained a long time at the level of e, must now be imagined as rising immediately before reaching f. If now the air that has descended in an anticyclone again flows toward a new depression then will it (under the assumption of the same conditions in this as in the first cyclone), by reason of a continuous acquisition of aqueous vapor, pass through conditions that are represented in the diagram (Fig. 34) by the line $f\,a$. This line we have to imagine as slowly rising, so that the diagram here drawn presents in fact the projection of a closed line.

(3) *The interchange of air between cyclone and anti-cyclone in winter.*

In winter the diagram for this process of interchange has a figure essentially different from that in Summer. First, all changes in condition, at least insofar as concerns the initial and final conditions (see Fig. 35), take place nearer to the coördinate axes since the temperatures that come into consideration do not rise so high as in summer, and since, corresponding to this, the isotherms that lie far from the axis are not attained. Again, we have here lower pressure and higher temperature at the starting point a, but at the end d higher pressure and lower temperature, so that d is to be sought to the left and above a.

* *Meteorologische Zeitschrift*, 1888, vol. v, page 15.

Furthermore, the lines whose projections are here considered are not so far from the coördinate plane as in summer, because the absolute capacity for moisture remains always slight.

If now we follow more accurately the change of condition during ascent in the cyclone, we may at first assume that the process up to the attainment of the upper limit of the cloud stratum very nearly agrees with the adiabatic expansion, since below this limit radiation, either to or from, can only play an unimportant part. If however a departure from the adiabatic process does occur then it can be only in the opposite direction to that which occurs in summer, that is to say, the lines will descend more decidedly than in summer.

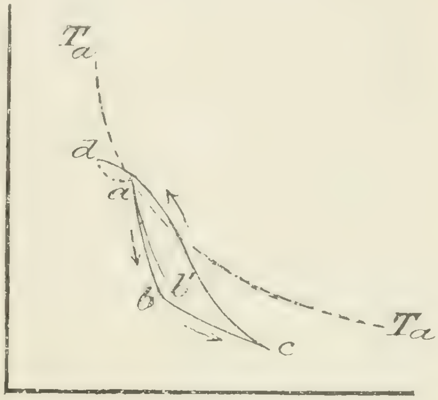

Fig. 35.

In Fig. 35 this latter case is assumed, as also that the passage out of the dry stage into the snow stage takes place immediately. From this point onwards the curve of condition again sinks more gradually, but with steadily increasing gradient in consequence of the overpowering cooling that certainly occurs at higher altitudes, until finally the turning point is attained and compression takes the place of expansion. The entire course of the change of condition to this point is presented by the curve *abc*. From this point onwards in consequence of the compression, the curve of condition must gradually advance to the point *d*. So far as our knowledge of the actual conditions of the atmosphere has attained hitherto, this gradual return to the point *d* occurs in such a way that at greater altitudes the compression proceeds adiabatically according to the adiabatic of the dry stage, whereas on approaching the ground the cooling by radiation that prevails there causes a deviation of the curve of condition from the adiabatic toward the axis of ordinates, and corresponding thereto the curve shows a course like *cd*. This curve however is nothing but the graphic expression for the well-known inversion that occurs on clear winter days in the vertical distribution of temperature. By reason of this inversion the curve near *d* approaches the dew-point curve, and can even pass it, so that condensation must occur and in the form of ground fog. But with the beginning of the formation of fog the radiation increases materially and corresponding to it the temperature diminution becomes always more intense with the proximity to the earth of the descending current of air.

Whether the passage from *c* to *d* be also possible by some other path by which from the very beginning of the compression the cooling and therewith the departure of the curve from the adiabatic makes itself felt, is a question that can be decided only after an accurate test com-

putation with the appropriate numerical data. At any rate such possible process would assume that in the anticyclone, at a certain height above the ground, exactly the same pressure and the same temperature prevail as at less altitudes above the base of the cyclone, since the projection of the curve of condition in this case must possess a double point.

These few examples, given only in their outlines, will suffice to enable one to realize the varied and useful applications that the method of graphic presentation here developed is capable of. By a further completion and development of the numerical side this method will give not only an excellent auxiliary means for the discussion and evaluation of existing data of observation, but above all will afford an indication as to the direction towards which material is to be collected in order to afford a deeper insight into the thermo-dynamics of the atmosphere.

If anything should seem especially suited to enable us to recognize the importance of the method of consideration here developed, it is the abundance of questions that press upon us at the first step we take in this way and that can at present be scarcely enumerated. I am thinking now, not only of the further development of theoretical consequences, therefore especially of the meaning of the thermal changes that occur in the atmosphere (especially the application of the second theorem of the mechanical theory of heat to these processes which may be developed in subsequent communications), but also, above all, of the stimuli that are to be derived therefrom to the observations of mountain stations, and especially in balloon voyages. For the latter it is full of meaning that in thermo-dynamic investigations the knowledge of the altitude above the sea can be entirely dispensed with and that it is entirely sufficient if we know the simultaneous values of the pressure, temperature, and moisture.

XVI.

ON THE THERMO-DYNAMICS OF THE ATMOSPHERE.*

(SECOND COMMUNICATION.)

By Prof. WILHELM VON BEZOLD.

In a memoir published several months since,† I made an attempt to so extend the Clapeyron method of graphic presentation of thermodynamic processes as to allow of its application to atmospheric changes. At the same time I showed by some examples how with the assistance of this method of representation even complicated phenomena can be studied with comparative ease, and how by means of it we are put in the position of being able to draw most important conclusions almost like child's play. In the following, the same method will be applied to other questions not then or only lightly touched upon.

First, I will treat of a conception that has lately been introduced into meteorology by von Helmholtz,‡ and which appears to me to possess great significance in this science. This is the idea of "wärmegehalt," or total amount of heat contained within a body. Helmholtz measures the heat contained in a mass of air by the absolute temperature that this same mass will assume when it is brought adiabatically to the normal pressure. The quantity that we here deal with is therefore not as one might easily have believed a quantity of heat but a temperature, and therefore it seemed to me, upon my first study of the memoir in question, desirable to replace the term "wärmegehalt" by another. In a conversation upon this matter von Helmholtz recognized the objection expressed by me as proper, and proposed that the word "wärmegehalt" should be replaced by the evidently much more proper expression "potential temperature." This latter expression will therefore be used exclusively in the following memoir, but at first this idea itself will be more accurately considered. Its presentation in a diagram will be attempted and a general theorem deduced from it.

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* Read before the Academy of Sciences, Berlin, November 15, 1888. (Translated from the *Sitzungsberichte der König. Preuss. Akademie der Wissenschaften zu Berlin*, 1888, vol. XLVI, pp. 1189–1206.)

† [See the preceding number of this collection of Translations.]

‡ "On Movements in the Atmosphere," *Sitzb. Berlin Akad.*, 1888, vol. XLVI, p. 647. [See No. V of this collection of Translations.]

I. THE POTENTIAL TEMPERATURE.

According to what has just been said the potential temperature is that absolute temperature that a body assumes when without gain or loss of heat it is adiabatically or pseudo-adiabatically reduced to the normal pressure. I intentionally give this definition the form here chosen since we are here concerned with the application of the idea to meteorological processes, and since in our case the processes without increase or loss of heat do not need to be strictly adiabatic in the ordinary sense of the word. As I have shown in the previous memoir we have only to do with adiabatic processes when the water formed by condensation does not fall to the earth but is carried along with the air, a condition that is only fulfilled in exceptional cases. As soon as water is lost, and this is generally the rule, even though no heat be gained or lost, we have to do with a process that is only pseudo adiabatic. When therefore in the following, mention is made of adiabatic changes, the pseudo-adiabatic will always be included therein in so far as this class is not excluded by the special term "strictly adiabatic."

This much being premised we may now first investigate whether and how the potential temperature can be represented in a diagram. The answer to this question is extremely simple. From the equation of condition for the dry stage

$$vp = R^* \, T$$

there results

$$v = \frac{R^*}{p} \cdot T$$

or if we substitute for p the normal pressure p_0

$$v = \frac{R^*}{p_0} \cdot T.$$

Therefore under constant pressure the absolute temperature is simply proportional to the volume, that is to say to the abscissa. But this absolute temperature under the pressure p_0 is the "potential temperature" for all other conditions that find their representation on the adiabatic passing through the point whose coördinates are v and p_0. We therefore obtain the following rule:

If a condition is given that is represented in the diagram, Fig. 36, by the point a, then we find the corresponding potential temperature by drawing an adiabatic line through a and seeking its point of intersection N' with a straight line $P_0 N$ drawn parallel to the axis of abscissas and distant therefrom by p_0. The distance of this point of intersection N' from the axis of ordinates, namely, the abscissa of N' (or $N' P_0$) is now a measure of the potential temperature.

We find the numerical values of v and T belonging to p_0 (and which I will now designate by v' and T' corresponding to the point N', while I

designate by v' and T' those corresponding to the initial condition a) by combining the equation of the adiabatic

$$p_a v^{\kappa}{}_a = p_0 v'^{\kappa},$$

with the equation of elasticity

$$\frac{p_a v_a}{T_a} = \frac{p_0 v'}{T'} = R^*$$

and we thus obtain

$$T' = \left(\frac{p_0}{p_a}\right)^{\frac{\kappa-1}{\kappa}} T_a$$

where $\varkappa = 1.41$.*

But this simple method of consideration is only allowable so long as the changes of condition take place within the dry stage. If this stage is left then the potential temperature belonging to a definite intial point has no longer a constant value, but it increases with the quantity of precipitation that is lost. A glance at the figure suffices to show this:

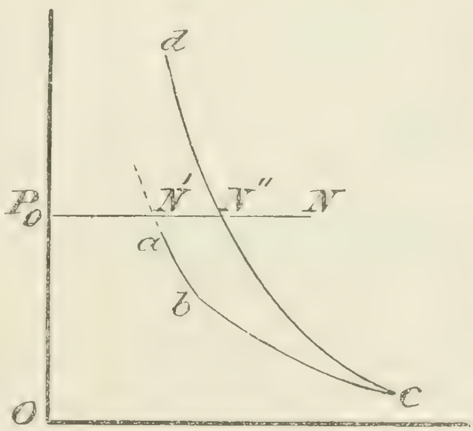

FIG. 36.

Assuming that the adiabatic of the dry stage drawn through a intersects the dew-point curve (which for simplicity is not shown in the figure) in b and that we now allow the air to still further expand, then one has to pass from b down along the adiabatic (or pseudo-adiabatic) of the rain or snow stage, that is to say along bc.

If now we seek the potential temperature for a point, c, of this line (in order to simplify the figure I have drawn the line bc only just to this point), in that we bring it again adiabatically to the normal pressure, then one ought not to run back along the curve bc, since on account of the precipitated water the conditions represented by this portion of the line are not again attainable, but on the other hand one can only attain to the line of normal pressure by following the adiabatic cd corresponding to the dry stage, but a dry stage with less quantity of aqueous vapor than before.

If we indicate by N'' the point at which this occurs or at which the normal pressure is thus attained, then as the measure of the potential

* In the previous memoir, in consequence of an oversight, k was used instead of $\varkappa$ by von Bezold, but at his request this has been changed in the present translation.

temperature we have the length $P_0N''>P_0N'$; that is to say, the potential temperature T'', as attained by adiabatic change after passing into the condensation stage and after precipitation of some water, is higher than the potential temperature T' of the initial condition and of all the conditions previously passed through in the dry stage 0.

Analytically this may be proved in the following manner:

For the transition from a to b the following equation obtains

$$p_a v_a^\kappa = p_b v_b^\kappa = C'$$

If this equation remains in force after crossing over the curve of saturation, then we obtain for the pressure proper to the volume v_c a value $p_\gamma < p_c$ where p_c is the pressure that in the condensation stage actually corresponds to the volume v_c.

But

$$p_c v_c^\kappa = p_d v_d^\kappa = C''$$

and since

$$p_\gamma v_c^\kappa = p_b v_b^\kappa = C'$$

and since also

$$p_\gamma < p_c$$

therefore

$$C' < C''.$$

But from this it further follows that $v''>v'$ and $T''>T'$ where v' and v'' are the volumes corresponding to the normal pressure p_0 on the adiabatics ab and cd; hence,

$$p_0 v'^\kappa = C'$$

and

$$p_0 v''^\kappa = C'',$$

beside which the following equation holds good:

$$v':v'' = T':T''.$$

Thus we attain to the theorem

In adiabatic changes of condition in moist air the potential temperature remains unchanged so long as the dry stage continues, but it rises with the occurrence of condensation and so much the more in proportion as more water is discharged.

Since in the free atmosphere, in general, evaporation does not occur and since also the carrying along of all the water that is formed, at least in the case of heavy condensation, must be considered as an exceptional case only, therefore, this theorem can also be brought into the following form:

Adiabatic changes of condition in the free atmosphere, assuming that there is no evaporation, either leave the potential temperature unchanged or elevate it.

From this theorem, which in its latter form reminds one of the theorem of Clausius in respect to the entropy, "The entropy strives towards a maximum," though not identical with it, one can draw consequences of the greatest importance. The next two sections will be devoted to these.

VII. THE VERTICAL TEMPERATURE GRADIENT.

All motions in the atmosphere can be considered as analyzed into vertical and horizontal components. The latter, in so far as they do not closely follow the irregularities of the earth's surface, are subject in only a slight degree to thermo-dynamic changes. On the other hand, in consequence of the expansion or compression in ascending and descending currents, the thermo-dynamic cooling or warming plays a very important rôle. The horizontal movements will therefore for the present be left entirely out of consideration, but the processes going on in the vertical currents will be thoroughly investigated. The changes of condition going on within ascending and descending currents must be considered in the free atmosphere as adiabatic so long as we content ourselves with a first approximation, and that we must do at first, since in the free air there is only a small opportunity given for active radiation and absorption. On the other hand the increase and diminution of heat will always make themselves felt decisively either where the absorbtivity and emissivity are remarkably increased or where the air comes into direct contact with bodies which themselves can strongly emit and absorb or otherwise take in or give out heat. This is the case:

(*a*) In the neighborhood of the earth's surface, where besides the increase in absorbtivity and emissivity of the air due to cloud or fog, the warming and cooling of the ground by radiation, as well as the evaporation, the formation of dew or frost, the thawing and freezing, have a powerful influence.

(*b*) In fog or cloud, which also possess a special power of absorbtion and emission, and where moreover evaporation can occur; and especially is it the upper limiting layer of clouds that one has to take into consideration.

In so far therefore as one can leave out of consideration the special localities just indicated, as also the mixture with other masses of air, one can approximately consider the processes in ascending and descending air currents as adiabatic. Even taking into consideration the special locations above mentioned, one can consider a scheme drawn up under the assumption of adiabatic change as to a certain extent an average or normal scheme, since such a scheme always occupies an intermediate position between those where the incoming radiation and those where the outgoing radiation prevails. How such a prevalence of either radiation must show itself has already been indicated in the previous communication [*p.* 212], where the interchange of air

between cyclone and anticyclone in summer and winter was investigated, at least in its principal features.

But in this study it is not necessary to limit oneself to the summer or the winter, but rather one can apply the scheme for the summer generally to all cases where the radiation is in excess, that is to say, not only to the summer time in general, but to the day-time and the hot zone; the scheme for the winter, on the other hand, is applicable not only to the winter season, but to the night-time and the cold zones of the earth. This normal scheme for the ascending and descending currents will therefore appear as shown in Fig. 36. The portion *a b* has reference to the ascending current in the dry stage, *b c* is its continuation in the condensation stage, finally *c d* is the portion of the curve that corresponds to the descending current.

This scheme differs only a little from that communicated in the first memoir. (For the case of the foehn, see page 240.) We can not expect it to be otherwise, since in the foehn one has also to do with an ascending and descending current of air in which the velocity with which the whole process goes on affords only a small opportunity for the gain and loss of heat. However, the diagram given in figure 36 as the "normal scheme" differs from that which obtains for the foehn in this respect, that the branch *cd* is longer. This is due to the fact that in the ordinary interchange between cyclone and anticyclone there always prevails a higher pressure at the base of the latter than at the base of the former; that is to say, the ending point *d* in the normal scheme must always lie higher than the starting point *a*, which is not the case in the foehn diagram. In general, one has to consider the process in the foehn as only a feature inserted into the normal interchange between anticyclone and cyclone. In the foehn the passage over the mountain chain forces the air in its normal interchange to describe an antecedent ascent and a subsequent descent which is only then followed by the definitive ascent in the cyclone. This being premised, the processes in the interchange, according to the normal scheme, will now be more precisely considered.

If we introduce the conception of the potential temperature, we attain the following theorems without any difficulty:

(*a*) In the ascending branch* the potential temperature increases steadily from the beginning of the condensation; in the descending branch it remains constant at the maximum value attained in the whole process. This maximum value corresponds also to the highest point to which the air has risen in its path.

(*b*) The potential temperature of the upper strata of the atmosphere is in general higher than that of the lower.

The first of these two theorems results directly from the diagram; the second follows from the fact that in the lower stratum the potential

* By the ascending branch is meant the portion *ab* which corresponds to the ascent in the atmosphere; the portion *cd* is considered as the descending branch.

temperature must, in the continuous interchange between cyclone and anticyclone, retain an average value that lies between the maximum value T'' and the smaller value T' corresponding to the base of the cyclone; that is to say, to the point a on the diagram. This average value is, however, certainly smaller than the maximum value T'' corresponding to the highest point of the path, and therefore to the condition c, and thus the theorem (b) is proven. Hence it follows that in nature the diminution of temperature for a constant elevation, or we will rather say, for 100 metres, that is to say; the so-called vertical temperature gradient, is, in general, smaller than results from the theory of the dry stage. As is well known, this gradient is 0.993 for the latter stage, that is to say, under the assumption of adiabatic change one would expect in the dry stage a diminution of 1° centigrade in temperature for an ascent of 100 metres.

This value 0.993 I will call ν.

The above given theorems concerning the potential temperature show at once that under the assumption of adiabatic exchange the real value of the temperature gradient must be less than ν.

We reach this conclusion from the following considerations:

Let t_a and t_d be the temperatures at the bases of the cyclone and anticyclone respectively (that is to say, at the starting and resting points of the ascending and descending currents) then, under the assumption of perfect adiabatic change, these will not greatly differ from the potential temperatures T' and T'', as these correspond to the ascending and descending branches in the dry stage, that is to say, to the conditions represented by the curved portions ab and cd in figure 36. In this process the departures from these temperatures are always of such a nature that $t_a < T'$ and $t_d > T''$. For, since the pressure p_a at the base of the cyclone is certainly smaller than the normal pressure, but the pressure p_b at the base of the anticyclone greater than it (at least when a normal pressure is chosen appropriate to this case, and therefore lying between p_a and p_b), therefore the temperature t_a is increased by referring it back to this pressure, while t_d by the corresponding process is diminished. Since the statement is thus proven that $t_a < T'$ and $t_d > T''$, and since, moreover, $T'' > T'$, therefore, by so much the more must $t_d > t_a$.

At the highest point of its path, such as corresponds to the point c of the diagram, the particle of air has a potential temperature T'' that is to say, precisely the same as at the end.

If now it be assumed that this point lies $100h$ metres above the earth's surface, then there results as temperature gradient for the descending branch that is to say, as the increase of temperature for each 100 metres of descent, the well-known value

$$n'' = \frac{t_d - t_c}{h} = \nu.$$

On the other hand, for the ascending branch we obtain a value

$$n' = \frac{t_a - t_e}{h},$$

if for the sake of simplicity the difference of temperature prevailing above and below be equally distributed throughout the whole height.

This simplification is, of course, not strictly correct since the ascending branch of the two stages certainly includes in itself several stages, *e. g.*, the dry stage, the rain or snow stage, and perhaps also the hail stage, or all together. Still the method of computation of the average gradient as given here in the formula is the only method that we can apply when we have only one upper and one lower station. The following considerations however remain applicable at least in a general way when we can apply more rigorous formula.

Namely, for purely adiabatic change in any case we have $t_a < t_d$, and therefore also

$$n' < n''.$$

We attain to the same result also when we simply consider that the vertical gradient within the condensation stage is materially smaller than in the dry stage. When, therefore, the greatest gradient coming into consideration in the ascending branch is $n'' = \nu$, then the average of all must certainly be smaller.

Therefore, in purely adiabatic ascent and descent and passage into the condensation stage the mean vertical temperature gradient in the ascending branch is always smaller than in the descending.

If now we imagine regions of ascending and descending currents alternately passing over one and the same point of the earth's surface, we thus obtain for the mean vertical temperature gradient above that point a value n that certainly lies between n' and n'' therefore satisfies the condition,

$$n' < n < n'',$$

wherein $n'' = \nu$ is nearly constant, n' however varies within wide limits according to the initial temperature and the initial quantity of aqueous vapor contained in the air.

Therefore, under the assumption of adiabatic changes, in moist air that reaches the point of condensation, the mean vertical temperature gradient is always smaller than in dry air.

We see from this that the consideration of the condensation alone already suffices to explain at least the direction of the departure of the observed vertical temperature gradients from those computed under the assumption of dry air, even if we retain the assumption of purely adiabatic changes. But this latter assumption is in fact certainly never fulfilled exactly, and it is therefore necessary to examine more accurately the influence that the departure to one side or the other from this normal process may have upon the vertical temperature gradient.

This again is most simply done by the introduction of the idea of the potential temperature. We can, namely, bring together all of the considerations just expounded into the following theorems:

(1) *If the potential temperature above and below is the same* i. e., *constant throughout the whole layer of air under consideration, then the vertical temperature gradient has the well-known value $n = \nu$.*

(2) *If the potential temperature in the upper stratum is higher than in the lower stratum (and this is in general the case), then is the temperature gradient smaller, and smaller in proportion as for a given difference in altitude, the difference of the potential temperatures is larger.*

If we indicate the potential temperature of the upper stratum by T_s and that of the lower stratum by T_i, then for $T_s > T_i$ we shall always have $n < \nu$ and in fact the differences $T_s - T_i$ and $\nu - n$ always increase simultaneously.

A decided cooling in the lowest stratum always causes a diminution of T_i and with it also a diminution of n, whereby even a change in the sign of n may occur within moderate altitudes. In the latter case, the temperature below is lower than in somewhat higher layers, and in that case we have the so-called inversion of temperature. If the cooling is not sufficiently strong to bring about an actual inversion of the temperature, still it causes a diminution of the gradient. Such decided cooling always takes place in the lowest stratum at the time of increased radiation, therefore especially in the region of the anti-cyclone, *i. e.*, under a clear sky, in winter, and in the night time. Therefore in the winter and in the night-time the vertical temperature gradient must be smaller than during the summer and day-time, even if inversion in the distribution of temperature does not occur. This result agrees perfectly with observations, as is especially proven by the many facts that Hann and others have collected from the Alpine regions.

On the other hand the investigation here carried out teaches that the inversion of temperature and the diminution of vertical gradient connected therewith are to be treated not as phenomena peculiar only to the mountain regions, but that we are to expect them also above the plains, and even above the ocean, at least insofar as the more violent movements of the air do not interfere therewith.

We are therefore obliged to agree with Woeikoff* when he from a few data draws the conclusion that this inversion is also to be expected in the region of the great winter anti-cyclone of eastern Siberia.

On the other hand I can not agree with him when he deduces from this the consequence that Messrs. Wild and Hann should have considered this circumstance in drawing their isotherms, and I consider the standpoint taken by them as perfectly justified.†

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* Woeikoff, *Klimate der Erde*, German edition, 1887, Bd. II, p. 322; *Meteorologische Zeitschrift*, 1884, Bd. I, p. 443.

† Hann, *Atlas der Met.*, 1887, p. 5. Wild, *Repert.*, 1888, Bd. XI, Nr. 14.

A direct proof of the inversion of temperature above the lowlands can only be expected from balloon observations.

To what extent radiation causes the inversion or at least the diminution of the gradient we shall learn from a work now soon to be published, that Sühring* has executed at my recommendation, and in which the vertical gradients of temperature between the Eichberg and the Schneekoppe, as well as between Neuenburg and Chaumont, are investigated according to the separate percentages of cloudiness.

It is not improbable that also above the ocean, and even at the time of the stronger insolation, a diminution of gradient, if not even an inversion of temperature, occurs, since over the sea the rapid evaporation in connection with the mobility of the water puts an impassable limit to the rise of temperature. The stability of the Atlantic anticyclone during the summer months may be based upon this circumstance.

The cases in which an increase of heat occurs at the earth's surface need no special consideration in the questions here considered. The gradient can only for a short time exceed the value ν, as determined for the expansion or compression of dry air. If this case occurs, then, according to the investigations of Reye and others, we have unstable equilibrium or a condition that can only exist temporarily, as in whirlwinds or thunderstorms. Therefore, even for the strongest insolation, the considerations above given continue to hold good.

On the other hand the fact must excite great consideration that, not only on the average of all cases, but also when we investigate only the region of ascending currents (and of these only those that are below the limit of clouds, that is to say, for moderate elevation of the upper station) we find that the vertical gradient is always decidedly smaller than ν. The reason of this is principally to be sought in the fact that the above views as presented by me, as also by other investigators in this direction, all rest upon an implied assumption that is only allowable to a very limited extent. They are based namely upon the assumption that the air ascending from the earth experiences no change in its constitution, except that due to the loss of water consequent on the adiabatic expansion, *i.e.*, that it experiences no mixture with masses of air of other temperature or other degrees of moisture, as also that every particle of air considered in the interchange between cyclones and anti-cyclones describes the whole path from the earth's surface to the limit of the temperature and back again.

But this is by no means the case. Only a small fraction of the air under consideration actually comes in contact with or even in close proximity to the earth's surface; and similarly with the ascent to the limit of the atmosphere or at least to the highest stratum that at any time takes part in the process under consideration. Moreover in the

* Sühring, *Die vertikale Temperaturabnahme.* Inaugural Dissertation d. Universität, Berlin, 1890

ascending whirl, masses of air are always drawn in from one side that had not yet sunk to the earth's surface and had remained correspondingly unaffected by the radiation and absorption that have their seat in that stratum, and which also had had no opportunity to take up water from the earth's surface. Since these masses of air coming from the upper portions of the anticyclone have in general higher potential and therefore also higher absolute temperature than the portions of the cyclone lying at equal altitudes above sea level, therefore the mixture of these will diminish the cooling of the ascending air and both thereby as also by reason of the lesser quantity of water that they possess, will delay the occurrence of condensation.

Therefore in the cyclone itself the vertical temperature gradient even beneath the clouds will not be so large as one would expect according to the law of the adiabatic changes for the dry stadium without mixture of foreign masses of air. Similar relations obtain, although not to an equally great extent, with regard to the descending current, which in its upper half is also fed by portions of the cyclone in which the condensation has not yet gone so far and has not yet attained the high potential temperature of the highest stratum concerned in the whole process. Therefore in reality both the ascending and the descending branches of the curve deviate from the schema of Figure 36, and in both of them the vertical gradient will more or less approximate the average as we find it when we consider the ascent and descent as a connected whole.

These considerations are entirely in accord with observed facts. Even when we deduce the vertical temperature gradient from observations at stations of which the upper one is not so high that it is frequently within the clouds, we attain to temperature gradients that in general are far less than that computed for the dry stage; this result is in great part only explicable as due to the above described mixture. The observations of the clouds also agree perfectly with what has been said, both with regard to the temperature conditions and the moisture.

Only the central part of the cyclone is to any considerable extent fed by masses of air that have flowed along the surface of the earth itself, as one can easily convince himself by a simple diagram;* whereas the periphery receives more and more air from the higher strata, whereby its lower boundary surface is raised but its power must be diminished. In fact also the clouds at the center of the cyclone hang down the lowest and are higher near the circumference, exactly as is demanded by the moisture conditions and the higher potential temperature of the intermixed masses of air. The fringe of clouds that we perceive beneath the layer of clouds that covers the sky (especially on wooded hills during the prevalence of a cyclone) and in which we can clearly follow the ascent of air in inclined paths, gives in connection with the

* See, for example, Mohn, *Grundzüge*, 3d edition, 1883, p. 261.

ragged clouds surrounding the border of the continuous cloud cover, an excellent picture of the mixture just described.

Of course it is understood that all these considerations relate only to the conditions that ordinarily occur in the interchange of air between cyclone and anti-cyclone.

Processes in which we have to do with unstable equilibrium (such as occur, for instance, in the great thunderstorms in front of an advancing current of air, where a whirl with a long horizontal axis rolls rapidly forward and brings simultaneously on the side of the descending current heavy rain-fall and great cooling with higher barometric pressure, while on the front or ascending side the cloudiness is just beginning)—such processes demand a very special investigation that may be postponed to some future occasion. At present only one more consequence will be drawn from the propositions relative to potential temperature which seems to me calculated to throw a new light on the interchange of heat in the atmosphere, and that especially demands consideration from a climatological point of view.

III. ON COMPLEX CONVECTION.

It has been shown above that in the adiabatic transfer of air out of the cyclone into the anti-cyclone, the potential temperature in the descending branch is higher than in the ascending. Hence it follows that in the descending branch a higher temperature prevails after attaining the initial pressure than prevails at the initial point, and a still higher temperature prevails at the end of the descending branch, that is to say on the ground in the anti-cyclone where, according to experience as well as for mechanical reasons, the pressure is always higher. Therefore in this transfer of air we are concerned not only with a simple transfer of the quantity of heat belonging to the air at the base of the cyclone, which we can here temporarily call the original quantity of contained heat, but this quantity of heat is increased by that heat of condensation which in the condensation stage did a part of the work of expansion and thereby diminished the cooling to a smaller quantity than it otherwise would be.

Even when in consequence of the stronger abstraction of heat at the base of the anti-cyclone the air is finally colder than it would have been in purely adiabatic interchange; and even when temperature inversion has occurred, still the temperature at the end of the process is still always higher than if the transportation of the air had taken place at the level of the earth's surface and the cooling influences had remained the same.

The heat of condensation or negative heat of evaporation, or as it was formerly called the liberated latent heat, accrues to the advantage of that region in which the descending current has reached the earth's surface.

We can therefore compare the whole process with that of a steam heater.

Moist air rises in the cyclone, attains the condensation stage and cools from that time on less rapidly since the heat of condensation does a part of the necessary work. The heat thus saved then enters into the descending current and finally is carried to the point at which the descending current reaches the earth's surface.

I consider it proper to designate by a special word those transfers of heat in which, besides the transport of warm or cooled bodies, changes of the condition of aggregation also occur, and therefore propose the name "complex convection" or "complex transfer." Such complex convection is met with when vapor is formed at one place and precipitated at another, or when ice falls as snow or hail, or when it is transported in the form of icebergs by ocean currents. If we apply this designation to the above-given considerations we obtain the following proposition:

"*In consequence of complex convection the temperature in anti-cyclonal regions is always higher than would be the case in simple convection.*"

The application of this proposition to the warm zone is of very special interest (I designedly avoid saying Tropical Zone since I can not consider the warm zone as limited by the Tropics) that is to say to the calm zone and the rings of higher atmospheric pressure that border it on either side, of which rings however the northern one is frequently interrupted.

The proposition just enunciated teaches that these two rings in consequence of complex convection are much warmer than would be the case if in the whole interchange one had only to do with dry air or with movements on one level. The warm zone is therefore hereby broadened and at the same time there is found within it a diminution of the temperature gradients.

In the calm zone itself much heat is used in evaporation and hence, in connection with the diminution of insolation by the covering of clouds, as also by reason of the water precipitated from colder regions above, the rise of temperature above a given limit is prevented. The heat consumed by evaporation at the earth's surface or at the ocean's surface does its work at a greater altitude in the region of the clouds when liberated by the condensation, and thus diminishes the cooling of the ascending current only to again reappear below in both the belts of descending currents.

A further development of the climatological consequences deducible from these considerations does not belong here. But this much we see at once, that the conclusions drawn from the mechanical theory of heat without any hypothesis whatever stand in direct contradiction to the older meteorological views. Formerly it was taught that the descending trade wind by cooling delivers to higher latitudes the water brought with it from the calm zone. Similarly it was taught that the heat liberated during the condensation raised the temperature, and that this

higher temperature inured to the places at or above which the condensation occurred.

The mechanical theory of heat shows that the current ascending in the calm zone must precipitate its water right there in the form of tropical showers, and that then it must descend as a drier and also as a warmer current (except in so far as it does not experience any material cooling, especially at the earth's surface). This theory further shows that the heat of condensation, in so far as super-saturation proper does not come into consideration, never shows itself as actually warming but only as diminishing the cooling that accompanies the ascent of the air, so that the current arrives at the upper limit warmer than it would without the accompanying condensation, and that the heat thus economized benefits the point at which the descending current reaches the earth's surface.

The considerations here developed can of course only be considered as approximate steps that still await additions and corrections. To my eye they play a rôle similar to that of the investigation of the so-called solar climate in climatology. Moreover some of these have no claim to complete novelty, but will be found here and there in connection with other special investigations.

On the other hand, they have never as yet been developed in such general—and never in such a simple—manner as is here done with the help of the idea of "potential temperature" and of the theorems that it was possible to deduce from this as to the potential temperature of the different layers of air. The consequences that can be deduced from this as to the static relations of the atmosphere, especially with reference to the fundamentally different behavior of cyclones and anti-cyclones in winter and in summer, both in respect to their intensity and their duration, are delayed to a later communication.

[An Appendix as published in the original memoir by von Bezold is omitted from this translation, as it has been at the author's request incorporated in its proper place in the latter portion of his first communication.]

XVII.

ON THE THERMO-DYNAMICS OF THE ATMOSPHERE.*

(THIRD COMMUNICATION.)

By Prof. WILHELM VON BEZOLD.

In the two papers previously published on the above subject the restrictive assumption has been always made that the masses of air under consideration experience no mixture with similar masses having other temperatures and other degrees of moisture. At the same time however it was shown that such mixtures must frequently occur in nature and that the investigations in question could possess only a restricted application so long as we neglect these processes.

For this reason therefore it is now necessary to extend the previous investigations in this direction.

But investigations on this subject have also a special interest because for a long time we formerly attributed too much importance to the mixing of masses of air of unequal temperature and near the point of saturation, whereas in more recent times we have gone to the opposite extreme and attributed to it scarcely any importance at all.

Following the example of James Hutton,† the mixture of such masses of air was, until within a few decades of years, considered as the principal cause of atmospheric precipitation.

Wettstein was (so far as I know) the first to antagonize this view‡ which however even to-day is still widely accepted.

He however fell into the opposite error in that he contended that, in general, precipitation never could occur by mixing.

Here, as in so many other points of modern meteorology, Hann§ first made the matter clear in that he, in the year 1874, proved that by mixture condensation could be indeed produced, but that the former method of computing the quantity of precipitation was affected by an error in principle after correcting for which the values obtained are so small

* Read before the Academy of Sciences at Berlin, October 17, 1889. [Translated from the *Sitzungsberichte der König. Preus. Akad. der Wissenschaften zu Berlin*, 1890, pp. 355–390.]

† *Roy. Soc. Edinb. Trans.*, 1788, Vol. I, pp. 41–86.

‡ *Vierteljahrss. d. naturf. Gesell. Zürich*, 1869, XIV, pp. 60–103.

§ *Ztschft. Oesterr. Gesell. Met.*, 1874, Vol. IX, pp. 292–296. [*Rep. Smithson.*, 1877, p. 385.]

that the production of a moderately heavy precipitation in this way is impossible.

At the same time he showed that the adiabatic expansion in this respect played an entirely different and much more important rôle, and that, in it we have to recognize the source of all considerable precipitations.

In this paper, so far as it concerned mixture Hann confined himself to the computation of an example from which it appeared that even under very improbable assumptions there could in this way only be realized very slight quantities of precipitations.

Pernter many years later* contributed to the solution of the problem in that he brought it into an exact mathematical form and at the same time also computed small numerical tables in order to facilitate the comprehension of the quantities that enter into the question.

But since the empiric formula for the tension of aqueous vapor enters into the expression given by Pernter, therefore the latter is rather complex and is not especially clear.

It seems therefore to me not only desirable but really necessary to take up the question anew and if possible prosecute it to a definite conclusion. This is the object of the following lines.

It will be shown how graphic methods give with extraordinary ease an insight into the whole theory of the mixture of air and how in such methods we possess at the same time the simplest means for the numerical evaluation of the quantities that enter into the question.

Various tables—some of which may also be welcome for other investigations—will also facilitate a general survey as well as the exact computations. After these preparatory sections there will be considered the various causes of the formation of precipitation, namely, direct cooling, adiabatic expansion, and mixture, in their relative importance and it will be shown how that only by the consideration of all these causes is it possible to obtain a deeper insight into the methods of the formation of clouds.

(*a.*) THE MIXTURE OF QUANTITIES OF AIR OF UNEQUAL TEMPERATURE AND MOISTURE.

Before we proceed to the mathematical treatment of this problem we must first come to a clear understanding as to whether definite masses or definite volumes shall be made the basis of the computation.

At the first view it would seem appropriate to adopt the volume, since we can from well-known tables obtain directly the quantity of water which corresponds to the saturation of one unit of volume.

This is doubtless the reason why in the older investigations of this subject based on Hutton's theory, one always started with the consideration of the unit of volume, and why Hann—when he would

* *Zeitschft. Oesterr. Gesell. Met.*, 1882, Vol. XVII, pp. 421–426.

demonstrate the imperfections of this theory in his considerations on this subject, followed the earlier method of treatment, and adopted the volume as a basis.

This is also quite justifiable so far as concerns the first estimates, and I also recently have made the same application in a popular lecture.

But when one wishes to obtain exact formulæ this method brings him into difficulties. These arise from the fact that the capacity for heat of a unit of volume, the so-called volume capacity, even without the consideration of the intermixed vapor of water, is to a high degree affected by pressure and temperature, so that no forms of approximation are allowable. The capacity for heat of the unit of mass of moist air, therefore its capacity for heat in the ordinary sense of the word, is entirely independent of the above mentioned quantities and is also so little influenced by the contained water within the limits that occur in meteorology that, as will later be more accurately shown, we can in the present question simply consider it as constant.

In order however not to lose the advantage that inures from the utilization of existing tables, I have computed for different pressures and successive degrees the quantity of aqueous vapor that is contained in a kilogram of saturated moist air for such pressures and temperatures as occur in the atmosphere and have communicated the table thus formed in an appendix to this paper (see page 287).

This table not only facilitates very considerably the solution of the questions that refer to the mixture of moist air, but it can also be applied with profit to many other investigations. After this preface the problem itself is to be considered more closely, and to this end an appropriate notation is first to be introduced.

Let there be

m_1 and m_2, the quantities expressed in kilograms, of air to be mixed together;

t_1 and t_2, their temperatures;

y_1 and y_2, the quantities expressed in grams, of vapor actually contained in a kilogram of moist air;

y'_1 and y'_2, the corresponding values of contained moisture in a kilogram of air at t_1 and t_2 in the saturated condition;

R_1 and R_2, the accompanying values in per cent. of the relative humidity.

ρ_1 and ρ_2, the same quantities expressed as fractions of unity, that is to say

$$\rho_1 = \frac{R_1}{100} \text{ and } \rho_2 = \frac{R_2}{100}.$$

t_3, y_3, y'_3, R_3, and ρ_3, the various values of the same above-named quantities in the mixture, in so far as the limit of saturation has not been exceeded, or at least no water has been lost, that is to say, true saturation exists.

t, y, y', R, and ρ, the corresponding values after mixture and after the loss of the quantity of water that exceeds the normal quantity for saturation, or also, in general, any given group of the same quantities belonging together.

The pressure expressed in millimetres of mercury will as before be expressed by β; the maximum of the elastic force of the vapor will in a corresponding manner be expressed by ε. The pressure β can be considered as constant during the process of mixing. This is allowable since, where mixture actually occurs, the two masses of air must necessarily exist under very nearly the same pressure and must also retain this [in the free air] even when on account of the mixing a change occurs in the total volume, which in general is very unimportant.

The problem of mixture becomes extremely simple so long as no precipitation of water occurs, that is to say so long as the quantities obtained by the mixture are to be indicated as in the above notation by the subscript 3.

In this case

$$y_3(m_1+m_2)=y_1m_1+y_2m_2$$

or

$$m_1(y_3-y_1)=m_2(y_2-y_3) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and further

$$c_1m_1(t_3-t_1)=m_2c_2(t_2-t_3)$$

where by c_1 and c_2 we understand the thermal capacities of the quantities of air to be mixed,* or since these quantities are to be considered equal

$$m_1(t_3-t_1)=m_2(t_2-t_3) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If we combine the equations (1) and (2) we obtain (the mixing ratio)

$$\frac{y_3-y_1}{y_2-y_3}=\frac{t_3-t_1}{t_2-t_3}=\frac{m_2}{m_1}$$

which is the well known equation that holds good for the mixture of two quantities of the fluid in question, having two different temperatures.

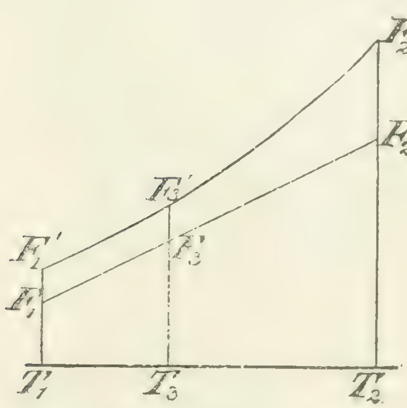

FIG. 37.

Since the graphic method will be chosen in the further development, therefore first of all this simple formula must be translated into a geometrical form.

To this end, in a rectangular system of coördinates, Fig. 37, the temperatures (t) are taken as abscissas, the quantities of moisture (y) as ordinates, and these are designated in the ordinary manner by OT_1, OT_2 . . . .

*Strictly speaking we should use mean values computed by a special formula between the above named c_1 and c_2 and that of the mixture c_3. Since, however, the values of c scarcely differ from each other for the different temperatures and pressures, we can therefore omit this refinement.

T_1F_1, T_2F_2, etc.; in the figure the origin O is omitted. We see at once that F_3 lies on the straight line drawn through F_1 and F_2 and that

$$\frac{T_1\,T_3}{T_2\,T_3}=\frac{T_3\,F_3-T_1\,F_1}{T_2\,F_2-T_3\,F_3}=\frac{m_2}{m_1}$$

In order now to obtain a decision as to the degree of saturation, we must also introduce into the diagram, as ordinates, along with the values of y_1, y_2, and y_3, also the values of y_1', y_2', and y_3', corresponding to complete saturation. The ends of these ordinates, which are represented by F_1', F_2', and F_3' in the diagram, all lie upon a curve that with increasing t rises rapidly, and the equation* of which is

$$y=623\,\frac{\varepsilon}{\beta-0.377\,\varepsilon}$$

when for β we insert the proper constant pressure.

With the assistance of this equation, or with the approximate formula obtained by development,

$$y=623\,\frac{\varepsilon}{\beta}+234.88\left(\frac{\varepsilon}{\beta}\right)^2$$

the tables communicated in the appendix [page 287] have been computed, by the help of which the curves can be easily constructed directly for the pressures therein considered, and which we can designate as curves of the quantity of vapor needed for saturation at the pressure β [or for brevity, *the saturation curve*].

It will now suffice to cast a glance at the figure in order at once to obtain the following propositions:

(1) So long as for given temperatures t_1 and t_2, the values $\frac{y_1}{y_1'}=\rho_1$ and $\frac{y_2}{y_2'}=\rho_2$, remain within given limits, the straight line $F_1\,F_2$ passes entirely beneath the saturation curve, and therefore there can be no mixing-ratio for which condensation can occur.

(2) When ρ_1 and ρ_2 increase so much that the straight line $F_1\,F_2$ touches or cuts the saturation curve, as in figure (38), then there occurs either one or many mixing-ratios that may bring about condensation.

(3) When $R_1=R_2=100$, *i. e.*, when the two quantities of air to be mixed are saturated, then the straight line $F_1\,F_2$ coincides with the curve $F_1'\,F_2'$, and then for every mixture there occurs super-saturation or condensation.

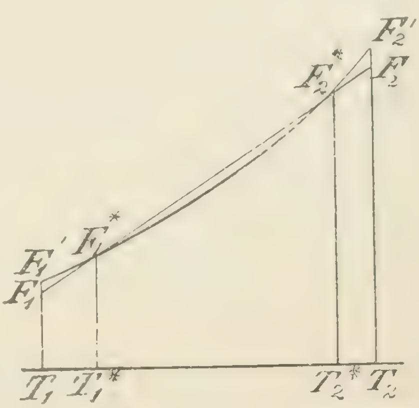

FIG. 38.

*Hann, *Zeit. Oesterr. Gesell. Met.*, 1874, vol. IX, p. 324. [Smithson Rep., 1877, p. 399.]

The investigation of the cases included in 2 can always be referred to case 3, since the points F_1^* and F_2^*, in which the straight line $F_1 F_2$ cuts the curve, $F_1' F_2'$ play precisely the same rôle in the second case as F_1 and F_2 in the third case.

If we consider more closely the propositions just enunciated, then we shall involuntarily be led to seek certain limiting values, the knowledge of which leads to the solution of the fundamental question whether, under given conditions, condensation will be possible or not.

The questions that obtrude in this connection are as follows:

(1) What limit must the relative humidity exceed for a given temperature of the components, or at least for one of them, in order that condensation may be possible for a properly chosen mixing-ratio?

(2) What limiting value must the relative humidity of one component exceed when the value of the other is given, and when also condensation is to become possible for a properly chosen mixing-ratio?

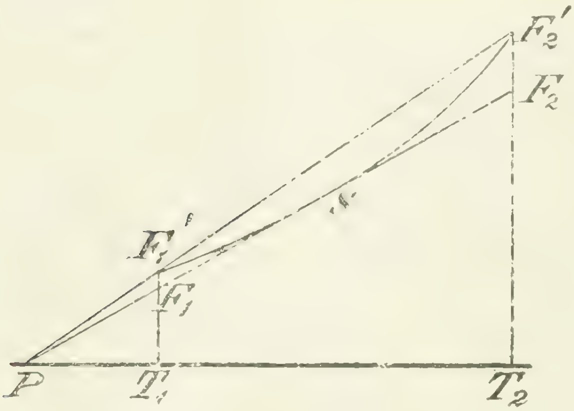

FIG. 39.

The first of these two questions can be expressed in the following form: When the limit of saturation is to be attained for an appropriate mixing ratio, and the relative humidity of both components is to be the same, what is the minimum value of this relative humidity?

That the knowledge of this minimum value is also a solution of question 1, we see most easily when we more accurately examine the answer to the question as last formulated.

We obtain this latter answer very easily through the following consideration: If R_1 is to equal R_2, then the straight line $F_1 F_2$ must cut the axis of abscissæ at the same point P Fig. 39, as does the prolongation of the chord $F_1' F_2'$. For if this condition is fulfilled then—

$$\frac{T_1 F_1}{T_1 F_1'} = \frac{T_2 F_2}{T_2 F_2'}$$

but now

$$\frac{T_1 F_1}{T_1 F_1'} = \frac{y_1}{y_1'} = \rho_1 = \frac{R_1}{100}$$

and

$$\frac{T_2 F_2}{T_2 F_2'} = \frac{y_2}{y_2'} = \rho_2 = \frac{R_2}{100}$$

and consequently, also

$$R_1 = R_2,$$

If now for a given value of $R_1 = R_2$, which may be called R_0, the point of saturation is to be just attained by proper mixing, then the straight line $P\ F_1\ F_2$ must just touch the saturation curve $F_1'\ F_2'$.

The point of tangency S gives therefore the temperature of the mixture for which saturation will be just attained, and hence also the mixing ratio.

But the value R_0, as the figure shows at the first glance, must be exceeded by at least one of the components when condensation is to become possible, and it therefore is precisely that limiting value that is desired in question No. (1).

It is easily seen that the knowledge of these boundary values is of high importance, it is therefore carefully considered in tables to be subsequently communicated. Equally simple is the solution of the second question, which, however, will here be considered only under the special assumptions that R_1 or R_2 is equal to 100.

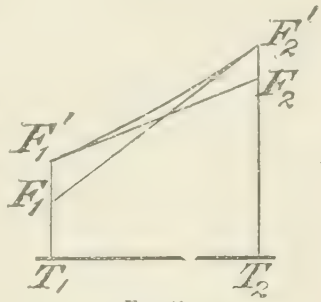

FIG. 40.

If $R_1 = 100$, that is to say, if the cooler of the two components is in the state of complete saturation, then we obtain the minimum value of R_2, when we, as in Fig. 40, draw at F_1' a tangent to the saturation curve, and prolong this until it cuts the ordinate $F_2'\ T_2$ at the point F_2. The desired value is $R_2 = 100\ \frac{F_2 T_2}{F_2' T_2}$. As soon as R_2 exceeds this limit condensation occurs on mixing, provided that there is sufficient of the colder component, that is to say, provided $\frac{m_1}{m_2}$ is large enough.

If, however, we consider the other case as given and assume that $R_2 = 100$, that is to say, that the warmer component is saturated, then we find R_1 when at F_2' we draw a tangent to the saturation curve and seek the intersection of it with the ordinate $F_1'\ T_1$.

Thus it becomes at once apparent to the eye that R_1 is always smaller than R_2, so that for sufficiently great distance between T_1 and T_2 the quantity R_1 can even attain a negative value, if such were imaginable.

The physical interpretation of this is that when warm saturated air is mixed with colder the latter can have a high degree of dryness and still condensation may occur for a proper mixing ratio; in many cases even the cooler air may be absolutely dry; it might even have a negative R_1 corresponding to its containing a certain mass of hygroscopic substance, if only there is sufficient quantity of warmer air, that is to say, if only $\frac{m_2}{m_1}$ is large enough.

In such cases, therefore, in place of the minimum value R_1 there occurs a limiting value of $\mu = \frac{m_2}{m_1}$, which must be exceeded if condensation is to occur.

These considerations show that mixtures of saturated warmer with unsaturated cooler air gives rise to condensations much more easily than do mixtures of saturated cooler with drier and warmer air.

The flow of a jet of saturated warmer air into a cool space must therefore be accompanied by much more powerful condensation than is the inflow of saturated colder air into a space filled with unsaturated warmer air.

The fact that clouds of vapor so easily arise over every open vessel filled with warm water, while the formation of fog near very cold bodies in warmer regions is much more rarely to be observed, gives an assurance of the correctness of this principle.

Whenever during moderately cool weather the door of a wash-house is opened great clouds of vapor pour out, but the opening of an ice cellar on a hot day has not a similar result.

Now that the limits have been determined within which, in general, mixture can occur, it is proper to give the quantity that can be precipitated by the condensation. Such precipitation occurs whenever the point F_3 lies above the saturation curve. For then the limit of saturation is exceeded, and by a quantity that is represented by the length $F_3 F_3' = y_3 - y_3'$.

This quantity, which will be designated by a_3, is that of which, before the writings of Wettstein and Hann, it was assumed that it was precipitated as water as the result of the mixing.

To what extent one was led into error by this assumption is most easily seen from the figure by the following considerations:

Let it be assumed that at first actual saturation occurs in the mixture, and let the whole quantity y_3 be actually present in the form of vapor or aqueous gas, then will the gradual precipitation of the vapor be accompanied by a simultaneous warming.

The increase of temperature hereby brought about is found from the equation

$$1000\, cdt = -rdy,$$

where c is the capacity for heat of the moist air under constant pressure, and r is the latent heat of evaporation, and where c is to be multiplied by 1,000, since we have taken a kilogram of the mixture, whereas y is expressed in grams.

Since now, as will subsequently become evident, the temperature t rises only a few degrees even for a very considerable supersaturation, therefore we can consider $\frac{c}{r}$ as constant in each individual case, and corresponding to this we obtain

$$y_3 - y = \frac{10^3 c}{r}(t - t_3) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which y and t represent those values that are obtained after the precipitation of the water that is in excess of the limit of saturation.

In Fig. 41, therefore, we find this temperature t in a very simple manner in that we draw through F_3 a straight line that makes with the axis of abscissas an angle

$$\alpha = \text{arc tang } \frac{10^3 c}{r}.$$

The point F, in which this straight line cuts the saturation curve, has the desired coördinates t and y, whereas the quantity of precipitated water $a = y_3 - y$ is a quantity that is represented in the figure by the short line $F_3 i$. According to the old theory t_3 and t, as well as y_3' and y', or, what is the same, y_3' and y, were considered respectively as the same. But now we see, as Hann had already shown in a special example, that this is not the case, but that $t > t_3$ and $y < y'_3$, and that correspondingly the actual quantity of water that can be precipitated in the most favorable case by mixing is

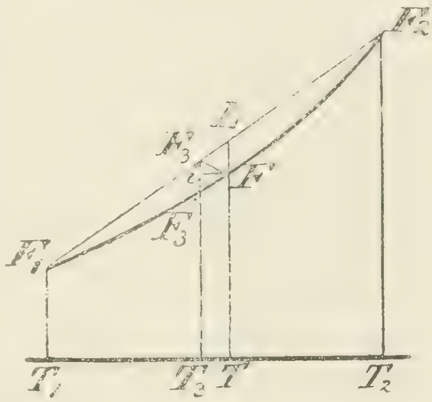

FIG. 41.

$$a = y_3 - y < a_3$$

that is to say less than any one has hitherto computed.

Since now $y = f(t)$ we can also put equation (3) in the form

$$t = t_3 + K(y_3 - f(t)),$$

where

$$K = \frac{r}{1000c} = \cot \alpha$$

and an empirical expression is to be substituted for $f(t)$.

This latter can, with the accuracy here desired, always be written under the form

$$f(t) = y_3 + A\,(t - t_3) + B\,(t - t_3)^2$$

so that we have only to consider the solution of an equation of the second degree.

However, the computation would be rather tedious and it is therefore decidedly preferable to execute this solution graphically, since this can be done rapidly and easily and preserves all the accuracy practically needed.

Of special importance is the circumstance that K can, in general, be considered as a constant, to which only two different values have to be given, acording as it relates to values above or below 0°.

Hitherto we have implicitly assumed that the temperatures lay above 0°; if this is not the case, then, in place of $\frac{r}{c}$, the value $\frac{r+l}{c}$ is to be sub-

stituted where l is the latent heat of melting ice. But so long as the melting point of ice is not exceeded, we can safely consider K as constant, as the following consideration shows. The following equation,* the extremely simple deduction of which may here be omitted, gives the value of c:

$$c = 0.2375 + 0.00024\ y.$$

Now a glance at the table given in the appendix shows that c will not exceed the value 0.2447, such as corresponds to a temperature 32° C. under 760 milimetres pressure. Since however on the other hand, for temperatures between 0° and 32° according to Regnault's figures,† r is confined between the limits 606.5 and 584.2, therefore the extreme values that $\frac{r}{1000c}$ can have for a pressure of 760 milimetres are 2.55 for $t = 0°$ and 2.39 for $t = 32°$.

For lower pressures (that is to say at greater altitudes), c is larger for a given temperature; but at the same time it is precisely under these conditions that only lower temperatures occur, and therefore only the higher values of r are to be considered, so that K still remains nearly within the same limits.

On account of the remarkably slight influence that the change of one unit in the first decimal place in the value of K has on the final result we can for brevity put $K = 2.5$, so long as $t > 0°$.

If $t < 0°$, then we have to add the quantity 80 [calories] to the value of r. If we consider this and then compute K for 0° and for $-30°$, first for $\beta = 760$ milimetres, and next for $\beta = 400$ milimetres we obtain as extreme values 2.87 and 2.98, so that here with even more right we can assume K to be constant and as we in fact will do equal to 2.9.

According to this, without important error, we may consider the lines $F_3 F$, in general, as parallel straight lines which experience only a slight bend at the point corresponding to 0°.

In the actual application of the above-explained graphic method we do best to place upon the system of coördinates, on which we have entered the saturation curve, a group of straight lines representing the series $F_3 F$, of which those on the left of the zero coördinate are inclined to the axis of abscissas so that $\tan \alpha = \frac{1}{2.9}$, but those to the right of the zero coördinate have $\tan \alpha = \frac{1}{2.5}$.

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* Hann, *Zeit. Oest. Gesell. Met.*, 1874, vol. IX, p. 324. [Smithson. Rep., 1877, p, 399.]

† According to the investigations of Dieterici (Wiedemann's *Annalen*, 1889, XXXVII, pp. 494–508), as well as according to those of Ekholm (*Bihang K. Svenska Vet. Akad. Handl.*, 1889, XV, Part I, No. 6.) these numbers are indeed not quite free from criticism. Since however on the one hand, the correction of these numbers scarcely comes into consideration in the final result here desired, and since on the other hand the value of the capacity for heat of dry air here adopted is based on the calorie used by Regnault, it appeared to me proper, if not even necessary, also to make use of the older value for r.

Special interest attends the question: In what ratio two quantities of air of given temperature and humidity must be mixed in order to obtain the greatest possible precipitation? The solution of this problem is given by a glance at Fig. 41. Since the quantity of precipitation is

$$a = F_3 F \sin \alpha,$$

therefore a will be a maximum when $F_3 F$ has its greatest value. But this is evidently the case when the tangent at the point F on the curve is parallel to the straight line $F_1 F_2$, or $F_1' F_2'$.

The point at which this tangent touches the curve can be determined either by construction and trial or, in case we have at hand a table of quantities of saturation, such as that in the appendix, computed for the barometric pressure in question, we have then to seek from it a value of t such that

$$\frac{dy}{dt} = \frac{y_2 - y_1}{t_2 - t_1}$$

which is not difficult to do after constructing a corresponding supplementary table of differences for each tenth of a degree.

Having found the point F we move further parallel to the previously mentioned group of straight lines until we strike the line $F_1 F_2$, and thus determine the point F_3, which on its part gives the point T_3, and thus the distances $T_1 T_3$ and $T_3 T_2$, whence results the mixing ratio that corresponds to the maximum precipitation. The precipitation itself we obtain from the above-given formula,

$$a = y_3 - y.$$

But we can also adopt another and purely numerical method for obtaining these quantities. For it is not difficult to see that FL (Fig. 41) is also a maximum at the same time with $F_3 F$, where we designate by L the point in which the prolongation of the ordinate FT intersects the straight line $F_1 F_2$.

Moreover when we represent the line FL by l, we have

$$\begin{aligned} l &= y_1 + (t - t_1) \tan \beta - y \\ &= t \tan \beta - y + y_1 - t_1 \tan \beta, \end{aligned}$$

where β represents the angle that the line $F_1 F_2$ makes with the axis of abscissas, that is to say,

$$\tan \beta = \frac{y_2 - y_1}{t_2 - t_1}$$

Since the value of y is not difficult to compute, when not taken directly from the table, one is therefore in condition to form a small auxiliary table for the value of the quantity l for certain values of t, such as lie in the neighborhood of the one desired, and from it take out

the maximum value of l or the value of t corresponding thereto. Then the value of a is given by the formula

$$a=l\frac{\tan\alpha}{\tan\alpha+\tan\beta}$$

whose deduction may here be omitted.

Thus both a numerical and a graphic method are at our disposal. If we follow the former, we can easily perceive that an extremely accurate knowledge of the quantity of vapor contained in a kilogram when in the condition of saturation is presupposed for an even moderately accurate determination of the value of a and t, as well as of the ratio $\frac{m_1}{m_2}$.

Because of the unreliability of the data at hand the values obtained by computation have in themselves a rather high degree of uncertainty, so that one can equally well make use of the far more convenient graphic method without thereby in fact losing any thing in accuracy.

In this latter way the following small tables have been computed, which give the limiting cases above treated as especially interesting for the pressures 700 and 400 *mm.* and for temperatures that proceed by steps of 10° and thereby makes possible a quick review of the various questions relative to mixtures of air.

The first horizontal line of each of these twelve tables relates to the case where both component masses are completely saturated, and gives in the column a the greatest precipitation that can occur * under these circumstances and under the most favorable mixing ratio $\frac{m_1}{m_2}$.

Therefore the a on the first line of each table, gives the maximum possible precipitation that can be brought about by mixture at the given temperatures.

The second line of each table gives the value of the relative humidity which must (at least for one of the components) be exceeded if precipitation by mixture is to be any way possible. We also find on this line under the headings t and $\frac{m_1}{m_2}$ the mixing temperature and the mixing ratio for which the point of saturation will be just attained when in both components the relative humidity has the minimum values, given under R_1 and R_2.

The third line shows the value of R_2 that must be exceeded by the relative humidity of the warmer component, if the cooler component is completely saturated and if precipitation is to become possible by mixture.

The fourth line gives the mixing ratio which must be exceeded if precipitation is to become possible by means of any proper mixing

* [Expressed in grams of water per kilogram of moist air.]

ratio when the cooler component is perfectly dry and the warmer component perfectly saturated.

The fifth line shows, under a, the maximum precipitation that is conceivable under the last mentioned condition of the components as to humidity as well as the mixing ratio and mixing temperature at which this maximum precipitation is attainable.

In many cases no precipitation is possible with perfect dryness of the cooler component. In such cases the fourth line is the analogue of the third since it gives the minimum value which the relative humidity of the colder component must exceed if in general precipitation is to become possible by mixture. Under these conditions in the nature of the case the fifth line becomes a blank.

The tables as here given relate only to the two pressures 700 and 400 millimetres. Since however these include all altitudes between 680 and 5,150 metres, that is to say, those altitudes in which the formation of cloud or at least precipitation proper principally occurs, and since the supplementing of these tables by means of the table given in the appendix is not difficult, I have thought that I might confine myself to these special cases.

At any rate these will suffice in order to give a general orientation as to the quantities coming into consideration, and therefore the tables themselves are now given, and it need only be stated that the figures must be considered only as approximations, since in general they are based upon the first differential quotients, but occasionally on the second differential quotients of the curve of vapor-pressure, so that very small changes in the experimental data or in the method of interpolation must make themselves very sensible.

t_1	t_2	R_1	R_2	a	t	$m_1 : m_2$
		$b=700^{mm}$; $t_2-t_1=20°$.				
		100	100	0.4	−9.0	102 : 98
		76	76	1/∞	−14.0	140 : 60
−20°	0°	100	52	1/∞	−20.0	1 : 0
		0	100	>0.0	>−11.8	<118 : 82
		0	100	<0.13	$\gtrless$−5.5	$\gtrless$60 : 140
		100	100	0.55	1.0	106 : 94
		81	81	1 ∞	−2.8	128 : 72
−10	+10	100	61	1/∞	−10.0	1 : 0
		0	100	>0.0	>−0.1	<1 : 1
		0	100	<0.2	$\gtrless$0.5	$\gtrless$54 : 146
		100	100	0.75	11.9	108 : 92
		86	86	1/∞	6.2	138 : 62
0	+20	100	62	1/∞	0.0	1 : 0
		0	100	>0.0	>12.2	<80 : 120
		0	100	<0.2	$\gtrless$16.7	$\gtrless$37 : 163

t_1	t_2	R_1	R_2	a	t	$m_1 : m_2$
			b=700mm; t_2-t_1=10°.			
		100	100	0.04	−15.5	57 : 43
		92	92	$1/\infty$	−16.0	60 : 40
−20	−10	100	82	$1/\infty$	−20.0	1 : 0
		55	100	$1/\infty$	−10.0	0 : 1
		100	100	0.11	−4.0	43 : 57
		94	94	$1/\infty$	−5.5	55 : 45
−10	0	100	85	$1/\infty$	−10.0	1 : 0
		47	100	$1/\infty$	0.0	0 : 1
		100	100	0.19	5.0	54 : 46
		94	94	$1/\infty$	4.5	55 : 45
0	+10	100	87	$1/\infty$	0.0	1 : 0
		64	100	$1/\infty$	10.0	0 : 1
		100	100	0.21	14.5	55 : 45
		94	94	$1/\infty$	14.0	60 : 40
+10	+20	100	87	$1/\infty$	10.0	1 : 0
		76	100	$1/\infty$	20.0	0 : 1
			b=400mm; t_2-t_1=20°.			
		100	100	0.50	−9.5	108 : 92
		76	76	$1/\infty$	−14.0	140 : 60
−20°	0°	100	58	$1/\infty$	−20.0	1 : 0
		0	100	$>1/\infty$	>-11.8	$<118 : 82$
		0	100	<0.2	$\gtrless -5.4$	$\gtrless 54 : 146$
		100	100	0.75	1.2	110 : 90
		80	80	$1/\infty$	−3.3	133 : 67
−10	+10	100	65	$1/\infty$	−10.0	1 : 0
		0	100	$>1/\infty$	>0.3	$<97 : 103$
		0	100	<0.2	$\gtrless 6.0$	$\gtrless 45 : 155$
			b=400mm; t_2-t_1=10°.			
		100	100	0.12	−15.5	58 : 42
		96	96	$1/\infty$	−16.0	60 : 40
−20	−10	100	85	$1/\infty$	−20.0	1 : 0
		48	100	$1/\infty$	−10.0	0 : 1
		100	100	0.17	−4.5	50 : 50
		94	94	$1/\infty$	−5.5	55 : 45
−10	0	100	88	$1/\infty$	−10.0	1 : 0
		52	100	$1/\infty$	−0.0	0 : 1
		100	100	0.20	6.0	47 : 53
		93	93	$1/\infty$	5.0	50 : 50
0	10	100	86	$1/\infty$	0.0	1 : 0
		65	100	$1/\infty$	10.0	0 : 1

In agreement with the previous results by Hann and Pernter, these tables show how small is the precipitation attainable by mixture when we consider components whose differences of temperature are even greater than ever occurs in nature.

Since on the other hand, according to the data recently collected by Hann,* quantities of water considerably greater than these can remain suspended in the air (as mist, fog, and cloud), therefore we see very plainly that, while the formation of cloud can be caused by mixture, yet the precipitation of rain or snow in any appreciable quantity can scarcely be brought about in this way.

At the same time the following diagram, which we here make use of for graphic computation, enables, in the most simple manner, to compare the quantity of precipitation formed by mixture with that which is produced by direct cooling as well as that produced by adiabatic expansion.

If we assume that by mixture under a favorable mixing ratio of saturated air at the temperature t_2 with other saturated air at the temperature t_1, the quantity of water a is precipitated (see Fig. 42), then we obtain the same quantity of precipitation when we directly cool the component y_2, from its temperature t_2 to a new temperature t_d, for which we have $y'_d = y'_2 - a$, but y'_d is the ordinate whose foot is T_d in Fig. 42.

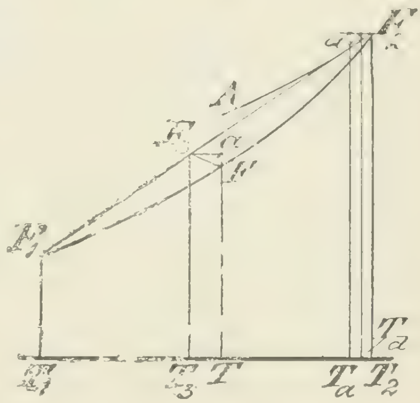

FIG. 42.

A glance at the general saturation curve suffices to show at once that the difference $t_2 - t_d$ is very much smaller than the difference $t_2 - t$; that is to say, that a very slight direct cooling affords as much precipitation as a considerable cooling by mixture with colder air, even when the latter is completely saturated.

The effect of adiabatic cooling is seen when in the diagram we draw the adiabatic curve as a function of the temperature and quantity of water contained in a kilogram of moist air.

Such an adiabatic curve sinks, as we easily perceive, rather more slowly from the right toward the left than the saturation curve. For since in this case the diminution of temperature goes hand in hand with the increase in volume, therefore, the quantity of moisture necessary for saturation will for falling temperatures be greater than it would be if the initial pressure were maintained; that is to say, than it would be by progressing along the saturation curve.

The adiabatic (which without any difficulty can be introduced into the diagram with sufficient accuracy with the aid of Hertz's Graphic Method*) will therefore have a path similar to that shown by the curve $F_2\,A$ in Fig. 42.

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* *Meteorologische Zeitschrift*, 1889, vol. VI, pp. 303–306.

* *Meteorologische Zeitschrift*, 1884, vol. I, pl. VII. [See No. XIV of this collection of Translations.]

But in this case the lowering of the temperature must be forced down to t_a, if the quantity of precipitated water is to be equal to a, since then the equation

$$y'_2 - y'_1 = a$$

holds good for y'_a, which represents the ordinate erected at T_a.

Here also the general course of the curve again shows that the fall of temperature necessary in order that a definite quantity of precipitation may be caused by adiabatic expansion is very much less than when the same quantity is to be produced by mixture.

A numerical example will best illustrate this principle: From the above-given small tables we see that at 700 millimetres pressure saturated air at 0° C. mixed with saturated air at 20° can precipitate at the most only 0.75 grams of water per kilogram of the mixture and that the final temperature of the mixture will be 11°.0; that is to say, for a cooling of the warmer component from 20° down to 11°.

By direct cooling, on the other hand, the same quantity of water would be precipitated from 1 kilogram of the warmer component when it is cooled from 20° down to 19°.2; whereas by adiabatic expansion a cooling of from 20° down to 18°.4 would be necessary; that is to say, a vertical ascent through a distance of about 310 metres.

This example shows in a very striking manner how slight need be the direct cooling by contact with cold objects, or by radiation, or even by adiabatic expansion, in order to produce quantities of precipitation, such as would by mixture be only obtainable in the extremest, scarcely imaginable cases.

With this the consideration of the mixture of masses of moist air may be brought to a close and only the single remark be made that the difference $t - t_3$ is smaller as the quantity a of the precipitated liquid decreases. The amount of this difference will therefore only exceed the value of 1° or 2° in such extreme cases as are assumed in the previous tables and generally will remain far within this limit.

Therefore in the majority of cases the mixing temperature may, without important error, be put equal to that which we obtain by mixing equal masses of dry air, whereby many computations experience a great simplification.

(b.) SUPER-SATURATED AIR.

In the foregoing solution of the problem of mixture it was assumed for the sake of simplicity, that in the cases where the formation of precipitation in this manner is really possible, super-saturation must first occur, and then precipitation follows.

This assumption was implied by Hann in his above-mentioned memoir* at a time when we still knew nothing as to whether aqueous vapor could actually exist in a supersaturated condition.

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* *Zeitschrift Oest. Gesell. Met.*, 1874, vol. IX. [Smithson. Rep., 1877, p. 397.]

But since the possibility of this has been demonstrated by the investigations of Aitken, Coulier, Mascart, Kiessling, and especially by Robert von Helmholtz,* it has some interest for us to make the precipitation from supersaturated air the object of a special investigation.

This precipitation, as is well known, occurs when super-saturated air (which can only exist when perfectly free from dust) is suddenly mixed with very fine particles of solid bodies, or possibly, also, when electric discharges take place through such supersaturated air.† We obtain directly from the above-given rules the amount of the precipitation, as also the rise in temperature.

We have only to omit the parts designated by the indices 1 and 2 in Fig 41, and to consider the condition indicated by the subscript index 3 as the starting point, then the ordinate $T_3\,F_3 = y_3$ gives the quantity of water in the state of supersaturation, while y again indicates as before the final remaining moisture; $y_3 - y$ indicates the quantity of moisture precipitated and $t - t_3$ the consequent rise of temperature.

This is, therefore, a method of formation of precipitation, in which one can actually speak of a liberation of latent heat (the latent heat of evaporation), as was formerly done in explaining the formation of precipitation in general.

In a certain sense this usage is allowable, even in the formation of precipitation by mixture, in so far as the temperature of the mixture comes out higher when water is precipitated than when this, under otherwise similar conditions, is not the case because of the insufficient quantity of water. This rise of temperature is however always a very unimportant one in consideration of the small quantity that can be condensed by mixture.

It is otherwise when true super-saturation is present. In such cases the rise of temperature can, according to the degree of super-saturation, be very considerable, as is easily seen from Fig. 41.

Still more considerable must the precipitation be that is caused by the sudden cessation of the super-saturation, namely: So soon as a sudden development of heat occurs at any one place in the atmosphere there follows a powerful ascent of the air which then, by adiabatic cooling, must always produce new formation of precipitation.

Under those conditions, when the vertical distribution of temperature approximates even distantly to that of convective equilibrium, then the sudden cessation of the condition of super-saturation causes this equilibrium to become unstable, and thus this cessation then affords the key to the explanation of a series of phenomena.

I consider it probable that it is in such processes, which indeed deserve a thorough investigation, that we have to seek the reason for the "cloud-bursts" properly so called. Of course, to establish this

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* Wiedemann's *Annalen*, 1886, XXVII, p. 527.

† R. von Helmholtz, Wiedemann's *Annalen*, 1887, XXXII, p. 4.

view the proof must first be given that the super-saturation, which we have hitherto only known in laboratory experiments, also occurs in the free atmosphere.

The mixture of super saturated air with other quantities of air scarcely needs a special consideration, since we at once see the result of such mixture when we imagine, in Fig. 41, one of the points, F_1 or F_2, transposed to the upper side of the curve $F' F'$, and then execute the further constructions according to the rules previously given.

(c.) MOIST AIR WITH INTERMIXED WATER OR ICE.

Water occurs in the atmosphere not only as vapor, but also in the form of drops of rain, crystals of ice, and particles of fog. The psychrometer and hygrometer teach us that the air is not necessarily saturated with vapor when water is mixed with it in this manner. Unfortunately we possess only very imperfect data as to how great a quantity of water can in this way be mechanically mixed with the atmosphere.* But there can be scarcely any doubt that the sum of the water mechanically mixed and that which is present in the form of vapor may together be smaller, or equal to, or even greater than the quantity corresponding to saturation for the given temperature. Corresponding to this statement, I will designate such mixtures as air which is "partly saturated mechanically," "wholly saturated mechanically," or "super-saturated mechanically." And now, first of all, we will investigate how such masses of air behave when mixed with ordinary air more or less moist.

By this investigation we shall come to learn the conditions under which the dissolution of fog or clouds or the evaporation of falling raindrops may occur. Such dissolution is, as we at once see, to be attained by mixture only when the intermixed air, which at first may be assumed to be the warmer component, is relatively dry.

Therefore, we will at first investigate the phenomena of mixture under the following conditions:

Let $R_1 > 100$ and composed of two parts, of which the one $\bar{R}_1$ is in the form of vapor and the other $\bar{\bar{R}}_1$ is liquid, and moreover let $\bar{R}_1 < 100$ while

$$\bar{R}_1 + \bar{\bar{R}}_1 = R_1.$$

Furthermore let $R_2 < 100$ and $t_2 > t_1$. This being assumed, the following formulæ hold good, using a notation which by analogy is intelligible of itself:

$$\bar{y}_1 + \bar{\bar{y}}_1 = y_1$$
$$y_1 > y_1'$$
$$\bar{y}_1 < y_1'.$$

*Hann, *Met. Zeit.* 1889, vol. VI, pp. 303–306.

In the accompanying Fig. 43, $\bar{y}_1$ is represented by $T_1\ \bar{F}_1$, and $\bar{\bar{y}}_1$ is represented by $\bar{F}_1\ F_1$, but the remaining lettering certainly needs no further explanation.

However, one thing may be especially noted, that the lines which represent the liquid or frozen * water are limited by two arrow points directed away from each other, since this facilitates a quick comprehension.

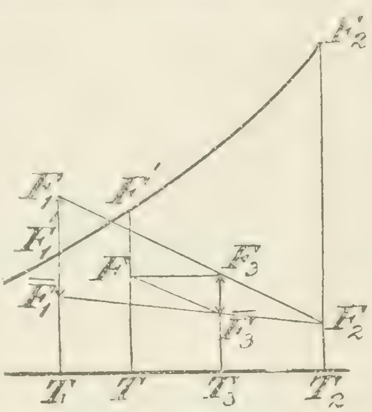

Fig. 43.

If now m_1 and m_2 are the quantities of the two components that enter into the mixture and we assume here also again that at first both the vapor and also the water are uniformly distributed in the mixture, and that evaporation of the water first occurs afterwards insofar as the saturation of the mixture with vapor allows of any such evaporation, then, just as before, we attain to a state of transition for which the corresponding quantities are appropriately indicated by the subscript 3.

The difference between this transition state and the air that is saturated by mixture consists in this, that in the present case the air actually passes through the transition state, whereas in the preceding article it was only imagined for convenience of computation.

In this transition state the quantities $\bar{y}_3$ of vapor and $\bar{\bar{y}}_3$ of water exist in one kilogram of the mixture before the dissolution occurs as is given by the equations

$$\frac{\bar{y}_3-\bar{y}_1}{y_2-\bar{y}_3}=\frac{t_3-t_1}{t_2-t_3}=\frac{m_2}{m_1}$$

and

$$\frac{\bar{\bar{y}}_1-\bar{\bar{y}}_3}{\bar{\bar{y}}_3}=\frac{t_3-t_1}{t_2-t_3}=\frac{m_2}{m_1}$$

which equations lose their apparent want of symmetry when we remember that $\bar{y}_2=y_2$ and that $\bar{\bar{y}}_2=0$.

Moreover, just as before, we have

$$\frac{y_1-y_3}{y_3-y_2}=\frac{t_3-t_1}{t_2-t_3}=\frac{m_2}{m_1}$$

In Fig. 43, $\bar{y}_3$ is represented by the line $T_3\ \bar{F}_3$ and $\bar{\bar{y}}_3$ by the line $\bar{F}_3\ F_3$.

* In general I assume in what follows that the temperature is above zero, since it is not difficult to modify the considerations appropriately for lower temperatures. But if we would also consider those cases in which water and ice exist alongside of each other or where water is present at temperatures below zero, then the investigation would become inordinately complicated.

The quantity $\bar{\bar{y}}_3$ that still remains liquid will now dissolve, in so far as the mixture is not saturated, or so much of it will dissolve as is needed for saturation. This of course can only occur in that the mixture itself cools, and that too by 2°.6 or 2°.9 for each gram that is evaporated, since we exclude the cases where heat is communicated from without.

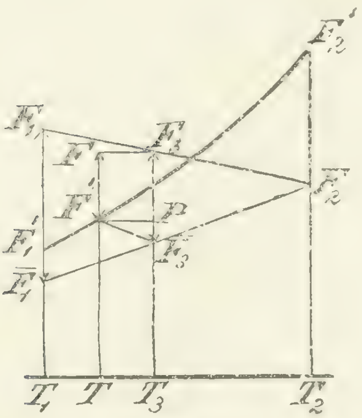

FIG. 44.

The final temperature T is therefore found by passing upward in Fig. 43 from $\bar{F}_3$ toward the left, parallel to the guide-line F_1 F_2, until at F we reach the same height as F_3, or at least until we reach the curve F'_1 F'_2, in case the line $\bar{F}_3$ F' so drawn would need to cross over the curve F'_1 F'_2.

In this latter case, which is represented in Fig. 44, all the water is not dissolved but only a portion $(y' - \bar{y}_3)$ as is represented in Fig. 44 by the distance $\bar{F}_3$ P.

The first of these two cases can be easily handled numerically, since under these conditions we have

$$t = t_3 - K\bar{\bar{y}}_3$$

$$= t_3 - K\bar{\bar{y}}_1 \frac{m_2}{m_1 + m_2}$$

$$= \frac{m_1 t_1 + m_2 t_2}{m_1 + m_2} - K\bar{\bar{y}}_1 \frac{m_2}{m_1 + m_2}$$

$$= \frac{m_1 t_1 + m_2 t_2 - K\bar{\bar{y}}_1 m_2}{m_1 + m_2}.$$

But the computation is as simple as this only when all the water is really evaporated; in the second case where mechanical supersaturation still continues it is better to apply the graphic method. An especial interest pertains here again to the investigation of the limiting cases for which in general there can occur a complete dissolution of the water originally present as liquid in one of the components. Of such extreme cases there is an extraordinary variety according as we are at liberty to assume arbitrarily either the mixing-ratio or the humidity of one or the other of the components.

At present we shall consider only the question, what is the initial limit of the mixing-ratio for given components in order that complete dissolution must always follow. This limit is evidently obtained when F' F_3 lie at the same altitude above the axis of abscissas, that is to say, when $y = y' = y_3$, or when F and F' coincide. In this case F is the

apex of a right-angled triangle whose vertical side is $F_3\,\bar{F}_3$ and whose hypothenuse is parallel to the guide-line.

If now we imagine the point T_3 moving to and fro along the axis of abscissas, then the apex of the triangle erected in the given manner upon the vertical side $F_3\,\bar{F}_3$ will describe a straight line passing through the point F_2, which line we easily find when we erect such a triangle on the portion cut off by the straight lines $F_1\,F_2$ and $\bar{F}_1\,F_2$ from any arbitrary ordinate and then join this apex with F_2.

We can, for instance, as in Fig. 45, choose for this purpose the ordinate erected at T_1.

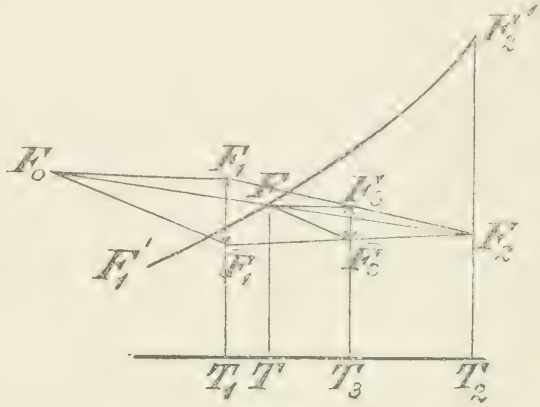

Fig. 45.

Then $\bar{F}_1 F_0 F_1$ is the triangle described and $F_0\ F_2$ is the straight line on which the desired point F must lie; but since it must also lie on the saturation curve, therefore it is at the intersection of F_0F_2 and the curve $F_1'F_2'$, and the desired limiting value of the mixing ratio is

$$\frac{T_2T_3}{T_1T_3}=\frac{m_1}{m_2}=\frac{1}{\mu}.$$

When the mixing ratio attains this limit or exceeds it on the side toward m_2, that is to say, as soon as $\frac{m_2}{m_1}=$ or $>\mu$ a complete dissolution of all the suspended water occurs.

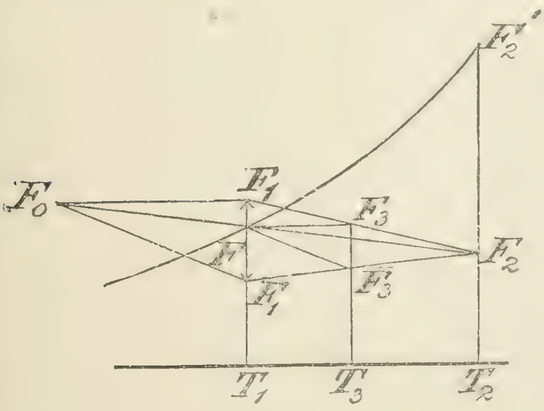

Fig. 46.

In such mixtures it can happen that the line $F\ F_3$ cuts the curve $F_1'\ F_2'$ on the left-hand side of $T_1\ F_1$. In such cases the temperature resulting from the completion of the mixture is lower than that of either component. The mixing ratio for which this phenomenon begins to occur is easily found by drawing, as in Fig. 46, through F (which is in this case identical with F_1') a line parallel to $F_0\ F_1$ (which is a guide line), and find its intersection with F_1F_2. The abscissa of this point is then the temperature t_3, which is produced by

mixture in this ratio before the subsequent dissolution. From this value of t_3 this mixing ratio itself can be determined.

We find by a very simple consideration that for this special value of t_3 the following equation holds good:

$$\frac{F_1\bar{F}_1}{F_3\bar{F}_3}=\frac{t_2-t_1}{t_2-t_3}$$

But since

$$F_3\bar{F}_3=\frac{t_3-t_1}{K}$$

and

$$F_1\bar{F}_1=y_1-\bar{y}_1$$

therefore

$$\frac{y_1-\bar{y}_1}{t_3-t_1}K=\frac{t_2-t_1}{t_2-t_3}$$

and consequently

$$\frac{y_1-\bar{y}_1}{t_2-t_1}K=\frac{t_3-t_1}{t_2-t_3}=\frac{m_2}{m_1}\mu_s.$$

Whenever $\mu<\mu_s$, that is to say, when the cooler "mechanically supersaturated air," or at least the saturated component enters into the mixture with greater weight, then will $t<t_1$ and of course $t<t_2$, or in other words, the finally resulting temperature will be lower than that of either component.

These considerations lead to the following apparently very paradoxical result: "*If warmer air is mixed with mechanically saturated or mechanically super-saturated air then a part of the suspended water can be evaporated and thereby a lower temperature produced.*"

"*If the given mechanically saturated air is hygroscopically unsaturated, that is to say, if the vapor is unsaturated, then this lowering of temperature will occur even by the mixing of saturated warmer air (of course in the proper ratio); if the air is saturated as to its vapor and the corresponding mechanical mixture is present as pure super-saturation, then the warmer air must possess a certain degree of dryness that is not difficult to determine.*"

The latter of these two propositions is evident as soon as we allow $\bar{F}_1$ to coincide with F as in Fig. 46, and then with F_2 still on the ordinate $T_2F'_2$ push so far downwards that F_0F_2 shall come to lie below FF_2, a condition which, however, is only to be satisfied so long as T_2 does not have too high a value.

The very paradoxical sentences just set forth lose their extraordinary appearance as soon as we recognize that a mixture of water and unsaturated moist air is not in a condition of equilibrium, but that in such a mixture evaporation must always occur unless by some special process the condition is kept stationary. Such mixtures are exemplified in the fogs and clouds, and during rain. The behavior of such mixtures has been implicitly investigated in the previous paragraphs and now only a few words further need be said.

Perhaps one might consider it a theoretical error that this behavior was not from the very first made the object of investigation but that the mixture of such masses with other air was chosen as the starting point of this study. But on the one hand this was the way by which I was actually led to the whole subject, and on the other hand abbreviations and simplifications are hereby rendered possible that seem to me sufficiently important to justify retaining this arrangement of the subject-matter.

In order to study the behavior of such mixtures when left to themselves we have only to choose as a starting point the condition represented by the ordinate $T_3\ F_3$ in Fig. 44, there considered as a state of transition, and we arrive then, according to the same rules as above, to the final condition $T\ F$ and thus also to the final temperature T.

Hence we recognize immediately "*that mixtures of water and unsaturated air as soon as left to themselves must cool and so much the more the further the vapor is from the point of saturation and the more liquid water or ice is mixed with it.*"

These considerations explain a phenomenon that I had frequently observed, but concerning which I was until recently not certain whether it might not be of a purely subjective nature.

It had frequently happened to me on passing through strata of fog such as fill the mountain valleys in the mornings of calm and subsequently clear days, that the impression of more severe cold occurs precisely when we attain the upper limit of the fog as we ascend the valley. Again it frequently occurred that just before the sun dissipates the morning fog that collects in valleys or spreads over the lowlands, the sensation of especial cold is experienced.

Such impressions of the sensations can of course very easily lead to error; but according to what has been above said, it is probable from theoretical grounds also, that the temperature just below the upper boundary of a dissolving layer of fog is lower than that of the layers above and below it. For when the sun begins to do its work at the upper boundary of the fog there then occurs, first, relative dryness immediately above this, and this relative dryness will, according to the velocity with which evaporation of the fog particles takes place, partly by diffusion, partly by direct radiation, also propagate itself to a certain, although very moderate depth, in the layer of fog.

But thereby (at least in many cases) the evaporation will be more accelerated than would correspond to the increase of heat by direct radiation, that is to say, the temperature must sink.

This expectation, grounded partly on the impression of one's feeling, partly on theoretical physical considerations, has now, during the writing of this memoir, received a confirmation by actual measurements. For the communication of these measurements I have to thank Mr. Bartsch von Sigsfeld, who, in a balloon constructed at his own expense, has already undertaken many aerial voyages for scientific purposes

and has also carried out meteorological observations when ascending mountains.

I will here first quote the results that von Sigsfeld obtained during a balloon trip from Augsburg on October 26, 1889, on which occasion the remarkably perfected aspiration psychrometer of Assmann of the newest construction * was used for the determination of temperatures and moisture.

The start occured as above remarked on October 26, at about 15 minutes before 10 A. M. The landing took place about 3 P. M., near Plochingen, on the railroad line between Ulm and Stuttgart. The general character of the weather on this day can be summarized as follows: While an extended barometric maximum with a center closely clinging to the Scandinavian peninsula, covered all of northern and central Europe there prevailed over southwestern and southern Europe a region of depression that starting from the southwest extended over all lands bordering on the Mediterranean and sent individual outrunners into southern Germany. The weather was almost everywhere cloudy with moderate motion of the air from the east.

Above Southern Germany itself there floated a layer of dry upper haze, whose lower indefinite boundary lay at an altitude of about 600 metres above the sea, whereas the upper, very well defined and flat boundary was found at an altitude of 1,200 metres. Up to this latter altitude the wind blew with moderate strength from the northeast, but above this it blew strongly from the scuth-southeast. Unfortunately only a few observations could be made, since von Sigsfeld was to a very large extent occupied with the navigation of the balloon, but notwithstanding this some important data were secured which I here reproduce:

Local time.	Altitude above sea level.	Barometric pressure.	Temperature.		Relative humidity.	Remarks.
			Dry bulb.	Wet bulb.		
h. m.	*Metres*	*mm.*	° *C.*	° *C.*	*Per cent.*	
9.47	471	723.3	7.5	6.2	83	At the starting place.†
10.45	471	....	....	....	....	Start: The temperature had changed very little since preceding observation.
(?)	(?)	....	3.0	2.9	98	Close under the upper boundary of the fog [or high dry haze].
(?)	1202	660.0	....	....	....	Upper boundary of the haze.
11.15	1615	630.0	5.3	2.0	55	
12.20	1358	647.3	5.5	3.5	73	After this a further rise of the balloon up to about 2,900 metres altitude and a steady diminution of the temperature of the dry bulb to about 3° C.

* Assmann. Das Aspirationspsychrometer. *Zeitschrift f. Luftschiffahrt*, 1890, IX, pp. 1-9 and 30-38.
† The barometer of the Augsburg Meteorological Station is at the altitude 499.6 metres.

These numbers, few as they are, still show that the layer of fog just under its upper boundary was the coolest portion of the whole path traversed by the balloon, and that close above this boundary the temperature shows a rapid rise, but the relative humidity a rapid fall.

Von Sigsfeld had obtained similar results even earlier, namely, October 5, 1887, on the occasion of an ascent of the Faulhorn, in Aargau, for the purpose of taking photographs.

I give the appropriate data in the following table:

Time.	Altitude.	Barometer.	Dry bulb.	Wet bulb.	Relative humidity.	Remarks.
h. m.	*Metres.*	*mm.*	°	°	*Per cent.*	
8 0 a. m.	808	692.0	4.6	....	....	Oberstdorf.*
8 0 a. m	1,058	671.5	2.1	1.9	96	Starting point, Riezlern; fog.
8 30 a. m.	1,203	659.1	0.7	0.7	100	Upper limit of fog.
9 10 a. m.	1,487	636.7	6.2	3.5	64	
10 05 a. m.	....	609.1	12.0	8.0	....	
10 40 a. m.	2,029	596.0	8.5	4.0	48	Faulhorn.
2 15 p. m.	2,031	595 1	4.6	4.4	97	Faulhorn; fog rises; upper boundary of fog attains and surpasses the summit of the mountain.
2 30 p. m.	2,029	594.8	3.6	2.8	89	Faulhorn; fog sinks.
2 35 p. m.	....	594.8	5.2	4.2	86	
2 40 p. m.	....	594.8	3.2	3.2	100	
3 12 p. m	....	594.8	2.5	2.5	100	Fog sinks; upper boundary descends to the summit of the mountain.
3 50 p. m.	1,950	600.0	2.0	2.0	100	Descending; fog still continues.
4 10 p. m.	1,785	612.3	2.0	2.0	100	Fog.
4 20 p. m.	1,668	621.0	2.8	2.8	100	Fog.
4 25 p. m.	1,615	625.0	3.5	3.4	98	Above the lower limit of fog.
4 25 p. m	....	625.0	4.5	4.0	93	Below the lower limit of fog.
5 10 p. m.	1,078?	668.?	6.7	6.2	93	At Riezlern.†

* According to Trautwein ("Southern Bavaria, etc.," seventh edition, Augsburg, 1884,) the altitude above the sea of Oberstdorf, which is that here adopted, is 808 metres; that of the summit of the Faulhorn is 2,033 metres, so that the value 2,031 metres is an excellent testimony to the reliability of the data.

† The morning observation gave the altitude of Riezlern as 1,058 metres, whereas the observation at 5 hours 10 minutes p. m. gave 1,078 metres, making use of the barometric pressure observed in the morning in Oberstdorf. But if we assume, as is required by the observations at Augsburg and Munich, that this reading had, during the intervening time, diminished by 1 millimetre, and furthermore adopt the very probable assumption that the aneroid could not perfectly follow the rapid changes of pressure during the descent, and therefore read about 2 millimetres too low, we shall obtain for Riezlern the altitude 1,062 metres above the sea, or a figure that agrees almost perfectly with that deduced from the morning observation.

The numbers above tabulated were given on the one hand with a well compared, quite reliable aneroid, and on the other so far as concerns the temperatures with an Assmann's aspiration psychrometer of the older construction.

From the above table we see very clearly that the upper boundary of the stratum of fog always shows a lower temperature than the neighboring strata above and below.

But whether this is as above assumed essentially the cold due to evaporation can not properly be decided. The high relative humidity

which was found even in the highest layer of fog raises some doubt in this direction. Some observations made by First Lieutenant Moedebeck and Lieutenant Gross on the occasion of a balloon voyage made on June 19, 1889, and which Lieutenant Gross has recently published * in a very interesting essay, apparently speak more clearly on this point, and certainly deserve a thorough scientific analysis. †

Here also the passage through thick clouds showed that the temperature at the upper boundary of these fell very low but immediately above this it rose at once suddenly. The observations of humidity also agree better with the theoretical views developed above. On this point Lieutenant Gross says with reference to a diagram given by him which shows the changes of the dry and wet thermometers, "We see from the comparison of the curves of the dry and wet thermometers that the moisture of the air rapidly increases with approach to the cloud, and that in the cloud itself where both curves coincide the air is completely saturated with aqueous vapor. But it is only in the lower part of the cloud that this is the case, and the moisture diminishes towards its upper part, an observation that I have already frequently made. This is certainly also explicable: In the upper part of the cloud the sun acts again as at first. Immediately above the cloud the wet thermometer makes a sudden rise. The air becomes suddenly very dry, as results without anything further, from the heat reflected back from the cloud."

That the lowest temperature should be observed immediately under the upper boundary of the cloud in spite of the influence of the sun seems to me explicable only by means of the cold due to evaporation in accordance with the manner above theoretically predicted.

One ought to be able to observe with all sharpness on the Eiffel tower the questions relating to the behavior of the upper surface of fog since it must frequently happen there that the boundary floats but a short distance above the meteorological instruments.

Perhaps also it will be possible there to establish at different heights self registering thermometers and psychrometers or hygrometers in order to obtain truly simultaneous observations immediately above and below the upper boundary of the fog (or mist).

(*d*.) THE FORMATION AND DISSOLUTION OF FOG AND CLOUD.

The preceding investigations into the formation of precipitation by mixture of quantities of air of unequal warmth and moisture show that

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* *Zeitschrift für Luftschiff fahrt*, 1889, VIII, p. 249.

† In referring to this essay I might also mention that Lieutenant Gross has also in the meantime confirmed the expectation expressed in my former communication [see pages 251–253] according to which the inversion of temperature in the region of the winter anti-cyclone is not a peculiarity of mountainous regions. On the occasion of a balloon voyage undertaken on December 19, 1878, from Berlin under the influence of such an anti-cyclone, the sling thermometer gave an increase of temperature of 8° in 1,000 metres of ascent between 1 P. M. and 4 P. M.

although such mixtures can not produce heavy rain or snow yet they can be of great importance in the formation of fog and cloud.

In accordance with this there are three processes that can, either by themselves alone or in conjunction, cause a condensation of the aqueous vapor in the atmosphere:

(*a*) Direct cooling, whether by contact with cold bodies or by radiation.

(*b*) Adiabatic expansion, or at least expansion with insufficient addition of heat.

(*c*) Mixture of masses of air of different temperatures.

In a corresponding manner the dissolution of fog and cloud already present may take place through the following processes:

(*a*) Direct warming, either by radiation or by contact with warmer bodies.

(*b*) Compression, whether adiabatic or at least with an insufficient abstraction of heat.

(*c*) Mixture with other masses of air having sufficient temperature and moisture.

Of these three different processes the one first mentioned is always the most effective.

In order to condense or dissolve a given quantity of water there need be only a relatively slight direct cooling or warming. When the condensation or dissolution of a certain quantity is to be accomplished by adiabatic expansion or compression the cooling or warming must be greater, that is to say, must cover a wider range of temperature than for direct cooling or warming.

Still larger temperature differences must come into play when the same quantity is to be condensed or evaporated by the process of mixture, in so far as this is any way possible.

The first pair of these processes, namely, the direct cooling or direct warming, comes especially into consideration in the formation of fog proper, which beginning at the earth's surface, extends upwards to greater or less altitudes. At times of excessive radiation the earth's surface first cools. When the cooling has reached the dew point there occurs condensation in the very lowest layer. Hereby the emissivity of this layer itself is increased. It then cools in its upper portion also by radiation, and thus the layer of fog grows upwards more and more until subsequently, at the time of increased inflow of heat, it dissolves itself in a precisely inverse manner.

No other considerable precipitation is formed by this method of condensation except the so-called drizzle. The reason of this undoubtedly is that the growth of the layer of fog upwards removes the possibility of further more intense radiation by the lower stratum. In the higher strata of the atmosphere such condensation by direct radiation can certainly only occur when cloudiness has already been produced in some other way, whether by mixture or by expansion or possibly by smoke.

At the upper limit of the cloud, especially in stratus clouds, the processes of growth and dissolution of the cloud by direct loss or gain of heat by radiation are carried on like the formation and dissolution of fog in the lowest strata of air.

The formation of clouds by adiabatic expansion as well as the dissolution by compression occurs wherever we have to do with ascending or descending currents of air. This process has in recent times been so frequently treated that the subject may here be treated very briefly. The cumulus clouds of summer with horizontal bases, the thunder cloud and the rain cloud, properly so called, owe their origin to this process. To what extent "nocturnal radiation" influences the upper layers of such clouds can only be made clear by further investigation.

Still more complicated than the two methods hitherto considered in the formation and dissolution of clouds and fog are the processes that accompany mixture. In both the above mentioned pairs of processes a steady increase of cooling or warming is accompanied by a steadily progressive condensation or dissolution. It is quite otherwise in mixtures. A process of mixture can progress in the same direction and yet cause at first condensation and in its subsequent stages dissolution. The breath which we exhale into the cool air leaves the mouth saturated but not yet in the condition of fog; only after the beginning of the mixing with the colder air does the formation of the cloud of vapor begin, which then through further mixture with colder, drier air, again dissolves. We see this process depicted in a strictly mathematical way in Fig. 38. If for instance we assume that a small quantity of air at the temperature t_1 is mixed with a larger quantity at the higher temperature t_2, then all possible mixing-ratios will occur from $\frac{m_2}{m_1}=0$ up to the final result, which we will assume to be greater than that which corresponds to the higher value y_2^*. In this case the quantity of contained water y passes through all values belonging to the ordinates of the line F_1F_2 until reaching the final value $y>y_2^*$. In this process condensation must occur as soon as the mixing-ratio exceeds the value which corresponds to the ordinate y_1^*; if it increases still further then beyond a definite point as it approaches towards the ordinate y_2^* dissolution again begins, which becomes complete for a mixing-ratio corresponding to the ordinate y_2^* and thus again results an unsaturated mixture.

If a smaller quantity of nearly saturated warmer air mixes with a larger quantity of colder air then will the mixture pass through its conditions in an inverse order, and again the initial condensation and the subsequent dissolution will occur under the conditions assumed in Fig. 38.

Although now in both cases condensation occurs first and then dissolution, still there is an important difference between them. For if we imagine the mixing-ratio to undergo steady change between the points

of condensation and of dissolution, that is to say, between the ordinates y_1^* and y_2^*, then will the resulting mean temperature $t = \frac{t_1^* + t_2^*}{2}$ be attained quicker when we go from y_1^* towards y_2^* than when we go from y_2^* towards y_1^*. For since $t > t_3$ therefore for $t = \frac{1}{2}(t_1^* + t_2^*)$ the mixing-ratio $\frac{m_1}{m_2} > 1$, that is to say, the mixture shows the average temperature, although so far as mass is concerned the colder component is in excess. According to this, if we mix saturated cooler air with steadily increasing quantities of saturated warmer air, then the warming of the mixture proceeds more rapidly at first than subsequently, whereas in the reverse process cooling proceeds more slowly at first and then steadily faster. The quantity condensed has also a similar relation; it also attains its maximum when there is an excess of the cooler component.

"*Therefore condensation begins sooner when a jet of cold moist air penetrates a large mass of warmer air than when a jet of warm moist air is blown into cooler air.*"

Therefore by the outward appearances of clouds that are forming and dissolving in this manner, one perceives whether warmer or colder air predominates.

From all the preceding we conclude that the following forms of fog and clouds may be considered as originating by mixture:

(1) The fog above warm moist surfaces, under the influence of colder air, therefore especially the fog over the sea in the cold season of the year or during the occurrence of cold winds.

(2) The "rank and file" clouds occurring on the boundary between two different strata of air flowing rapidly above each other, which von Helmholtz* has first recognized as a consequence of wave motion and designated by the name, atmospheric billows, in which however adiabatic condensation also comes into consideration at places where the air is thrown upward after the manner of the formation of crests and foam on ocean waves.

(3) The layers of stratus that also form at such separating surfaces and which frequently first appear as atmospheric billows and subsequently become denser.

(4) Cloud streamers that form and again dissolve at the summits of mountains or in narrow mountain passes when the form of the mountain is such as to make it possible for jets of warmer or colder masses of air to penetrate into similar masses of other temperatures.†

(5) The ragged clouds, or the disconnected clouds, such as one frequently observes during rapid motions of the air, perpetually changing

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* *Sitzungsberichte, König. Preus. Akad. Wissensch. zu Berlin:* Berlin, 1888, p. 661, and 1889, p. 503. [See also Nos. VI and VII of this collection.]

† Von Bezold, *Himmel und Erde*, 1889, vol. II, p. 7.

their form and appearing and disappearing, and such as also occur with clouds formed by adiabatic expansion, especially during thunder storms.

These different methods of cloud formation by direct cooling, by adiabatic expansion, and by mixture can of course also occur side by side in the most varied combinations, as is expressed in the extraordinary diversity of cloud forms.

It seems to me very important in the study of these forms to keep these different processes in view, since only then can we hope finally to attain a thorough knowledge of these forms.

Above all, as Hellmann has appropriately expressed it, it is necessary to lay the foundation for a "physiology of the clouds" before we can hope to attain to a truly satisfactory arrangement and nomenclature.*

But further work will still be necessary before this problem is solved, since on the one hand the question becomes more complicated the nearer we approach to it, and since on the other hand it appears so extraordinarily difficult to realize experimentally even approximately those conditions under which the formation and dissolution of clouds take place in the atmosphere.

Beautiful and praiseworthy as are the experiments that Vettin has made with clouds of smoke, still we must be very careful about the conclusions which we would draw from them as to the formation of the real clouds. All experiments with smoke, when looked at properly, give only pictures of the movements in dry air, since the condensation and evaporation as well as the processes of compression and expansion are excluded, and we therefore are working under conditions such that in the real atmosphere no formation of clouds would occur.

But it is precisely because of these processes (condensation, evaporation, compression, and expansion) that we can not consider the motion of a cloud as a measure of the motion of the air, for not only do clouds hang apparently motionless on the mountains, whereas in fact strong winds are streaming through them (*e. g.* Föhn cloud-bank, the Table-cloth of the Table mountain, the Cloud-cap of the Helm-wind) but it even happens to aëronauts that they pass through clouds while moving in a horizontal direction. This latter is however only possible when the cloud has a velocity different from that of the air in which it floats, since the balloon itself has only the power of vertical motion.

The cloud is in fact not a body that can be driven forward as such by the air unchanged, but is a form in a process of continuous formation and disappearance, and can have as a whole motions entirely different from those of the particles of which it consists.

On account of the increased interest with which at the present time we are studying the forms and motions of the clouds, it seemed to me important to call attention to all these points since we must have these in mind when we attempt from the external appearance of the clouds to draw any conclusion as to the processes which in individual cases determine their growth or dissolution and therefore also their form.

* Compare also O. Volger in *Gaea*, 1890, vol. II, pp. 65–75.

APPENDIX.

Table giving the quantity of water in grams that is contained as vapor in a kilogram of saturated air.

t	b=760^{mm}	b=700^{mm}	b=600^{mm}	b=500^{mm}	b=400^{mm}	b=300^{mm}	b=200^{mm}
−30	0.31	0.34	0.39	0.48	0.60	0.80	1.20
29	.34	.37	.43	.52	.65	.87	1.31
28	.38	.41	.48	57	.71	.95	1.43
−27	0.41	0.45	0.52	0.63	0.78	1.04	1.56
26	.45	.49	.57	.69	.86	1.14	1.71
25	.49	.54	.63	.75	.94	1.25	1.88
24	.54	.59	.69	.82	1.03	1.37	2.06
23	.59	.65	.75	.90	1.13	1.50	2.25
−22	0.65	0.71	0.82	0.99	1.23	1.63	2.46
21	.71	.77	.90	1.08	1.34	1.78	2.69
20	.77	.84	.98	1.18	1.46	1.94	2.94
19	.84	.92	1.07	1.28	1.60	2.12	3.21
18	.92	1.00	1.16	1.39	1.74	2.32	3.50
−17	1.00	1.09	1.26	1.52	1.90	2.53	3.81
16	1.09	1.18	1.37	1.65	2.07	2.75	4.14
15	1.19	1.28	1.49	1.79	2.24	2.99	4.49
14	1.28	1.39	1.62	1.94	2.43	3.24	4.87
13	1.39	1.51	1.76	2.11	2.64	3.52	5.28
−12	1.50	1.64	1.90	2.29	2.86	3.82	5.73
11	1.63	1.77	2.06	2.48	3.10	4.13	6.20
10	1.76	1.91	2.23	2.68	3.35	4.47	6.72
9	1.91	2.07	2.41	2.90	3.62	4.84	7.26
8	2.06	2.24	2.61	3.13	3.92	5.23	7.85
−7	2.23	2.42	2.82	3.38	4.24	5.65	8.49
6	2.40	2.61	3.04	3.65	4.58	6.10	9.16
5	2.59	2.81	3.28	3.94	4.94	6.58	9.88
4	2.79	3.03	3.54	4.25	5.32	7.09	10.66
3	3.01	3.27	3.81	4.58	5.72	7.64	11.49
−2	3.24	3.52	4.10	4.93	6.16	8.23	12.37
−1	3.48	3.78	4.42	5.30	6.63	8.85	13.32
0	3.75	4.07	4.75	5.71	7.13	9.52	14.33
+1	4.03	4.37	5.10	6.13	7.67	10.24	....
2	4.32	4.70	5.48	6.58	8.24	11.00	....
+3	4.64	5.04	5.88	7.07	8.85	11.81	....
4	4.98	5.41	6.31	7.58	9.49	12.68	....
5	5.34	5.80	6.77	8.13	10.18	13.60	....
6	5.71	6.22	7.26	8.72	10.91	....	....
7	6.13	6.66	7.77	9.34	11.69	....	....
+8	6.56	7.13	8.32	9.99	12.52	....	....
9	7.02	7.63	8.91	10.70	13.40	....	....
10	7.51	8.16	9.53	11.44	14.33	....	....
11	8.03	8.72	10.18	12.24	15.32	....	....
12	8.58	9.32	10.88	13.08	16.38	....	....
+13	9.16	9.95	11.62	13.97	17.50	....	....
14	9.78	10.62	12.41	14.91	18.[illegible]9	....	....
15	10.43	11.34	13.24	15.91	19.94	....	....
16	11.13	12.09	14.12	16.97	....	....	....
17	11.86	12.89	15.05	18.10	....	....	....

Table giving the quantity of water in grams that is contained as vapor in a kilogram of saturated air—Continued.

t	b=760mm	b=700mm	b=600mm	b=500mm	b=400mm	b=300mm	b=200mm
+18	12.64	13.73	16.04	19.29	....	....	....
19	13.46	14.62	17.09	20.55	....	....	....
20	14.33	15.57	18.20	21.88	....	....	....
21	15.25	16.57	19.37	....	....	....	....
22	16.22	17.63	20.59	....	....	....	....
+23	17.24	18.75	21.90	....	....	....	....
24	18.32	19.93	23.28	....	....	....	....
25	19.47	21.17	24.73	....	....	....	....
26	20.68	22.48	....	....	....	....	....
27	21.95	23.86	....	....	....	....	....
+28	23.29	25.31	....	....	....	....	....
29	24.70	26.84	....	....	....	....	....
30	26.18	28.47	....	....	....	....	....

In computing this table the vapor tensions of aqueous vapor have been adopted as given by Broch, Travaux et Memoirés, Bur. Internat. des Poids et Mesures, 1881, tome I.

XVIII.

ON THE VIBRATIONS OF AN ATMOSPHERE.*

By Lord Rayleigh.

In order to introduce greater precision into our ideas respecting the behavior of the earth's atmosphere, it seems advisable to solve any problems that may present themselves, even though the search for simplicity may lead us to stray rather far from the actual question. It is proposed here to consider the case of an atmosphere composed of gas which obeys Boyle's law, viz, such that the pressure is always proportional to the density. And in the first instance we shall neglect the curvature and rotation of the earth, supposing that the strata of equal density are parallel planes perpendicular to the direction in which gravity acts.

If p, σ be the equilibrium pressure and density at the height z, then

$$\frac{dp}{dz}=-\sigma g; \qquad (1)$$

and by Boyle's law,

$$p=a^2\sigma, \qquad (2)$$

where a is the velocity of sound. Hence

$$\frac{d\sigma}{\sigma dz}=-\frac{g}{a^2}, \qquad (3)$$

and

$$\sigma=\sigma_0 e^{-\frac{gz}{a^2}}, \qquad (4)$$

where σ_0 is the density at $z=0$. According to this law, as is well known, there is no limit to the height of the atmosphere.

Before proceeding further, let us pause for a moment to consider how the density at various heights would be affected by a small change of temperature, altering a for a', the whole quantity of air and there-

* From the *London, Edinburgh and Dublin Phil. Mag.*, Feb., 1890, fifth series, vol. XXIX, pp. 173–180.

fore the pressure p_0 at the surface remaining unchanged. If the dashes relate to the second state of things, we have

$$\sigma = \sigma_0 e^{\frac{-gz}{a^2}}, \quad \sigma' = \sigma'_0 e^{\frac{-gz}{a'^2}}.$$

$$p = p_0 e^{\frac{-gz}{a^2}}, \quad p' = p_0 e^{\frac{-gz}{a'^2}},$$

while

$$a^2 \sigma_0 = a'^2 \sigma'_0 .$$

If $a'^2 - a^2 = \delta a^2$, we may write approximately

$$\frac{p'-p}{p_0} = \frac{\delta a^2}{a^2} \frac{gz}{a^2} e^{\frac{-gz}{a^2}}.$$

The alteration of pressure vanishes when $z=0$, and also when $z=\infty$. The maximum occurs when $\frac{gz}{a^2}=1$, that is, when $p=\frac{p_0}{e}$. But $(p' - p_0)$ increases relatively to σ, continually with z.

Again, if ρ denote the proportional variation of density,

$$\rho = \frac{\sigma' - \sigma}{\sigma} = \frac{a^2}{a'^2}\left(e^{\frac{-gz}{a'^2} + \frac{gz}{a^2}} - 1 \right).$$

If $a'^2 > a^2$, ρ is negative when $z = 0$, and becomes $+\infty$ when $z = \infty$. The transition $\rho = 0$ occurs when $\frac{gz}{a^2} = 1$, that is, at the same place where $p' - p$ reaches a maximum.

In considering the small vibrations, the component velocities at any point are denoted by u, v, w, the original density σ becomes $(\sigma + \sigma\rho)$, and the increment of pressure is δp. On neglecting the squares of small quantities the equation of continuity is

$$\sigma \frac{d\rho}{dt} + \sigma \frac{du}{dx} + \sigma \frac{dv}{dy} + \sigma \frac{dw}{dz} + w \frac{d\sigma}{dz} = 0$$

or by (3)

$$\frac{d\rho}{dt} + \frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} - \frac{gw}{a^2} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The dynamical equations are

$$\frac{d\delta p}{dx} = -\sigma \frac{du}{dt}, \quad \frac{d\delta p}{dy} = -\sigma \frac{dv}{dt}, \quad \frac{d\delta p}{dz} = -g\sigma\rho - \sigma \frac{dw}{dt};$$

or by (3) since

$$\delta p = a^2 \sigma \rho,$$

$$a^2 \frac{d\rho}{dx} = -\frac{du}{dt}, \quad a^2 \frac{d\rho}{dy} = -\frac{dv}{dt}, \quad a^2 \frac{d\rho}{dz} = -\frac{dw}{dt} \quad . \quad . \quad . \quad . \quad (6)$$

We will consider first the case of one dimension, where u, v vanish, while ρ, w are functions of z and t only. From (5) and (6),

$$\frac{d\rho}{dt}+\frac{dw}{dz}-\frac{gw}{a^2}=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$a^2\frac{d\rho}{dz}=-\frac{dw}{dt}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

or by elimination of ρ,

$$\frac{1}{a^2}\cdot\frac{d^2w}{dt^2}=\frac{d^2w}{dz^2}-\frac{g}{a^2}\frac{dw}{dz} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The right-hand member of (9) may be written

$$\left(\frac{d}{dz}-\frac{g}{2a^2}\right)^2 w-\frac{g}{4a^4}w,$$

and in this the latter term may be neglected when the variation of w with respect to z is not too slow. If λ be of the nature of the wave-length, $\frac{dw}{dz}$ is comparable with $\frac{w}{\lambda}$; and the simplification is justifiable when a^2 is large in comparison with $g\lambda$, that is when the velocity of sound is great in comparison with that of gravity-waves (as upon water) of wave length λ. The equation then becomes

$$\frac{d^2w}{dt^2}=a^2\left(\frac{d}{dz}-\frac{g}{2a^2}\right)^2 w;$$

or, if

$$w=We^{\frac{gz}{2a^2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$\frac{d^2W}{dt^2}=a^2\frac{d^2W}{dz^2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

the ordinary equation of sound in a uniform medium. Waves of the kind contemplated are therefore propagated without change of type except for the effect of the exponential factor in (10), indicating the increase of motion as the waves pass upwards. This increase is necessary in order that the same amount of energy may be conveyed in spite of the growing attenuation of the medium. In fact $w^2\sigma$ must retain its value, as the waves pass on.

If w vary as e^{int}, the original equation (9) becomes

$$\frac{d^2w}{d^2z}-\frac{g}{a^2}\frac{dw}{dz}+\frac{n^2w}{a^2}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Let m_1, m_2 be the roots of

$$m^2-\frac{g}{a^2}m+\frac{n^2}{a^2}=0,$$

so that

$$m = \frac{g \pm \sqrt{(g^2 - 4n^2 a^2)}}{2a^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

then the solution of (12) is

$$w = Ae^{m_1 z} + Be^{m_2 z}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

A and B denoting arbitrary constants in which the factor e^{int} may be supposed to be included.

The case already considered corresponds to the neglect of g^2 in the radical of (13), so that

$$m = \frac{g \pm 2\, nai}{2\, a^2}$$

and

$$we^{-\frac{\frac{1}{2}gz}{a^2}} = Ae^{in\left(t + \frac{z}{a}\right)} + Be^{in\left(t - \frac{z}{a}\right)} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

A wave propagated upwards is thus

$$w = e^{\frac{\frac{1}{2}gz}{a^2}} \cos n\left(t - \frac{z}{a}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and there is nothing of the nature of reflection from the upper atmosphere.

A stationery wave would be of type

$$w = e^{\frac{\frac{1}{2}gz}{a^2}} \cos nt \sin \frac{nz}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

w being supposed to vanish with z. According to (17), the energy of vibration is the same in every wave length, not diminishing with elevation. The viscosity of the rarefied air in the upper regions would suffice to put a stop to such a motion, which can not therefore be taken to represent anything that could actually happen.

When $2\, na < g$, the values of m from (13) are real, and are both positive. We will suppose that m_1 is greater than m_2. If w vanish with z, we have from (14) as the expression of the stationary vibration

$$w = \cos nt \left(e^{m_1 z} - e^{m_2 z}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

which shows that w is of one sign throughout. Again by (8)

$$a^2 \rho = n \sin nt \left\{ \frac{e^{m_1 z}}{m_1} - \frac{e^{m_2 z}}{m_2} \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Hence $\frac{d\rho}{dz}$, proportional to w, is of one sign throughout; ρ itself is negative for small values of z, and positive for large values, vanishing once when

$$e^{(m_1 - m_2)z} = \frac{m_1}{m_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

When n is small we have approximately

$$\left.\begin{aligned} m_1 &= \frac{g}{a^2} - \frac{n^2}{g}, \\ m_2 &= \frac{n^2}{g} \end{aligned}\right\} \quad \dots\dots\dots \quad (21)$$

so that ρ vanishes when

$$e^{\frac{gz}{a^2}} = \frac{gz}{n^2 a^2} \quad \dots\dots\dots \quad (22)$$

or by (4) when

$$\frac{\sigma}{\sigma_0} = \frac{n^2 a^2}{g^2} \quad \dots\dots\dots \quad (23)$$

Below the point determined by (23) the variation of density is of one sign and above it of the contrary sign. The integrated variation of density, represented by $\int_0^\infty \sigma\,\rho\,dz$, vanishes, as of course it should do.

It may be of interest to give a numerical example of (23). Let us suppose that the period is one hour, so that in C. G. S. measure $n = \frac{2\pi}{3600}$. We take $a = 33 \times 10^4$, $g = 981$. Then

$$\frac{\sigma}{\sigma_0} = \frac{1}{290};$$

showing that even for this moderate period the change of sign does not occur until a high degree of rarefaction is reached.

In discarding the restriction to one dimension, we may suppose, without real loss of generality, that $v = 0$, and that u, w, ρ, are functions of x and z only. Further we may suppose that x occurs only in the factor e^{ikx}; that is, that the motion is periodical with respect to x in the wave-length $\frac{2\pi}{k}$; and that as before t occurs only in the factor e^{int}. Equations (5) and (6) then become

$$in\,\rho + iku + \frac{dw}{dz} - \frac{gw}{a^2} = 0 \quad \dots\dots \quad (24)$$

$$a^2 k\,\rho = -n\,u \quad \dots\dots\dots \quad (25)$$

$$a^2 \frac{d\rho}{dz} = -inw \quad \dots\dots\dots \quad (26)$$

from which if we eliminate u and w we get

$$\frac{d^2\rho}{dz^2} - \frac{g}{a^2} \cdot \frac{d\rho}{dz} + \left(\frac{n^2}{a^2} - k^2\right)\rho = 0 \quad \dots\dots \quad (27)$$

an equation which may be solved in the same form as (12).

One obvious solution of (27) is of importance. If $\frac{d\rho}{dz} = 0$, so that $w = 0$, the equations are satisfied by

$$n^2 = k^2 a^2 \quad \dots\dots\dots \quad (28)$$

Every horizontal stratum moves alike, and the *proportional* variation of density (ρ) is the same at all levels. The possibility of such a motion is evident beforehand, since on account of the assumption of Boyle's law the velocity of sound is the same throughout.

In the application to meteorology, the shortness of the more important periods of the vertical motion suggests that an "equilibrium theory" of this motion may be adequate. For vibrations like those of (28) there is no difficulty in taking account of the earth's curvature. For the motion is that of a simple spherical sheet of air, considered in my book upon the "Theory of Sound," § 333. If r be the radius of the earth, the equation determining the frequency of the vibration corresponding to the harmonic of order h is

$$n^2 r^2 = h\,(h+1)\,a^2 \qquad (29)$$

the actual frequency being $\frac{n}{2\pi}$. If τ be the period, we have

$$\tau = \frac{2\pi r}{a\sqrt{h\,(h+1)}} \qquad (30)$$

For $h=1$, corresponding to a swaying of the atmosphere from one side of the earth to the opposite,

$$\tau_1 = \frac{2\pi r}{a\sqrt{2}}, \qquad (31)$$

and in like manner for $h=2$.

$$\tau_2 = \frac{2\,\pi r}{\sqrt{6}} = \frac{\tau_1}{\sqrt{3}} \qquad (32)$$

To reduce these results to numbers we may take for the earth's quadrant

$$\frac{1}{2}\pi r = 10^6 \text{ centimeters};$$

and if we take for a the velocity of sound at 0° as ordinarily observed, or as calculated upon Laplace's theory, viz, $33 \times 10^3 \frac{\text{centimeters}}{\text{second}}$, we shall find

$$\tau_1 = \frac{4 \times 10^9}{\sqrt{2} \times 33 \times 10^3} \text{ seconds} = 23.8 \text{ hours}$$

on the same basis,

$$\tau_2 = 13.7 \text{ hours}.$$

It must however be remarked that the suitability of this value of a is very doubtful, and that the suppositions of the present paper are inconsistent with the use of Laplace's correction to Newton's theory of sound propagation. In a more elaborate treatment a difficult question would present itself as to whether the heat and cold developed during atmospheric vibrations could be supposed to remain undissipated. It

is evidently one thing to make this supposition for sonorous vibrations, and another for vibrations of about 24 hours period. If the dissipation were neither very rapid nor very slow in comparison with diurnal changes (and the latter alternative at least seems improbable), the vibrations would be subject to the damping action discussed by Stokes.*

In any case the near approach of τ_1 to 24 hours, and of τ_2 to 12 hours, may well be very important. Beforehand the diurnal variation of the barometer would have been expected to have been much more conspicuous than the semidiurnal. The relative magnitude of the latter, as observed at most parts of the earth's surface, is still a mystery, all the attempted explanations being illusory. It is difficult to see how the operative forces can be mainly semidiurnal in character: and if the effect is so, the readiest explanation would be in a near coincidence between the natural period and 12 hours. According to this view the semidiurnal barometric movement should be the same at the sea level all round the earth, varying (at the equinoxes) merely as the square of the cosine of the latitude, except in consequence of local disturbances due to want of uniformity in the condition of the earth's surface.

TERLING PLACE, WITHAM, *Dec.*, 1889.

***Phil. Mag.*, 1851 (4), vol. I, p. 305. Also, Rayleigh; "Theory of Sound," § 247.

XIX.

ON THE VIBRATIONS OF AN ATMOSPHERE PERIODICALLY HEATED.*

By Max Margules.

The computation of the variations of pressure in the atmosphere arising from periodic changes in temperature has a certain interest in connection with a problem of meteorology that, like all dynamic problems in this field, necessitates very extensive computations.

The daily variation of the barometer, freed from all non-periodic influences, can be represented very satisfactorily by the super-position of two waves, one of which has a whole day as its period; the other has the half day. The diurnal wave is undoubtedly an effect of the variation of temperature. It appears much stronger on clear days than on cloudy days; it is very slight at sea and shows on the land notable inequalities. The semi-diurnal wave is on the other hand of a regularity that is uncommon in meteorological phenomena. At places of the same latitude it is of very nearly equal amplitude and of the same phase in reference to the local time. If we consider this wave also as a consequence of the variations of temperature, then the connection seems to be obscure.

The mean daily temperature represented for any place by a curve, can like every such curve, be analyzed into a series of waves of twenty-four, twelve, eight, and six hour periods. Does the twenty-four-hour wave of pressure originate from the corresponding wave of temperature? Does the twelve-hour variation of pressure depend on the twelve-hour temperature variation? Why is the amplitude of the twelve-hour pressure term so large in comparison with the twenty-four-hour term, whereas the reverse is true for the temperature? Whence come the regularity of the one and the local variations of the other?

These questions have been asked repeatedly. In a memoir recently published,† Hann has given the most comprehensive and thorough description of the daily oscillation of the barometer, utilizing the rich

* Translated from the *Sitzungsberichte der Königlich Akademie der Wissenschaften zu Wien* (Math.), 1890, vol. XCIX, pp. 204–227. See, also, Exner's *Repertorium der Physik.* 1890, Band XXVI, pp. 613–633.

† "Unters. ü. d. tägliche Oscillation d. Barometers," *Vienna Denk.*, vol. 55, 1889.

observational material from all lands and oceans with the object of establishing a basis for a further mathematico-physical theory.

In order to attain this, one must first treat the phenomenon under assumptions that simplify the labor. I believed that some computations as to the variations of pressure in air that is periodically heated would contribute to the better understanding of the diurnal variation of the barometer. In the course of the work, it appeared that the computation must not be confined to the simplest cases if one would make it useful to a certain degree. For this reason the investigation has grown to a larger size than was desired by me.

Before giving the detailed computations let the review of certain results take precedence. Let T_0 and p_0 indicate the absolute temperature and the pressure of the air when at rest; $T_0\,(1+\tau)$ and $p_0\,(1+\varepsilon)$ indicate temperature and pressure of air in motion. When τ is given as a periodic function of the time t and of the locality x then ε will also appear as such a function.

Let a wave of temperature

$$\tau = A \sin 2\pi \left(\frac{t}{\Theta} + \frac{x}{L}\right)$$

with constant amplitude move in the direction $-x$ in a plane layer of air upon which no other forces are acting.

This will produce a wave of pressure

$$\varepsilon = A \frac{L^2}{L^2 - c^2\Theta^2} \sin 2\pi \left(\frac{t}{\Theta} + \frac{x}{L}\right)$$

where c indicates the velocity of propagation of a free vibration when the process is strictly isothermal: in air at the temperature 273° we have $c=280$ metres per second.

If we assume the length of the wave to equal the circumference of the equator then for a period whose duration is one day and for a pressure p_0 expressed as 760 millimetres of the barometer a variation of temperature of one degree will produce a variation of pressure of 4.4 millimetres.

Both temperature and pressure vibrations have the same phases when their velocity of propagation $\left(V = \frac{L}{\Theta}\right)$ is greater than c, but opposite phases when it is smaller than c. If $V=c$ then will ε be indefinitely large, as must occur in the case of a frictionless medium when the forced vibrations have the same period as the free. Again, let a wave of temperature similar to the preceding advance in a plane stratum of air, subject to the influence of constant gravity. The air now moves horizontally in the direction of the progress of the wave and also vertically. The pressure wave on the ground is given by an equation similar to the preceding only in the numerator $c^2\Theta^2$ is to be substituted for L^2. For the equator, the day and 760 millimetres, a

temperature variation of one degree gives a pressure variation of 1.3 millimetres.

If however the amplitude of the temperature variation is not uniform throughout the whole height, but diminishes with the height so that it diminishes by one-half for each ascent of 1,000 metres, then a temperature variation of ten degrees at the earth's surface gives a variation of pressure at the same level of only 2.4 millimetres.

In respect to the whole-day wave for continental tropical regions one could be satisfied with this result. The agreement, however, is only accidental. The twelve-hour wave of pressure at sea still remains entirely inexplicable. Even on the land one should expect that the amplitudes of the whole-day and half-day waves of pressure would have the same ratio as the corresponding temperature amplitudes, since ε remains unchanged when we put $\frac{1}{2}$ L and $\frac{1}{2}$ Θ in place of L and Θ.

The computation would hold good for a cylinder of great diameter equally as for a plane; even under certain restrictions it would also hold good for a mass of air within a circular boundary. But it can only be applied to the atmosphere when the air is divided into a number of zones by vertical walls parallel to the circles of latitude. The zones in the neighborhood of the latitude of 50° would have enormous variations of pressure, and there also two neighboring zones would have opposite phases; the amplitudes diminish thence toward both the pole and the equator.

From the great differences in pressure that are thus obtained for different zones, we see the necessity of reducing to calculation the condition of the air over the whole sphere without any partition walls. I pass over the formulæ for the sphere at rest in order to report upon that part of the computation that apparently offers useful results for the elucidation of the half-day wave of pressure. First, I will present some passages quoted already by Hann from a memoir of Sir William Thomson's.

After speaking of the disproportion between the whole and half day variation of the temperature on one hand and the pressure on the other, Thomson says:* "We must consider the atmosphere as a whole and investigate its vibrations with the help of the formulæ that Laplace has developed for the ocean in the *Mecanique Céleste*, and which, as he has shown, are also applicable to the atmosphere. When in the calculation of the tide-producing force, one introduces the influence of temperature instead of attraction, and develops the oscillations corresponding to the whole day and half day terms of the temperature curve, it will probably be found that in the first case the period of the free oscillations departs more from twenty-four hours than in the second case from twelve hours, wherefore for a relatively small amount of tide-producing force,

* "On the thermo-dynamic acceleration of the earth's rotation," *Proc. R. S. Edinburgh*, 1882, vol. XI. Sir William Thomson. "Mathematical and Physical Papers," London, 1890, vol, III, page 344.

the amplitude of the half day term will be greater than that of the whole-day term."

This prediction is completely verified. When we execute the computation for an atmosphere considered as a rotating spherical shell in which waves of temperature advance from meridian to meridian according to the equation

$$\tau = C \sin \omega \sin (nt+\lambda)$$

(where ω = Polar distance, λ = geographical longitude, n = velocity of the rotation of the earth), then we find for $T_0=273^0$ the wave of pressure

$$\varepsilon = C \sin (nt+\lambda) \,[1.146 \sin \omega - 0.423 \sin^3 \omega - 0.370 \sin^5 \omega - 0.106 \sin^7 \omega$$
$$- 0.018 \sin^9 \omega - 0.002 \sin^{11} \omega - \ . \ . \ . \ ]$$

When however at every place the wave of temperature repeats itself twice daily and we assume

$$\tau = C \sin^2 \omega \sin (2nt+2\lambda)$$

then there results

$$\varepsilon = -C \sin (2nt+2\lambda) \,[37.99 \sin^4 \omega + 23.06 \sin^6 \omega$$
$$+ 5.75 \sin^8 \omega + 0.81 \sin^{10} \omega + 0.07 \sin^{12} \omega + \ . \ . \ . \ ]$$

The law according to which the amplitude of the temperature wave diminishes from the equator toward the pole has been assumed different in the two cases only because of the easier computation; this however is of slight influence in the general result which is, that for equal variations of temperature the resulting variations of pressure become much greater in the double daily wave than in the single wave. The coefficients of the first sine series vary only very slowly with T_0 (or with n when we, as Thomson does, consider the period as the variable). It is otherwise in the half-day wave; here the factor of $\sin^4 \omega$ in the neighborhood of $T_0=268^\circ$ passes, from $-\infty$ over to $+\infty$ precisely as in the plane wave before considered when the velocity of propagation of the forced vibration is made equal to that of the free vibration. Thus slight semi-diurnal waves of temperature of scarcely appreciable amplitude are sufficient to produce great waves of pressure in frictionless air if we assume the temperature of the spherical shell to be in the neighborhood of 268°.

Thus far the computation. Its application to the daily variation of the barometer is only clear as to one point. The semi-diurnal wave of pressure may be considered as a consequence of a semi-diurnal wave of temperature of small amplitude. Thus is explained the relative magnitudes (of the diurnal and semi-diurnal temperature and pressure waves) but not the uniformity of the semi-diurnal variation of pressure over the land and the ocean. This uniformity has led Hann to seek the origin of the phenomenon in the absorption of heat by the upper strata of air. But the lower strata have also a semi-diurnal temperature va-

riation and one that varies with locality and with the condition as to cloudiness. It is a question whether the variations of pressure thence resulting are so small in comparison with the regular variations that they are not very noticeable in the averages.

The neglect of the friction and the vertical motion of the air in our last calculations, the assumption of a constant mean temperature for the whole mass of air, and the assumption that for equal latitudes we have equally large ranges of temperature and pressure, allow us to make only the most general application to the case of nature. A more perfect calculation, taking account of the actual distribution of land and water, would be as difficult to execute as would be the computation of the rise and fall of the tides for an ocean of irregular shape, or even for one bounded by meridians.

I. MOVEMENT OF THE AIR IN VERTICAL PLANES.

Notation. $u =$ horizontal velocity along the axis of x; $w =$ vertical velocity positive upwards along the axis of z; $\mu =$ density; $p =$ pressure; $T =$ absolute temperature; $t =$ time; $g =$ the acceleration of gravity; $R =$ a constant.

We imagine the earth as an infinite plane above which, in all east-west vertical planes, the movement of the air occurs in a similar manner. For slight velocities that allow us to neglect terms in the equations of motion that are of the second degree in u and w, these equations, together with the equations of continuity and of elasticity are as follows: *

$$\left.\begin{aligned} &\frac{\partial u}{\partial t} = -\frac{1}{\mu}\frac{\partial p}{\partial x} \\ &\frac{\partial w}{\partial t} = -\frac{1}{\mu}\frac{\partial p}{\partial z} - g \\ &\frac{\partial \mu}{\partial t} + \frac{\partial(\mu u)}{\partial x} + \frac{\partial(\mu w)}{\partial z} = 0 \\ &p = R\,\mu\,T. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (1)$$

If the atmosphere is at rest then p, μ, T, have the value p_o, μ_o, T_o, which are functions of the altitude only,

$$\left.\begin{aligned} &\frac{1}{p_o}\frac{d\,p_o}{dz} = -\frac{g}{R T_o} \\ &\frac{1}{\mu_o}\frac{d\,\mu_o}{dz} = -\frac{g}{R T_o} - \frac{1}{T_o}\frac{d\,T_o}{dz} \\ &p_o = R\,\mu_o\,T_o \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (2)$$

[* The expression "Zustands-Gleichung der Gase," which is applied in Germany to the equation $p\,v = RT$ has, I believe, no single equivalent in ordinary English scientific phraseology unless we adopt the very inelegant historical title Boyle-Mariotte-Gaylussac-Charles law. It is the law connecting density, temperature, volume, or pressure, and expresses the simple fact that the substance is truly gaseous. But the characteristic of a gas is its elasticity, and the equation gives the elastic pressure.—*C. A.*]

The motion is caused by small variations of temperature. Such variations will, as a rule, produce only slight variations of density and of pressure. If we put

$$p = p_0 (1 + \varepsilon)$$
$$\mu = \mu_0 (1 + \sigma)$$
$$T = T^0 (1 + \tau)$$

then ε, σ, τ, are small numbers whose products and squares we shall neglect.

From the following equations,

$$\left.\begin{aligned} \frac{\partial u}{\partial t} &= -RT_0 \frac{\partial \varepsilon}{\partial x} \\ \frac{\partial w}{\partial t} &= -RT_0 \frac{\partial \varepsilon}{\partial z} + g\tau \\ \frac{\partial \sigma}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} - w\left(\frac{g}{RT_0} + \frac{1}{T_0}\frac{dT_0}{dz}\right) &= 0 \\ \varepsilon &= \sigma + \tau \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which take the place of (1), we eliminate u, w, σ, by differentiating the first according to x, the second according to z, and the third according to t.

We thus obtain the following differential equation, in which τ is to be considered as a given function of x, z, and t, but ε as a function of x, z, and t that is still to be determined.

$$\left.\begin{aligned} &\frac{\partial^2 \varepsilon}{\partial x^2} + \frac{\partial^2 \varepsilon}{\partial z^2} - \frac{g}{RT_0}\frac{\partial \varepsilon}{\partial z} - \frac{1}{RT_0}\frac{\partial^2 \varepsilon}{\partial t^2} \\ &= \frac{g}{RT_0}\frac{\partial \tau}{\partial z} - \frac{g}{RT_0}\left(\frac{g}{RT_0} + \frac{1}{T_0}\frac{dT_0}{dz}\right)\tau - \frac{1}{RT_0}\frac{\partial^2 \tau}{\partial t^2} \end{aligned}\right\} \quad . \quad . \quad . \quad (4)$$

Before we treat the equation for motions in two dimensions we will consider the simplest case of linear vibrations.

II. LINEAR VARIATIONS.

When $g = 0$, and τ and ε depend only on t and x, equation (4) becomes

$$\frac{\partial^2 \varepsilon}{\partial t^2} - RT_0 \frac{\partial^2 \varepsilon}{\partial x^2} = \frac{\partial^2 \tau}{\partial t^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4a)$$

and when $\tau = 0$, this becomes the Newtonian equation for acoustic vibrations in the atmosphere which gives $c = \sqrt{RT_0}$ as the velocity of propagation.

If we consider—not the variation of temperature, but the flow of heat as known, then we have to introduce the relation.

$$dQ = C_v\, dT + p\, d\left(\frac{1}{\mu}\right) = C_v\, T_0\, d\tau - RT_0\, d\sigma = C_p\, T_0\, d\tau - RT_0\, d\varepsilon$$

where the change of kinetic energy is omitted, as being a quantity of

the second degree, in u; $dQ =$ the heat imparted to the unit mass of air during the time dt; $C_v =$ specific heat of air under constant volume; $C_p =$ specific heat under constant pressure

$$C_p = C_v + R$$

$$\tau = \frac{Q}{C_v T_0} + \frac{R}{C_v}\sigma = \frac{Q}{C_p T_0} + \frac{R}{C_p}\varepsilon.$$

By combining this last equation with $(4a)$ we obtain

$$\frac{\partial^2 \varepsilon}{\partial t^2} - R T_0 \frac{C_p}{C_v}\frac{\partial^2 \varepsilon}{\partial x^2} = \frac{1}{C_v T_0}\frac{\partial^2 Q}{\partial t^2}, \quad \ldots \ldots \ldots \quad (4b)$$

which converts into the Laplacian equation when $Q = 0$. In this the temperature variations of the air for rapid acoustic vibrations produced by adiabatic compressions and expansions are considered, and the velocity of propagation is therefore

$$c' = \sqrt{R T_0 \frac{C_p}{C_v}}$$

For our purpose it will be more convenient to consider the pressure variations as a consequence of the temperature variations not as a consequence of the variable flow of heat. We therefore return to equation $(4a)$.

III. WAVE OF TEMPERATURE.

A progressive wave of temperature

$$\tau = A \sin (nt + mx) = A \sin 2\pi \left(\frac{t}{\Theta} + \frac{x}{L}\right) \quad \ldots \quad (5)$$

causes a wave of pressure

$$\left.\begin{aligned} \varepsilon &= B \sin (nt + mx) \\ B &= \frac{L^2}{L^2 - c^2\Theta^2} A \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (6)$$

advancing in the same direction.

$\frac{L}{\Theta} = V$ is the velocity of the progress of both of these waves. The phases of the waves are the same or opposite according as V is larger or smaller than c. But $V = c$ leads to an infinitely large value of B, a result to which we must always come when in a frictionless medium the period of the forced vibrations agrees with those of the free.

For the atmosphere we have

$$R = \frac{10333 \times 9.806}{273 \times 1.293} = 287.0.$$

Here, and in the following, we adopt as units the metre, the kilogram, the second of time, the degree of the Centigrade thermometer, and for pressures the barometric scale. For $T_0=273^\circ$ we have $c=279.9$.

The values $L=4\times10^7$ or the circumference of the equator, $\Theta=24\times60\times60$ or 1 day and $T_0=273^\circ$ gives a wave of pressure whose maximum coincides with the maximum of temperature, and also gives $B=1.576\times A$. A temperature variation of 1° C. produces a pressure variation $\frac{p_0\times1.576}{273}$ or 4.4 millimetres of mercury when p_0 is 760 on the barometer scale.

When we desire to obtain pure horizontal vibrations in a layer of appreciable altitude without neglecting force of gravity we should have to introduce a function (A) of the altitude as we see from the equations (3), that shall satisfy the condition

$$\frac{1}{A}\frac{dA}{dz}=\frac{g}{c^2}\frac{L^2-c^2\Theta^2}{L^2}$$

For isothermal vibrations in a vertical column of air the conditions are

$$\tau=0$$
$$\frac{\partial\varepsilon}{\partial x}=0$$

and equation (4) becomes

$$\frac{\partial^2\varepsilon}{\partial z^2}-\frac{g}{RT_0}\frac{\partial\varepsilon}{\partial z}-\frac{1}{RT_0}\frac{\partial^2\varepsilon}{\partial t^2}=0.$$

This equation or the corresponding equation in w has recently been discussed at length by Lord Rayleigh (*Phil. Mag.*, Feb., 1890).*

IV. VIBRATIONS OF THE AIR WHEN A WAVE OF TEMPERATURE ADVANCES HORIZONTALLY, TAKING INTO CONSIDERATION THE FORCE OF GRAVITY.

With a constant value of T_0 and putting $\tau=A\sin(mx+nt)$ the differential equation (4) becomes

$$\frac{\partial^2\varepsilon}{\partial x^2}+\frac{\partial^2\varepsilon}{\partial t^2}-\alpha\frac{\partial\varepsilon}{\partial z}-\frac{\alpha}{g}\frac{\partial^2\varepsilon}{\partial t^2}=-\frac{\alpha}{g}\frac{\partial^2\tau}{\partial t^2}-\alpha^2\tau \quad . \quad . \quad . \quad . \quad (4c)$$

$$\left[\alpha=\frac{g}{RT_0}\right]$$

The wave of pressure will be of the form $\varepsilon=F(z)\sin(mx+nt)$.

* [See also No. XVIII of this present collection of Translations.]

The notation and solution are as follows :

$$\frac{d^2F}{dz^2}-\alpha\frac{dF}{dz}+hF=\left(\frac{\alpha}{g}n^2-\alpha^2\right)A$$

$$\left[h=\frac{\alpha}{g}n^2-m^2\right]$$

$$F(z)=B+K_1e^{k_1z}+K_2e^{k_2z}$$

$$B=\frac{A}{h}\left(\frac{\alpha}{g}n^2-\alpha^2\right)$$

$$k_1=\frac{\alpha}{2}-\sqrt{\frac{\alpha^2}{4}-h} \qquad k_2=\frac{\alpha}{2}+\sqrt{\frac{\alpha^2}{4}-h}$$

In order to determine the constants of integration K_1 and K_2 whose factors in the expression for ε represent free vibrations we note that $w=0$ when $z=0$ and also when z has a very large value$=Z$ which corresponds to a fictitious upper plane bounding the atmosphere. From the second of equations (3) we obtain

$$w=\frac{g}{\alpha n}\left(K_1k_1e^{k_1z}+K_2k_2e^{k_2z}-\alpha A\right)\cos(mx+nt)$$

The boundary conditions give

$$K_1k_1+K_2k_2=\alpha A$$

$$K_1k_1e^{k_1Z}+K_2k_2e^{k_2Z}=\alpha A$$

$$K_1k_1=\alpha A\,\frac{e^{k_2Z}-1}{e^{k_2Z}-e^{k_1Z}}$$

$$K_2k_2=\alpha A\,\frac{1-e^{k_1Z}}{e^{k_2Z}-e^{k_1Z}}$$

If now, as in our example (where the wave length is the circumference of the earth and the period is one day), h is very small compared with α^2, then is k very small, and k_2 nearly equal to α. Hence, K_2 will be smaller in proportion as Z is larger. If we desire to apply the resulting formula only to altitudes that are slight in comparison with Z, then will $K_2e^{k_2z}$. With this limitation we put $K_2=0$ and $K_1k_1=\alpha A$, and obtain

$$w=A\frac{g}{n}\left(e^{k_1z}-1\right)\cos(mx+nt)$$

$$\varepsilon=A\left(\frac{\alpha n^2}{gh}-\frac{\alpha^2}{h}+\frac{\alpha}{k_1}e^{k_1z}\right)\sin(mx+nt)$$

Under the assumption that $\frac{h}{\alpha^2}$ is a small quantity we have

$$k_1=\alpha\left(\frac{h}{\alpha^2}+\frac{h^2}{\alpha^4}\right)$$

$$\frac{\alpha}{k_1}=\frac{\alpha^2}{h}-1,$$

and when we retain only the first two terms of the exponential series we obtain

$$\varepsilon=A\left(\frac{m^2}{h}+\alpha z\right)\sin(mx+nt)=A\left(\frac{c^2\Theta^2}{L^2-c^2\Theta^2}+\alpha z\right)\sin 2\pi\left(\frac{t}{\Theta}+\frac{x}{L}\right).$$

For $L=4\times 10^7$, $\Theta=24\times 60\times 60$, we obtain

$$\varepsilon=A\ (0.576+0.000125z)\sin(mx+nt).$$

The relative variations of pressure near the earth's surface increase very slowly with the altitude. At the surface of the earth itself the variations of pressure are appreciably smaller in the ratio of $\frac{0.576}{1.576}$ than in the example of the third section, where purely horizontal vibrations occurred. A daily variation of temperature of 1° C. would in the present case cause a pressure variation of 1.6$^{\text{mm}}$. The phases of both vibrations occur simultaneously when $L > c\Theta$.

V. A SIMILAR COMPUTATION FOR THE CASE WHEN THE AMPLITUDE OF THE TEMPERATURE VIBRATION DIMINISHES WITH THE ALTITUDE.

The differential equation (4) becomes

$$\frac{\partial^2\varepsilon}{\partial x^2}+\frac{\partial^2\varepsilon}{\partial z^2}-\alpha\frac{\partial\varepsilon}{\partial z}-\frac{\alpha}{g}\frac{\partial^2\varepsilon}{\partial t^2}=\alpha\frac{\partial\tau}{\partial z}-\frac{\alpha}{g}\frac{\partial^2\tau}{\partial t^2}-\alpha^2\tau \quad . \quad . \quad . \qquad (4d$$

To the assumption $\tau=Ae^{-sz}\sin(mx+nt)$ there corresponds

$$\varepsilon=(Be^{-sz}+Ke^{kz})\sin(mx+nt)$$

$$B(s^2+\alpha s+h)=A\left(\frac{\alpha n^2}{g}-\alpha^2-\alpha s\right)$$

$$k=\frac{\alpha}{2}-\sqrt{\frac{\alpha^2}{4}-h}$$

h has the same meaning as before. K stands for K_1 and K_2 disappears under the same limitations as before, (namely, that the result is to be applied only to altitudes that are slight in comparison to Z).

From the condition $w=0$ when $z=0$, there follows $Kk=Bs+A\alpha$, hence

$$\varepsilon=\frac{A}{s^2+\alpha s+h}\left\{\left(\frac{\alpha}{g}n^2-\alpha^2-\alpha s\right)e^{-sz}+\frac{\alpha}{k}\left(\frac{n^2}{g}s+h\right)e^{kz}\right\}\sin(mx+nt)$$

If $\frac{h}{\alpha^2}$ is very small, and s of the same order of magnitude as α, or even much larger, then for values of z that are not too large, this last equation becomes

$$\varepsilon=A\left(\frac{\alpha}{s+\alpha}\,\frac{m^2}{h}+\alpha z\right)\sin(mx+nt)$$

$$=A\left(\frac{\alpha}{s+\alpha}\,\frac{c^2\,\Theta^2}{L^2-c^2\,\Theta^2}+\alpha z\right)\sin 2n\left(\frac{x}{L}+\frac{t}{\Theta}\right)$$

If we put $s=0.000693$, then, at an altitude of 1,000 metres, the variation of temperature will be half as large as at the surface of the earth. With this value, and the same values of L and Θ as above, there results

$$\varepsilon=A\,(0.153\times 0.576+0.000125z)\sin(mx+nt)$$

Hence, for a mean temperature of 273°, a barometric variation of 2.45 millimetres is produced by a daily variation of 10° in temperature at the surface of the earth.

VI. TRANSFORMATION OF THE DIFFERENTIAL EQUATIONS FOR SPHERICAL COÖRDINATES.

Instead of the rectilinear coördinates x, y, z, the spherical coördinates ($r=$ radius; $\omega=$ polar distance; $\lambda=$ east longitude from adopted meridian), are to be introduced

$$x=r\sin\omega\cos\lambda,$$
$$y=r\sin\omega\sin\lambda,$$
$$z=r\cos\omega.$$

The equations of motion of a point on which the forces X, Y, Z are acting along the rectilinear axes, which are $X=\frac{d^2x}{dt^2}$, etc., are thus transformed into the following:

$$\left.\begin{aligned}
P&=\frac{d^2r}{dt^2}-r\left(\frac{d\omega}{dt}\right)^2-r\sin^2\omega\left(\frac{d\lambda}{dt}\right)^2\\
\Omega&=r\frac{d^2\omega}{dt^2}+2\frac{dr}{dt}\frac{d\omega}{dt}-r\cos\omega\sin\omega\left(\frac{d\lambda}{dt}\right)^2\\
\Lambda&=r\sin\omega\frac{d^2\lambda}{dt^2}+2\sin\omega\frac{dr}{dt}\frac{d\lambda}{dt}+2r\cos\omega\frac{d\omega}{dt}\frac{d\lambda}{dt}
\end{aligned}\right\}\quad\ldots\quad(7)$$

where P, Ω, Λ are the components of the forces in the directions of the new coördinates dr, $r\,d\omega$, $r\sin\omega\,d\lambda$. If the velocities are so small that we can neglect their squares and products, then only the first term will remain on the right-hand side of each of these equations. If we put

$$\frac{dr}{dt}=a, \qquad r\frac{d\omega}{dt}=b, \qquad r\sin\omega\frac{d\lambda}{dt}=c$$

we have

$$P=\frac{da}{dt}, \qquad \Omega=\frac{db}{dt}, \qquad \Lambda=\frac{dc}{dt}.$$

Therefore the equations of motion of a fluid that is only under the influence of a constant force of gravity positive in the direction of the diminishing radius, are

$$\left.\begin{aligned} -g-\frac{1}{\mu}\frac{\partial p}{\partial r}&=\frac{\partial a}{\partial t}\\ -\frac{1}{\mu}\frac{\partial p}{r\partial\omega}&=\frac{\partial b}{\partial t}\\ -\frac{1}{\mu}\frac{\partial p}{r\sin\omega\,\partial\lambda}&=\frac{\partial c}{\partial t}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

These equations are applicable to the motion on a sphere at rest. In order to investigate the relative motion on the rotating terrestrial sphere, we modify equation (7) in that we put $\nu t+\lambda$ in place of λ where ν is the velocity of rotation of the earth. In place of $\frac{d\lambda}{dt}$ in equation (7) there now occurs $\frac{\nu+d\lambda}{dt}$. If, again, we put c in place of the new $r\sin\omega\frac{d\lambda}{dt}$, if we retain the products νa, νb, νc, and if on the other hand we omit the terms in ν^2, which indicate only a slight change in the force of gravity, then we obtain the equations for the motion of a fluid on a rotating sphere. On the right-hand sides of the equations (8) the terms $-2\nu c\sin\omega$, $-2\nu c\cos\omega$ and $+2\nu a\sin\omega+2\nu b\cos\omega$ are to be added respectively.

The equation of continuity has the same form for the sphere at rest as for the rotating sphere.

$$\frac{\partial\mu}{\partial t}+\frac{\partial(\mu r^2 a)}{r^2\partial r}+\frac{\partial(\mu b\sin\omega)}{r\sin\omega\,\partial\omega}+\frac{\partial(\mu c)}{r\sin\omega\,\partial\lambda}=0 \quad . \quad . \quad . \quad . \quad (9)$$

Introducing the notation

$$p=p_0(1+\varepsilon), \qquad T=T_0(1+\tau)$$

allied to that above used, we obtain the following differential equations

for the motion of the atmosphere on the rotating sphere that result from small variations of the temperature τ

$$\left.\begin{aligned}
&g\tau - RT_0\frac{\partial\varepsilon}{\partial r} = \frac{\partial a}{\partial t} - 2\nu c\sin\omega \\
&-RT_0\frac{\partial\varepsilon}{r\partial\omega} = \frac{\partial b}{\partial t} - 2\nu c\cos\omega \\
&-RT_0\frac{\partial\varepsilon}{r\sin\omega\partial\lambda} = \frac{\partial c}{\partial t} + 2\nu a\sin\omega + 2\nu b\cos\omega \\
&\frac{\partial\varepsilon}{\partial t} - \frac{\partial\tau}{\partial t} + \left(\frac{2}{r} - \frac{g}{RT_0}\right)a + \frac{\partial a}{\partial r} + \frac{\partial(b\sin\omega)}{r\sin\omega\partial\omega} + \frac{\partial c}{r\sin\omega\partial\lambda} = 0
\end{aligned}\right\} \quad (10)$$

If $\nu = 0$, these give the corresponding equations for the sphere at rest.

VII. THE ATMOSPHERE WITHIN A SPHERICAL SHELL AT REST.

As in the first computation in the second section for the case of a plane we shall assume only horizontal motions. Moreover the radius of the sphere S will be assumed very large in proportion to the height of the stratum of air. If in equation (10) we substitute S instead of r, put $a=0$ and $\nu=0$ and eliminate b and c from the last three equations, there results

$$\frac{S^2}{RT_0}\left(\frac{\partial^2\tau}{\partial t^2} - \frac{\partial^2\varepsilon}{\partial t^2}\right) + \frac{1}{\sin\omega}\frac{\partial}{\partial\omega}\left(\frac{\partial\varepsilon}{\partial\omega}\sin\omega\right) + \frac{\partial^2\varepsilon}{\sin^2\omega\,\partial\lambda^2} = 0 \quad . \quad (11)$$

Single daily wave. The wave of temperature

$$\tau = A\sin\omega\sin(nt + \lambda)$$

causes a wave of pressure

$$\varepsilon = B\sin\omega\sin(nt + \lambda)$$

where A and B have the relation

$$B\left(\frac{n^2 S^2}{RT_0} - 2\right) = A\frac{n^2 S^2}{RT_0}$$

With $T_0 = 273°$, $n = \dfrac{2\pi}{24 \times 60 \times 60}$, S = radius of the earth, and p_0 = 760 *mm.*, a variation of temperature of 1° on the equator will produce a variation of pressure at the equator of 10.4 *mm.* B will be equally large for the spherical shell as for a plane wave of the same periodic time, when we assume the wave length for the plane to be equal to the circumference of the circle of 45° latitude on the sphere.

Double daily wave. For the temperature wave

$$\tau = A\sin^2\omega\sin(2nt + 2\lambda)$$

we obtain the pressure wave

$$\varepsilon = B\sin^2\omega\sin(2nt + 2\lambda)$$

with the following relation between A and B

$$B\left(\frac{4n^2S^2}{RT_0}-6\right)=A\frac{4n^2S^2}{RT_0}$$

With the same constants as before 1° variation of temperature on the equator gives 6.2 *mm.* variation of pressure.

On the occasion of the computation for the rotating sphere we shall again have opportunity to explain that the particular integrals that we, in both cases, have given as the solution of the differential equation (11) contain the complete solution for the whole spherical shell.

If we put Θ_1 for the duration of the vibration for single waves for which B is infinitely large, and similarily Θ_2 for the double wave, then we have

$$\Theta_1=\frac{2\pi}{n_1}=\frac{2\pi S}{\sqrt{2RT_0}}$$

$$\Theta=\frac{2\pi}{2n_2}=\frac{2\pi S}{\sqrt{6RT_0}}$$

These are the values of the periods of free vibrations of a spherical shell. Lord Rayleigh (*L. E. D. Phil. Mag.* Feb. 1890) investigates only such and finds (by putting $\sqrt{RT_0\frac{C_p}{C_v}}$ for the velocity of propagation instead of $\sqrt{RT_0}$) for the atmosphere on the earth at rest $\Theta_1=23.8$ hours and $\Theta_2=13.7$ hours; therefore the first is much nearer to 24 than the second is to 12 hours. He remarks however that it is doubtful whether one ought to adopt the Laplacian velocity of propagation for vibration of such long duration.

Therefore the relative magnitudes of the semi-diurnal variation of the barometer still remains a riddle. But this is so only so long as we confine the calculations to the sphere at rest.

VIII. CALCULATION FOR A ROTATING SPHERE.

Diurnal wave.—In this case also the calculation will be carried out only for air in a spherical shell whose thickness is small in comparison with the radius S of the sphere, and also under the further assumption that the movements are horizontal, and that therefore $a=0$. [This latter assumption and the omission of the first of equations (10) are certainly not unobjectionable; they are imitated from the analogous processes in the theory of the tides.] The difference between the sidereal day and the solar day is not considered, and $\nu=n$.

$$\left.\begin{aligned}-\frac{RT_0}{S}\frac{\partial\varepsilon}{\partial\omega}&=\frac{\partial b}{\partial t}-2nc\cos\omega\\-\frac{RT_0}{S}\frac{\partial\varepsilon}{\sin\omega\partial\lambda}&=\frac{\partial c}{\partial t}+2nb\cos\omega\\S\left(\frac{\partial\varepsilon}{\partial t}-\frac{\partial\tau}{\partial t}\right)+\frac{1}{\sin\omega}&\left\{\frac{\partial(b\sin\omega)}{\partial\omega}+\frac{\partial c}{\partial\lambda}\right\}=0\end{aligned}\right\}\quad\ldots\quad(10a)$$

When $\tau=A\ (\omega)\sin(nt+\lambda)$, then ε, b, c, are to be sought in expressions of the following form:

$$\varepsilon=E\ (\omega)\sin(nt+\lambda),$$
$$b=\varphi\ (\omega)\cos(nt+\lambda),$$
$$c=\psi\ (\omega)\sin(nt+\lambda),$$

wherefore the last of equations (10 a) becomes

$$nS\ (E-A)+\frac{1}{\sin\omega}\left\{\frac{d(\varphi\sin\omega)}{d\omega}+\psi\right\}=0,$$

whilst the first two give

$$\varphi=\frac{RT_0}{nS}\frac{\frac{dE}{d\omega}+E\frac{2\cos\omega}{\sin\omega}}{1-4\cos^2\omega}$$

$$\psi=-\frac{RT_0}{nS}\frac{\frac{dE}{d\omega}2\cos\omega+\frac{E}{\sin\omega}}{1-4\cos^2\omega}$$

These latter values substituted in the preceding equation lead to a relation between E and A only, or between ε and τ. It will be convenient for the further computation to introduce an auxiliary function, $\Phi\ (\omega)$;

$$\left.\begin{aligned}
&\Phi\ (\omega)=\frac{nS}{RT_0}\varphi\ (\omega)\sin(\omega)\\
&(1-4\cos^2\omega)\ \Phi\omega=\frac{1}{\sin\omega}\frac{d\,(E\sin^2\omega)}{d\omega}\\
&E=\frac{1}{\sin^2\omega}\int\Phi\ (\omega)\sin\omega\,(4\sin^2\omega-3)\,d\omega\\
&\frac{n^2S^2}{RT_0}(E-A)+\frac{1}{\sin\omega}\left\{\frac{d\Phi}{d\omega}-\Phi\frac{2\cos\omega}{\sin\omega}-\frac{E}{\sin\omega}\right\}=0
\end{aligned}\right\}\quad .\ .\ (11.)$$

If we assume Φ to have the following form:

$$\Phi\ (\omega)=\cos\omega\,(a_1\sin\omega+a_3\sin^3\omega+a_5\sin^5\omega+\quad .\ .\ .\ )$$

then

$$E\ (\omega)=b_1\sin\omega+b_3\sin^3\omega+b_5\sin^5\omega+\quad .\ .\ .$$

$$b_1=a_1,\ b_3=\frac{4a_1-3a_3}{5},\ b_5=\frac{4a_3-3a_5}{7},\quad .\ .\ .$$

Let the temperature amplitude diminish from the equator to the pole according to the cosine of the latitude or

$$A\ (\omega)=C\sin(\omega)$$

and for brevity put

$$k=n^2\frac{S^2}{RT_0}$$

then we obtain the following equations for the determination of the constants

$$\left.\begin{aligned} &\left(1+\frac{3}{5}\right) a_3 - \left(k+\frac{4}{5}\right) a_1 - k\,C = 0 \\ &\left(3+\frac{3}{7}\right) a_5 - \left(\frac{3}{5} k + \frac{4}{7} + 2\right) a_3 + \frac{4}{5}\,k\,a_1 = 0 \\ &\left(i-2+\frac{3}{i+2}\right) a_i - \left(\frac{3}{i} k + \frac{4}{i+2} + i - 3\right) a_{i-2} + \frac{4}{i} k\, a_{i-4} = 0 \\ &\qquad\qquad i = 5, 7, 9, \ldots \end{aligned}\right\} \quad . \; . \; (11a)$$

Apparently a_1 remains undetermined; for the computation of the others, following the lead of Laplace, we write

$$\frac{a_{i-2}}{a_{i-4}} = \frac{4\,k\,(i+2)}{3\,k\,(i+2) + (i-2)\,i\,(i+2) - (i-1)\,i\,(i+1)\dfrac{a_i}{a_{i-2}}}$$

By the interchange of i with $i+2$ a similar expression is formed for $\frac{a_i}{a_{i-2}}$ and then $\frac{a_{i+2}}{a_i}$ and in a similar manner for the subsequent terms of the series, and by substituting these values in the above equation we obtain a continued rapidly converging fraction.

$$q_3 = \frac{a_5}{a_3} = \cfrac{4\,k\,9}{N_3 - \cfrac{Z_5}{N_5 - \cfrac{Z_7}{N_7 - \cdots}}}$$

$$N_1 = 3\,k.\,7 + 3.\,5.\,6,$$

$$N_3 = 3\,k.\,9 + 5.\,7.\,8,$$

$$Z_3 = 4\,k.\,4.\,5.\,6.\,9$$

$$q_5 = \frac{a_7}{a_5} = \cfrac{4\,k\,11}{N_5 - \cfrac{Z_7}{N_7 - \cfrac{Z_9}{N_9 - \cdots}}}$$

$$N_5 = 3\,k.\,11 + 7.\,9.\,10,$$

$$Z_5 = 4\,k.\,6.\,7.\,8.\,11,$$

If in the second of equations (11a) we put $a_5 = q_3\, a_3$, then will $\frac{a_3}{a_1}$ also be determined, and the quotient has the same value as if it were computed from the serial fraction

$$q_1 = \frac{a_3}{a_1} = \cfrac{4\,k.\,7}{N_1 - \cfrac{Z_3}{N_3 - \cfrac{Z_5}{N_5 - \cdots}}}$$

By the first of equations ($11a$) we obtain also the value of a_1; consequently that of

$$a_3 = q_1\, a_1$$
$$a_5 = q_1\, q_3\, a_1, \text{ etc.}$$

If, in the computation of q_1 we take a sufficient number of fractions, as, for instance, up to N_{19}, we have thereby also performed the greater part of the numerical computation for q_3, q_5, and q_7.

This remarkable method of determining the constants was by Laplace applied to the theory of the tides. Its true importance was first recognized again by Sir William Thomson, who defended it against Airy.* Without Thomson's commentary Laplace would not be easy to understand. In our case the matter presents itself very similarly. The differential equation (11), when we replace φ by E, is of the second order, and should have an integral with two arbitrary constants. These can be determined when on two arbitrary circles of latitude, certain conditions are to be fulfilled, such for instance as $\varepsilon=0$, or $b=0$. One constant drops out when we let one of the parallel circles coincide with the pole; the other is in this case to be determined as if the second parallel was the equator itself. At the equator, on account of the symmetry, we must have $b=0$. The equatorial plane is to be considered as a fixed partition.

The computation assumes that $\dfrac{a_i}{a_{i-2}}$ converges towards 0 as i increases. If we assume for a_1 not the value that results from the computation of the continued fraction but some other arbitrary one, and therewith compute a_3, a_5, etc., by equation (11a), we obtain a series that diverges for the equator, where $\sin\,\omega = 1$.

I have computed the constants with two values of k. First,

$$k = 2.5 \qquad S = \frac{4 \times 10^7}{2\pi} \qquad R = 287.0$$
$$n = \frac{2\pi}{24 \times 60 \times 60} \qquad\qquad T_0 = 298.7^\circ$$

And second, for

$$k = 2.7352 \qquad\qquad T_0 = 273^\circ$$

If we also write $\left\{\begin{array}{l} \alpha_1 C \text{ instead of } a_1, \\ \alpha_3 C \text{ instead of } a_3, \\ \beta_1 C \text{ instead of } b_1, \\ \beta_3 C \text{ instead of } b_3, \end{array}\right.$

We find—

$$\left.\begin{array}{l} \tau = C \sin\,\omega\,(nt + \lambda), \\ \Phi = C \cos\,\omega\,(\alpha_1 \sin\,\omega + \alpha_3 \sin^3\,\omega + \ \ .\ \ .\ \ .) \\ \varepsilon = C \sin\,(nt + \lambda)\,[\beta_1 \sin\,\omega + \beta_3 \sin^3\,\omega + \beta_5 \sin^5\,\omega + \ \ .\ \ .\ \ .] \end{array}\right\} \quad (12)$$

* Airy: "On an Alleged Error in Laplace's Theory of Tides." *Phil. Mag.*, 1875 (4), vol. L., p. 227.

	α_1	α_3	α_5	α_7	α_9
$k = 2.5$	-1.119	-0.745	-0.232	-0.040	-0.004
$k = 2.7352$	-1.146	-0.823	-0.279	-0.053	-0.006

	β_1	β_3	β_5	β_7	β_9
$k = 2.5$	1.119	-0.448	-0.326	-0.090	-0.013
$k = 2.7352$	1.146	-0.423	-0.370	-0.106	-0.018

With the value of $k = 2.7352$ we obtain as the sum of the series of sines within the [ ] in the value of ε:

On the equator	0.23
At latitude 30°	0.50
At latitude 45°	0.58
At latitude 60°	0.51

Therefore the variation of pressure has a maximum in the neighborhood of 45° when we assume the variation of temperature to be proportional to the cosine of the latitude. For $2C = \frac{1}{273}$, *i. e.*, for a variation of temperature of 1° at the equator there results a variation of pressure of 0.64 millimetres at the equator, but 1.6 millimetres at latitude 45°.

In order to investigate how the result is affected when we assume that the temperature amplitude diminishes more rapidly from the equator to the pole, we will carry out the computation for still another case, namely—

$$A(\omega) = C \sin^3 \omega,$$

which gives for the determination of a the equations—

$$\left(1 + \frac{3}{5}\right) a_3 - \left(k + \frac{4}{5}\right) a_1 = 0.$$

$$\left(3 + \frac{3}{7}\right) a_5 - \left(2 + \frac{4}{7} + \frac{3}{5} k\right) a_3 + \frac{4}{5} k a_1 = kC.$$

$$\left(5 + \frac{3}{9}\right) a_7 - \left(4 + \frac{4}{9} + \frac{3}{7} k\right) a_5 + \frac{4}{7} k a_3 = 0. \tag{11b}$$

The ratio $\frac{a_3}{a_1}$ is given from the first equation, but q_3, q_5, etc., retain the same values as before. The second equation determines the value of a_1. As before we have—

$$\left.\begin{aligned} \tau &= C \sin^3 \omega \sin(nt + \lambda) \\ \varepsilon &= C \sin(nt + \lambda) [\beta_1 \sin \omega + \beta_3 \sin^3 \omega + \beta_5 \sin^5 \omega + \ . \ . \ .] \end{aligned}\right\} \tag{12b}$$

For $k = 2.7352$ we have—

$\beta_1 = 0.601$	$\beta_7 = -0.172$
$\beta_3 = 0.316$	$\beta_9 = -0.030$
$\beta_5 = -0.566$	$\beta_{11} = -0.003$

The sum of the series of sines in the value of ε is—

For the equator	0.15
For latitude 30°	0.38
For latitude 45°	0.42
For latitude 60°	0.32

Again we find a minimum at the equator; the maximum of the pressure amplitude lies between latitudes 30° and 45°; the diminution in the higher latitudes is greater than in the previous examples, but still slow in comparison with the diminution of the temperature amplitude. According to equations (12) and (12b) the greatest pressure and highest temperature occur simultaneously.

IX. ROTATING SPHERE: SEMI-DIURNAL WAVE.

If in the differential equations (10a), for the horizontal motions on a rotating sphere, we put

$$\begin{aligned} \tau &= A\,(\omega)\sin(2nt+2\lambda) \\ \varepsilon &= E\,(\omega)\sin(2nt+2\lambda) \\ b &= \varphi\,(\omega)\cos(2nt+2\lambda) \\ c &= \psi\,(\omega)\sin(2nt+2\lambda) \end{aligned}$$

there results:

$$\varphi=\frac{RT_0}{2nS}\,\frac{\dfrac{dE}{d\omega}+E\dfrac{2\cos\omega}{\sin\omega}}{\sin^2\omega}$$

$$\psi=-\frac{RT_0}{2nS}\,\frac{\dfrac{dE}{d\omega}\cos\omega+\dfrac{2E}{\sin\omega}}{\sin^2\omega}$$

$$2nS(E-A)+\frac{1}{\sin\omega}\left\{\frac{d(\varphi\sin\omega)}{d\omega}+2\psi\right\}=0$$

After the elimination of φ and ψ, and when we again put $k=\dfrac{n^2S^2}{RT_0}$ there remains

$$\frac{d^2E}{d\omega^2}\sin^2\omega-\frac{dE}{d\omega}\sin\omega\cos\omega+E(4k\sin^4\omega+2\sin^2\omega-8)=4kA(\omega)\sin^4\omega\,. \quad (13)$$

If we assume that $A(\omega)=C\sin^2\omega$, we have then to do with the same problem as in the computation of the semidiurnal tide in an ocean of constant depth. Assuming

$$E(\omega)=a_0+a_2\sin^2\omega+a_4\sin^4\omega+a_6\sin^6\omega+\;.\;.\;.\;.\;.$$

there results

$a_0=0$, $a_2=0$, a_4 apparently undetermined,

$$\left.\begin{aligned} (4\times6-8)a_6-(3\times4-2)a_4-4kC&=0 \\ (i^2+6i)a_{i+4}-(i^2+3i)a_{i+2}+4ka_i&=0 \\ i=4,\,6,\,8\;\;.\;\;.\;\;. \end{aligned}\right\} \quad .\;.\;.\;.\; (13a)$$

$$q_i=\frac{a_{i+2}}{a_i}=\frac{4k}{i(i+3)-i(i+6)\dfrac{a_{i+4}}{a_{i+2}}}$$

From this we develop the continued fraction as before, and compute the ratios of the constants. But α_4 is not now indeterminate, but its value is immediately found to be $-Cq_2$; hence [see Ferrel, p. 320]

$$\alpha_6 = -Cq_2q_4$$
$$\alpha_8 = -Cq_2q_4q_6 \quad . \quad . \quad .$$

$$\left.\begin{array}{l}\tau = C\sin^2\omega\sin(2nt+2\lambda) \\ \varepsilon = C\sin(2nt+2\lambda)[\alpha_4\sin^4\omega+\alpha_6\sin^6\omega+\alpha_8\sin^8\omega+..]\end{array}\right\} \quad . \quad (14)$$

For $4k = 40, =10, =5$, Laplace has computed the constants. Only the middle value of these is of interest for our problem. I have in addition executed the computation for some neighboring values of k.

$4k$ = 10.	10.94	11.	11.1	11.2	12.
T_0=298°.7	273°.0	271°.5	269°.1	266°.7	248°.9
α_4 −6.196	−37.99	−55.00	−247.8	101.8	8.270
α_6 −3.247	−23.06	−33.68	−154.2	64.3	5.919
α_8 −0.724	− 5.75	− 8.46	− 39.2	16.5	1.662
α_{10} −0.092	− 0.81	− 1.20	− 5.6	2.4	0.260
α_{12} −0.008	− 0.07	− 0.11	− 0.5	0.2	0.026

These numbers confirm Thomson's expectations, that the period of the free vibrations of this kind, for a rotating spherical atmosphere of ordinary temperature, lies very near 12 hours. Instead of so determining the velocity of rotation of the earth that the period shall agree exactly with a half-day, we can choose a corresponding temperature. It lies near to 268°. At this point α_4 passes from $-\infty$ over to $+\infty$. In the neighborhood of this value forced vibrations must lead to enormously great amplitudes. Therefore a slight semi-diurnal wave of temperature would suffice to produce a very great wave of pressure of the same period. At temperatures below 268° the phases of both are in agreement; in other cases they are opposed.

For $4k$=10, or T_0=298.°7, we obtain at the equator

$$\varepsilon = -10.26\ C\sin(2nt+2\lambda)$$

Therefore, a temperature amplitude of $0.038° = \frac{298.7}{760 \times 10.26}$ would suffice in order to produce a pressure amplitude of 1 *mm*.

The comparison of the atmosphere with a spherical shell having a constant temperature of 298°.7 gives, as we shall see, the lunar tide on the equator much larger than it is, as deduced from observations. Similarly one must require corresponding large temperature amplitudes in order to produce the observed semidiurnal pressure amplitude of 1 *mm* at the equator. In view of the great imperfections in the assumptions no importance can be attached to the numerical values.

This computation only shows that in order to produce semidiurnal variations of pressure of the same amount as the diurnal variation much smaller temperature variations will suffice.

X. TIDAL EBB AND FLOW OF THE ATMOSPHERE.

In order to facilitate the comparison of the problems treated in Sections VIII and IX with the computations that have been made for the tidal ebb and flow, I will allow myself to add some things that do not properly belong to the subject of this investigation. The following formulæ differ from the ordinary ones only in the notation, and in the fact that the velocities are retained in place of the displacements.*

In the rotating spherical shell of radius S, and of constant temperature T, the attraction of the sun produces motions for which the following equations, deduced from equations (7) and (10a), hold good:

$$\left.\begin{aligned} \frac{\partial (V - R\,T\,\varepsilon)}{S\,\partial\omega} &= \frac{\partial b}{\partial t} - 2\,n\,c\cos\omega \\ \frac{\partial (V - R\,T\,\varepsilon)}{S\sin\omega\,\partial\lambda} &= \frac{\partial c}{\partial t} + 2\,n\,b\cos\omega \\ \frac{\partial\varepsilon}{\partial t} + \frac{1}{S\sin\omega}\left(\frac{\partial (b\sin\omega)}{\partial\omega} + \frac{\partial c}{\partial\lambda}\right) &= 0 \end{aligned}\right\} \quad \ldots\ldots \quad 15.$$

V indicates the potential of the sun at the point (ω, λ) of the rotating spherical shell. When the sun stands over the equator, its distance from the earth being P, its mass M, the constant of attraction $\varkappa$, we have then for the potential

$$\varkappa\,M\,[P^2 - 2\,P\,S\sin\omega\cos(nt + \lambda) + S^2]^{-\frac{1}{2}}$$

This being developed according to the powers of $\frac{S}{P}$ we obtain at first terms that have no, or at least very slight, import for the tidal ebb and flow; then come those that are to be subtracted when we consider the motion of the fluid as relative only to the center of gravity of the earth. That part of the potential which causes the semidiurnal tide we designate by V in order to substitute it in the equations (15).

$$V = \frac{3}{4}\frac{\varkappa\,M\,S^2}{P^3}\sin^2\omega\cos(2\,nt + 2\,\lambda) = H(\omega)\cos(2\,nt + 2\,\lambda)$$

Put also

$$\begin{aligned} \varepsilon &= E(\omega)\cos(2\,nt + 2\,\lambda) \\ b &= \varphi(\omega)\sin(2\,nt + 2\,\lambda) \\ c &= \psi(\omega)\cos(2\,nt + 2\,\lambda) \end{aligned}$$

and

$$H - R\,T \cdot E = G(\omega)$$

and eliminate φ, ψ from equations (15) we thus obtain

$$\begin{aligned} \frac{d^2 G}{d\omega^2}\sin^2\omega - \frac{d\,G}{d\omega}\sin\omega\cos\omega + G\,(4\,k\sin^4\omega + 2\sin^2\omega - 8) \\ = 4\,k\,H\sin^4\omega \quad \ldots\ldots\ldots\ldots \quad (16) \end{aligned}$$

* Compare, for example, the concise presentation by G. H. Darwin in the Encyclopædia Britannica, 9th edition, article "Tides."

This is the same as equation (13) of the previous section, only here G replaces E, and H replaces A (ω).

$$G=\frac{3}{4}\frac{\varkappa M S^2}{P^3}(\alpha_4 \sin^4 \omega + \alpha_6 \sin^6 \alpha + \quad . \quad . \quad . \quad)$$

$$E=\frac{1}{R T}\frac{3}{4}\frac{\varkappa M S^2}{P^3}(\sin^2 \omega - \alpha_4 \sin^4 \omega - \alpha_6 \sin^6 \omega - \quad . \quad . \quad . \quad) \quad . \quad . \quad (17)$$

For a given value of T therefore, α_4, α_6, etc., are the same constants as in Section IX.

m is the mass of the earth; $\frac{\varkappa m}{S^2} = g$; $M = 355000\, m.$; $P = 24000\, S$;

$$\frac{3}{4}\frac{\varkappa M S^2}{P^3} = 1.203$$

Hence on the equator when $4\,k = 10$, or $T = 298.7$, we have

$$760\,\varepsilon = \frac{760}{287 \times 298.7} \times 1.203 \times 11.26 \times \cos(2\,nt \times 2\lambda)$$

$$= 0.12\ (mm) \cos(2\,nt + 2\lambda)$$

Thus by the sun's attraction a semi-diurnal variation of the barometer of 0.24 *mm.* would arise at the equator; but through the moon's action one that is three times greater, 0.7 *mm.*

Laplace, in *Mécanique Céléste*, book IV, chapter 5, computed the atmospheric tide with the same value of k, but for an atmosphere over an ocean of constant depth, whose tides influence those of the air, whereas here the atmosphere over a rigid earth is alone considered. For our case the same formulæ obtain as for an ocean of uniform depth equal to l. In the equations (15) and subsequently, we have only to put gl in place of $R\,T$, and gy in place of $R\,T\,\varepsilon$, when y is the elevation of the surface of the sea above the mean level.

The lunar tides computed from equations (17) with any allowable value of T are very much too large in comparison with those deduced from the barometer observations.* One can scarely wonder at this

* Besides the observations of Bouvard mentioned in Book XIII, *Mécanique Celéste* and which, arranged by syzygies and quadratures, show scarcely any difference in the daily variation of the barometer (note that only the observations of 9 A. M. and 3 P. M. were used), there are at hand for later dates computations of series of hourly observations for certain tropical stations that Professor Hann has pointed out to me. These give the following barometer variations produced by the lunar tides:

	Latitude.	Altitude.	Barometric variation.	Authority.
	° ′	*metres.*	*mm.*	
Singapore	1 11	....	0.16	Elliot, *Fortsch. d. Ph.*, 1852, p. 703.
Batavia	6 11	....	0.115	Bergsma, *Amsterdam Academy*, 1870. Vander Stok, *Batavia observations*, 1885, vol. 6. Sixteen accordant years.
St. Helena	15 57	540	0.10	Sabine, *Fortsch. d. Ph.*, 1848, p. 402.

The results for Singapore and St. Helena are remarkable in that the maxima occur precisely at the moment of lunar culmination; at Batavia the high tide is 50 minutes late.

when he reflects that all the hypotheses introduced into the computation (the neglect of the vertical motion, of the friction, and of the difference between a day and the interval between two lunar culminations) contribute to increase the computed tidal ebb and flow. With reference to the last mentioned difference, I might further remark that it is easily introduced into the computation. The ratio 3 to 1 between the lunar tide and the solar tide as assumed by Laplace holds good only so long as the value $4k$ (in which the depth of the ocean or the temperature of the air enters in the respective problems) is far from a certain critical value which lies between 11.1 and 11.2. With $4k=10$ the ratio in question is 2.2 to 1, but with $4k=11.1$ the ratio becomes 1 to 5.

I do not carry out the computation here, because it seems too hypothetical to compare the atmosphere with a spherical shell of perfectly definite temperature, and under this assumption then to consider this semi-diurnal variation of pressure as a consequence of the solar attraction. Much more probable is it that it arises from a regular constituent of the semi-diurnal variation of temperature.

XX.

LAPLACE'S SOLUTION OF THE TIDAL EQUATIONS.*

By William Ferrel.

In this paper (supplementary to that under the same heading in vol. IX, No. 6, of the *Astronomical Journal*), it is proposed to explain more fully a certain point in the latter (which did not appear clear to a correspondent some time since), by presenting the matter more in detail, and also to clear up some doubts held by some with regard to the convergency of the series in the tidal expression.

In Darwin's Equation No. (34),† we have the following differential equation to be satisfied, which is equivalent to that of Laplace:

$$\nu^2(1-\nu^2)\frac{d^2u}{d\nu^2}-\nu\frac{du}{d\nu}-(8-2\nu^2-\beta\nu^4)\,u+\beta E\nu^6=0 \quad . \quad . \quad . \quad . \quad (1)$$

[Darwin's Eq. (33).]

in which u is the difference between the real amplitude of the tide and that given by the equilibrium theory, $\nu=\sin\vartheta$ is the sine of the geographical polar distance ϑ, $E\nu^2$ is the amplitude of the equilibrium tide, and

$$\beta=\frac{4n^2}{gl} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which $\frac{n^2}{g}=\frac{1}{289}$ and l is the depth of the ocean, supposed to be uniform, in terms of the earth's radius.

Putting

$$u=K_2\nu^2+K_4\nu^4+K_6\nu^6 \ldots\ldots\ldots\ldots K_n\nu^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which n is any even number, corresponding with the exponent, and substituting this value of u and its derivatives in (1) above, we get, by equating the coefficients of like powers of ν to 0,

$$K_2=0. \quad 12K_4-12K_4=0. \quad 16K_6+\beta E=0, \text{ etc.,}$$

* From Gould's *Astronomical Journal*, 1890, vol. x, pp. 121–125.

† Encyclopedia Britannica, 9th ed. art. "Tides," § 16, vol. XXIII, p. 359.

$$u=(K^2-E)\,\nu^2+K_4\,\nu^4+K_6\,\nu^6+\ldots K_{2i}\,\nu^{2i} \quad . \quad . \quad . \quad (34.)$$

and generally after K_6.

$$(n(n-2)-8)K_n-[(n-2)(n-3)-2]K_{n-2}+\beta K_{n-4}=0.$$

From these equations we get the following expressions of K_n:

$$\left.\begin{array}{l} K_2=0 \\ K_4=K_4 \\ K_6=\frac{5}{8}K_4-\frac{1}{16}\beta E \end{array}\right\} \quad \ldots\ldots\ldots \quad (4)$$

$$K_8=\left(\frac{7}{16}-\frac{1}{40}\beta\right)K_4-\frac{7}{160}\beta E.$$

$$K_{10}=\left(\frac{21}{64}-\frac{79}{2880}\beta\right)K_4-\left(\frac{21}{640}-\frac{1}{1152}\beta\right)\beta E.$$

and generally, after K_6,

$$K_n=\frac{n-1}{n+2}K_{n-2}-\frac{\beta}{(n+2)(n-4)}K_{n-4} \quad \ldots\ldots \quad (5)$$

This general expression is equivalent to Laplace's and Darwin's law as given in my preceding paper, equation (2), but is more simple and convenient in deducing any coefficient K_n from the last two preceding. The one is reducible to the other by putting $n=2i+4$. The general law of (5) does not hold until after K_6, but K_4 and K_6 being obtained from the direct equation of the coefficients of ν^4 and ν^6, then by means of these, K_8 is obtained, either directly from the equation of the coefficients, or from the general expression of (5), and this law can be extended forward, but not backward. For instance, K_6 is not obtainable from K_4 and K_2. As is usual in such cases, the general law is not obtained until after several equations of the coefficients, and when the values of K_n are given directly in this way, and not by the general law, the former must be taken, and the general law, which is a relation found between the coefficients after K_6 only, can not be extended back.

Putting h for the amplitude of the real tide, we have, from what has been stated above,

$$h=E\nu^2+u=E\nu^2+K_4\nu^4+K_6\nu^6 \quad \ldots\ldots \quad +K_n\nu^n \quad . \quad . \quad (6.)$$

Laplace extended the relation above, found to exist between the coefficients of ν in (3), and after K_6 only, back so as to make it, by means of the continued fraction, determine the value of K_4 and so the relation between $E\nu^2$ and u. This makes K_4 a determinate quantity, whereas the equation of the coefficients of ν^4 gives $K_4=K_4$, an indeterminate quantity. It is evident that any value of K_4 satisfies the differential equation, and so, with the other coefficients depending upon it, is a solution of the tidal equation.

The extension of the general relation of (5) back so as to make it determine K_4, and the relation between $E\nu^2$ and u in (6), was regarded by the writer in his previous paper as an extension of the law back where

it is not applicable, and this is what was not clearly understood by his correspondent.

From (4) it is seen that the tidal expression consists of two parts, one of which depends upon K_4, and is independent of the tidal forces contained in E, and the latter depends upon these forces. It is evident that the former can exist without the latter. Also that being independent of the forces, and dependent simply upon certain initial motions which the sea may be supposed to have independent of the forces, it must vanish when there is friction, and so K_4 must be put equal to 0 in the real case of nature.

We come now to the second part of what we have proposed to consider here, namely, the convergency of the series in the expression of u in (3). Inasmuch as the vanishing ratio between consecutive values of K_n is unity, as is readily seen from an inspection of (5), it has been said that the device of Laplace in the use of the continued fraction was necessary to make the expression of u convergent at the equator where $\nu = 1$, so as to give a finite value of u. It is true that the expression at first is more convergent with a large value of K_4, such as is given by the continued fraction, but still the vanishing ratio in any case is unity. But it can be shown that the expression gives a finite value of u when we put $K_4 = 0$.

We get by development,

$$(1-\nu^2)^{\frac{1}{2}}=1-\frac{1}{2}\nu^2-\frac{1}{8}\nu^4-\frac{1}{16}\nu^6 \quad . \quad . \quad . \quad . \quad -A_n\nu^n=1+\Sigma_2^\infty A_n\nu_n \quad . \quad . \quad (7).$$

in which the relation between each coefficient A_n and the preceding one, commencing with $-\frac{1}{2}$, is

$$A_n=\frac{n-3}{n}A_{n-2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8).$$

Hence we have, when $\nu=1$

$$\Sigma_2^\infty A_n=-1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9).$$

$$\Sigma_{n'+2}^{\infty'} A_n=-(1+\Sigma_2^{n'} A_n) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10).$$

in which n' is the exponent of any assumed term in the series.

The expression (5) above may be put into the form,

$$K_n=\frac{n-3}{n}K_{n-2}+\frac{6}{n(n+2)}K_{n-2}-\frac{\beta}{(n+2)(n-4)}K_{n-4} \quad . \quad . \quad (11).$$

From this, by means of (8), we get for any coefficient for which the characteristic is n',

$$K_{n'}=\frac{K_{n'-2}}{A_{n-2}}A_{n'}F_{n'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12).$$

in which,

$$F_{n'}=\left(1+\frac{6}{(n'+2)(n'-3)}-\frac{n'\beta}{(n'+2)(n'-4)(n'-3)}\right)\frac{K_{n'-4}}{K_{n'-2}} \quad . \quad . \quad (13)$$

and putting $n'+2$ for n' in (12) we get

$$K_{n'+2}=\frac{K_{n'}}{A_{n'}}A_{n'+2}F_{n'+2}$$

$$K_{n'+4}=\frac{K_{n+2}}{A_{n+2}}A_{n'+4}F_{n'+4}.$$

This becomes by substituting for $\frac{K_{n'+2}}{A_{n'+2}}$ its value derived from the preceding expression

$$K_{n'+4}=\frac{K_{n'}}{A_{n'}}A_{n'+4}F_{n'+2}F_{n'+4}$$

In like manner we get generally

$$K_{n'+i}=\frac{K_{n'}}{A_{n'}}A_{n'+i}F_{n'+2}F_{n'+4} \quad . \quad . \quad . \quad . \quad F_{n'+i} \quad . \quad . \quad . \quad (14).$$

in which the values of the factors $F_{n'+2}$, $F_{n'+4}$, $F_{n'+i}$, are given by (13) by adding 2, 4, and i respectively to n' in that expression.

Now, all these factors are finite, and hence putting now K_n for its equivalent, $K_{n'+i}$ and A_n for $A_{n'+i}$, we have

$$\Sigma_{n'+2}^{\infty} K_n = \text{a finite quantity}$$

since by (10)

$$\Sigma_{n'+2}^{\infty} A_n = \text{a finite quantity}.$$

From (13) and (14) it is seen that any coefficient, taken without regard to signs,

$$K_{n'+i} < \frac{K_{n'}}{A_{n'}} A_{n'+i} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

when

$$\beta > \frac{6\,(n'+i-4)}{n'+i}\ \frac{K_{n'+i-2}}{K_{n'+i-4}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

since when this condition is satified all the factors $F_{n'+2}$, $F_{n'+4}$, . . . $F_{n'+i}$ are less than unity. Therefore, we have, putting n for $n'+i$,

$$\Sigma_{n'+2}^{\infty} K_n \quad . \quad . \quad . < \Sigma_{n'+2}^{\infty} \frac{K_{n'}}{A_{n'}}A_n\text{, or by (10),} < -\frac{K_{n'}}{A_{n'}}(1+\Sigma_2^{n'} A_n) \qquad (17)$$

when

$$\beta > 6 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

since this is what (16) becomes when i is infinitely great. This is simply the limiting condition in all cases, and the first number of (17) is generally less than the second when β is considerably less than 6.

We have from (3)

$$u = \Sigma_2^{\infty} K_n = P_{n'} + Q_{n'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

in which

$$\left.\begin{aligned} P_{n'} &= \Sigma_2^{n'} K_n, \\ Q_{n'} &= \Sigma_{n'+2}^{\infty} K_n < -\frac{K_{n'}}{A_{n'}}\left(1+\Sigma_2^{n'} A_n\right) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (20)$$

With the values of $P_{n'}$ and $Q_{n'}$ (19) gives u, and this in (6) gives h, the amplitude of the tide.

Laplace computed the values of $2h$, that is, the range of the tides at the equator, at the times of conjunction of the moon and sun, for the several values of β equal 40, 10, and 5, to which, by (2), correspond the several values of l, the depth of the ocean, equal to $\frac{1}{2890}$, $\frac{1}{722.2}$, and $\frac{1}{361.1}$ of the earth's radius, or approximately 1.4, 5.5, and 11 miles respectively.

Taking as an example the case in which β=10, we get from (4) and (5) by putting $K_4 = 0$, the following values of K_n in terms of E in the last column of the following table, and from (7) and (8) the corresponding values of A_n in the second column.

n	A_n	K_n
2	−0.50000	....
4	0.12500	....
6	0.06250	−0.6250
8	.03906	.4375
10	.02734	.2413
12	.02051	.1505
14	.01611	.1072
16	.01309	.0823
18	.01091	.0661
20	.00927	.0548
..	..	..
40	.00322	.0172
..	..	..
60	.00174	.0091

Putting n' equal 20, 40, 60, we get the following corresponding values from this table when complete for all the values of n from 2 to 60,

$$A_{20} = -\ .00927 \qquad 1+\Sigma_2^{20}\ A_n = .17621$$
$$A_{40} = -\ .00322 \qquad 1+\Sigma_2^{40}\ A_n = .12536$$
$$A_{60} = -\ .00174 \qquad 1+\Sigma_2^{60}\ A_n = .10254$$

From the values of K_n we likewise get

$$K_{20} = -.0548 \quad K_{20}\ /\ A_{20} = 5.91 \quad P_{20} = -1.7647$$
$$K_{40} = -.0172 \quad K_{40}\ /\ A_{40} = 5.18 \quad P_{40} = -2.0488$$
$$K_{60} = -.0091 \quad K_{60}\ /\ A_{60} = 5.23 \quad P_{60} = -2.1687$$

We therefore get from (19) and (20) with the preceeding data,

$$u < -1.7647 - 5.91 \times .17621 \text{ or } < -2.8061$$
$$u < -2.0488 - 5.18 \times .12536 \text{ or } < -2.7157$$
$$u < -2.1687 - 5.23 \times .10254 \text{ or } < -2.6870$$

and so on, according as we take $n'=20$, 40, 60, or still greater values. It is seen that the first value, in which we get the value of $P_{n'}$ from summing the actual values of K_n from $n=6$ to $n=n'$, and then get the sum of the remaining infinite number of terms approximately from the last of (20), differs but little from the last value, in which the value of $P_{n'}$ was obtained from summing the actual values of K_n up to $n'=60$, and then obtaining the sum of the remaining terms from the last of (20). It is evident that the real value of u must be only a very little less negatively than -2.6870. The several values of u differ the less, the more nearly the condition of (16) is satisfied, which, when the value of n' is large, is very nearly that of (18). In our example $\beta=10$, and so is too large to give equal values in the several cases of $n'=20$, 40, or 60. With $\beta=40$ there is much greater difference in the several values, and the uncertainty in the last value is consequently much greater, but the last number so obtained is always a limit below which the real value is.

Since our values of K_n have been computed in terms of E the value of u above must be multiplied into E. With this value, then, we get from (6) for the value of h at the equator, where $\nu=1$,

$$h=(1-2.687)\,E=-1.687\,E.$$

The value of E is that of the amplitude of the equilibrium tide at the equator, which in the case of the lunar tide, if we assume the moon's mass equal $\frac{1}{80}$, is 0.812 of a foot. Hence we get for the range of the lunar tide, approximately, at the equator,

$$2\,h=-2\times1.687\times0.812=-2.74 \text{ feet.}$$

Its being negative indicates that low water occurs at the time of the moon's meridian transit.

Laplace, in the same case, obtained for the range of the tide for the moon and sun in conjunction or opposition 11.05 metres, which, being positive, indicates that high water occurs at the time of meridian passage. But instead of $K_4=0$, he used $K_4=6.196$, obtained from his continued fraction. Besides, the mass of the moon which he used was much too large.